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The Conjugate Classes of Unipotent Elements of the Chevalley Groups E_7 and E_8

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§1. Introduction and notations.

Let K be an algebraically closed field and let G be a Chevalley group of type $E_n(n=6, 7$ or $8)$. When the characteristic $\mathrm{ch}(K)=p>0$, we denote by k a finite subfield of K and let q be the number of elements of k. Let $G(k)$ be the group of k-rational points in G. Then the conjugate classes of unipotent elements in $G(k)$ follow from the conjugate classes of G and the factor groups $Z_G(x)/Z_G(x)^\circ$ of the centralizers $Z_G(x)$ of unipotent elements x (T. A. Springer, R. Steinberg [12]).

When the characteristic $\mathrm{ch}(K)$ is zero or sufficiently large, the conjugate classes of unipotent elements are determined by E. B. Dynkin [2] and R. Bala, R. W. Carter [11]. Furthermore the structures of the connected centralizers $Z_G(x)^\circ$ of unipotent elements x are determined by G. B. Elkington [3].

We consider the unipotent classes (=the conjugate classes of unipotent elements) in G under no restriction with respect to the characteristic p. The main results are as follows:

1) We determine the unipotent classes in G.

2) We determine the unipotent classes in $G(k)$ when $\mathrm{ch}(K)=p>0$.

3) We determine the inclusion relations among the Zariski closures of unipotent classes in G.

(In the case $G=E_6(K)$, the results 1) and 2) are determined by K. Mizuno [6].)

This paper is organized as follows:

§2: The equivalent ralation of ideals.

§3: E_6.

§4: E_7.

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§ 5: E_8.

The results are listed up at the end of this paper. The tables are as follows:

Tables 1–3: the representatives of unipotent classes in G.

Tables 4–5: the representatives of unipotent classes in $G(k)$

$$(G=E_7(K) \quad \text{or} \quad E_8(K)\,, \quad \mathrm{ch}(K)>0)\,.$$

Tables 6–8: The inclusion relations among the Zariski closures of unipotent classes in G.

Tables 9–10: The structures of the centralizers of unipotent elements in G.

Table 11: The positive root system of type E_8.

Table 12: The structure constants $N_{\alpha,\beta}$.

Table 13: The root adjacency graph of type E_8.

For every unipotent element $x \in G$, it is shown that the factor group $Z_G(x)/Z_G(x)^\circ$ has the following property;

(*) every irreducible complex representation is realized over the rational number field $\boldsymbol{Q}$.

This can be used to verify the validity of (*) for Weyl groups as was shown by T. A. Springer [7].

From Tables 9 and 10, we see that Elkington's tables [3] contain a number of errors, which occur in type $E_7(L_1=(3A_1)', A_2, A_2+3A_1, 2A_2+A_1, (A_3+A_1)', A_4, D_5+A_1(a_{10}), (A_5)', D_5)$ and in type $E_8(L_1=2A_1, A_2+2A_1, 2A_2, 2A_2+A_1, A_3, 2A_2+2A_2, A_3+A_1, A_3+A_2, D_7(a_1), D_7)$.

I should like to thank Prof. G. Lusztig for careful reading of the manuscript and pointing out some errors in it.

NOTATIONS. Let L be a group and let M be a subgroup of L. For a subset S of L we denote by $M(S)=\{msm^{-1} \mid m \in M, s \in S\}$. For $s \in L$, we denote by $Z_G(s)=\{m \in M \mid ms=sm\}$. When L is a linear algebraic group, we denote by $\mathrm{Ru}(L)$ the unipotent radical and we denote by $D(L)$ the derived group of L. Let L be a connected reductive group such that $\dim Z(L)=m$. If $D(L)$ is a semisimple group of type Y, we denote by $L=mT_1+Y$. We denote by $|S|$ the number of the finite set S. We denote by Z_m a cyclic group of order m and we denote by S_m a symmetric group of degree m.

Let K be an algebraically closed field of characteristic p. When $p>0$, let k be a finite subfield of K and let q be the number of elements of k. Let ζ be a fixed generator of the multiplicative group k^* and let η(resp. τ, μ be a field element of k such that the polynomial

$X^2-X-\eta$ (resp. $X^3-X-\tau$, $X^4-X-\mu$) is irreducible over k. Let G be a simply connected Chevalley group of type $E_n(n=6, 7$ or $8)$ over the field K. We use the notations $x_\alpha(t)$, U, H, B, W, Σ, $\cdots$, etc. as defined in [1], [9].

We call a set I of positive roots an ideal if $\alpha\in I$, $\beta\in\Sigma^+$, $\alpha+\beta\in\Sigma^+$ implies $\alpha+\beta\in I$. For a subset S of Σ^+, we denote by $I(S)$ a unique minimal ideal of Σ^+ which contains S. We introduce a partial order $\leqq$ in Σ^+ defined by "$\alpha\leqq\beta\Leftrightarrow I(\beta)\subset I(\alpha)$". For a subset S of Σ^+, we denote by $\mathfrak{B}(S)$ the set of minimal roots in S with respect to the partial order in Σ^+. For an ideal I of Σ^+, we denote by $U(I)$ the unipotent subgroup generated by all $X_\alpha(\alpha\in I)$ and we denote by $G(I)=G(U(I))$. For an element x of U, we denote by $I(x)$ a unique minimal ideal I such that $U(I)$ contains x. We define an equivalent relation $\sim$ on ideals by "$I\sim J\Leftrightarrow G(I)=G(J)$". We put $G'(I)=G(I)-\cup_J G(J)$, where J runs over all ideals such that $G(J)\subsetneq G(I)$. Let x be an element of U. We say that a sequence (P, R, V, V_1) of subgroups of G gives a structure of $Z_G(x)$ if P, R, V, V_1 satisfy the following conditions:

1) $P\supset B=UH$, $x\in V$, $R\subset \mathrm{Ru}(P)$, $U\supset V\supset V_1\supset D(V)$, $Z_G(x)\subset P$,

2) R, V and V_1 are connected normal subgroups of P,

3) $R(x)=xV_1$,

4) $(P/R, V/V_1)$ is a prehomogeneous space which has $P(xV_1)$ as the open orbit.

The fundamental roots of Σ^+ are denoted by α_i with Dynkin diagram

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1 —— 3 —— 4 —— 5 —— 6(—— 7 —— 101)
          |
          2
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The other positive roots are defined in Table 11. The structure constants $N_{\alpha,\beta}$ $(\alpha, \beta\in\Sigma^+)$ are listed in Table 12.

§ 2. The equivalence relation $\sim$.

Let S be a subset of Σ^+. We say that S is a (r, s)-cube if S satisfies the following conditions:

1) $\mathfrak{B}(S)$ consists of a single element α_0,

2) There exist $r+s+1$ fundamental roots $\beta_i(i=1, 2, \cdots, r)$, $\gamma_j(j=1, 2, \cdots, s)$ and δ such that

$$\alpha_0+\beta_1,\ \alpha_0+\gamma_1,\ \alpha_0+\delta\in\Sigma^+ ,$$

$$\beta_i+\beta_{i+1}(1\leqq i\leqq r-1)\ ,\quad \gamma_j+\gamma_{j+1}(1\leqq j\leqq s-1)\in\Sigma^+\ ,$$
$$\tilde{\alpha}_0=\alpha_0+\delta+\sum_{i=1}^{r}\beta_i+\sum_{j=1}^{s}\gamma_j\in\Sigma^+\ ,$$

3) $S=\{\alpha\in\Sigma^+\mid\alpha_0\leqq\alpha\leqq\tilde{\alpha}_0\}$.
Since (r, s)-cube S is characterized by the $r+s+1$ fundamental roots β_i, γ_j, δ and the root α_0, we denote by

$$\mathrm{Cube}(\alpha_0;\beta_1,\cdots,\beta_r;\gamma_1,\cdots,\gamma_s;\delta)$$

the (r, s)-cube S. We denote by $\mathrm{Side}(S)=\{\beta_1,\cdots,\beta_r,\gamma_1,\cdots,\gamma_s,\delta\}$.

The series of following Lemmas 1-6 are useful properties of the equivalence relation $\sim$.

LEMMA 1. *Let I be an ideal of Σ^+ and let α be an element of $\mathfrak{B}(I)$. Suppose that w is a simple reflection such that $w(\alpha)>\alpha$ and $|I-w(I)|\leqq 1$. Then we get*

1) $I\sim I-\{\alpha\}$ *if* $I=w(I)$,
2) $I\sim(I-\{\alpha\})\cup\{w(\beta)\}$ *if* $\{\beta\}=I-w(I)$.

PROOF. Suppose $I=w(I)$ and $x\in U(I)-U(I-\{\alpha\})$. Then the element x is conjugate to an element of $U(I-\{\alpha\})$ in $B\langle w\rangle B$. This shows 1). Suppose $\{\beta\}=I-w(I)$. Then $(I-\{\alpha\})\cup\{w(\beta)\}$ is an ideal. Hence 2) follows from 1).

LEMMA 2. *Let I be an ideal of Σ^+ and let $S=\mathrm{Cube}(\alpha_0;\beta;\gamma;\delta)$ be an $(1, 1)$-cube in I such that $I-S$ is an ideal.*

1) *If $I=w_\beta(I)=w_\gamma(I)$, then $I\sim I-\{\alpha_0,\delta+\alpha_0\}$.*
2) *If $I=w_\alpha(I)$ for all $\alpha\in\mathrm{Side}(S)$, then $I\sim I-\{\alpha_0,\alpha_0+\beta,\alpha_0+\gamma\}$.*

PROOF. First, we consider the case $S=\mathrm{Cube}(\alpha_4;\alpha_2;\alpha_3;\alpha_5)$. Let x be an element of $U(I(S))$. Then the element x can be expressed as $\prod x_\alpha(t_\alpha)$. By Lemma 1, the element x is conjugate to an element of $U(I(S)-\{\alpha_4\})$. Hence we may assume $t_{\alpha_4}=0$. By the action of $x_{\alpha_2}(u_2)x_{\alpha_3}(u_3)$, the element x is conjugate to an element of the set $x_{\alpha_9}(t_9)x_{\alpha_{10}}(t_{10})x_{\alpha_{11}}(t_{11})x_{\alpha_{15}}(t_{15}+t_9u_3+t_{10}u_2)x_{\alpha_{16}}(t_{16}+t_{11}u_3)x_{\alpha_{17}}(t_{17}+t_{11}u_2)x_{\alpha_{22}}(t_{22}+t_{17}u_3+t_{16}u_2+t_{11}u_2u_3)U(I(S)-S)$. If $t_9=0$, the element x is conjugate to an element of $U(I(S)-\{\alpha_4,\alpha_{11}\})$ by the action of $\langle\omega_2,\mathfrak{X}_{\alpha_2}\rangle$. Similarly, if $t_{10}=0$, the element x is conjugate to an element of $U(I(S)-\{\alpha_4,\alpha_{11}\})$ by the action of $\langle\omega_3, X_{\alpha_3}\rangle$. If $t_9t_{10}t_{11}\neq 0$, the element x is conjugate to an element of the set $x_{\alpha_9}(t_9)x_{\alpha_{10}}(t_{10})x_{\alpha_{11}}(t_{11})x_{\alpha_{16}}(t'_{16})x_{\alpha_{17}}(t'_{17})U(I(S)-S)$ for some t'_{16} and t'_{17}. By the action of $\omega_2\omega_3$, the element x is conjugate to an element of $U(I(S)-\{\alpha_4,\alpha_{11}\})$. Since there exists some element w of W such that $w(S)=$

Cube$(\alpha_4; \alpha_2; \alpha_3; \alpha_5)$ and $w(\text{Side}(S))=\{\alpha_2, \alpha_3, \alpha_5\}$ for any (1, 1)-cube S, the assertion 1) is proved. The assertion 2) follows from 1) and Lemma 1.

LEMMA 3. *Let I be an ideal of Σ^+ and let $S=\text{Cube}(\alpha_0; \beta_1, \beta_2; \gamma_1, \gamma_2; \delta)$ be a (2, 2)-cube. Suppose that*

1) *$S-I=\{\alpha_0, \alpha_0+\beta_1, \alpha_0+\gamma_1, \alpha_0+\delta\}$ and $I-S$ is an ideal,*
2) *$I=w_\delta(I)=w_{\beta_2}(I)=w_{\gamma_2}(I)$.*

Then the ideal I is equivalent to the ideal $I-\{\alpha_0+\beta_1+\gamma_1\}$.

PROOF. First, we consider the case $S=\text{Cube}(\alpha_4; \alpha_3, \alpha_1; \alpha_5, \alpha_6; \alpha_2)$. Let x be an element of $U(I(S-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11}\}))$. Thus we put $x=\prod x_{\alpha_i}(t_i)$ $(i>0)$. By the action of the element $x_{\alpha_1}(u_1)x_{\alpha_2}(u_2)x_{\alpha_3}(u_6)$, the element x is conjugate to an element of the set $xx_{\alpha_{21}}(t_{16}u_1)x_{\alpha_{22}}(t_{16}u_2)x_{\alpha_{23}}(-t_{16}u_6)x_{\alpha_{20}}(t_{15}u_1+t_{14}u_2)x_{\alpha_{24}}(t_{18}u_2-t_{17}u_6)x_{\alpha_{26}}(t_{22}u_1+t_{21}u_2+t_{16}u_1u_2)x_{\alpha_{27}}(t_{23}u_1-t_{21}u_6-t_{16}u_1u_6)x_{\alpha_{29}}(t_{23}u_2-t_{22}u_6-t_{16}u_2u_6)x_{\alpha_{33}}(t_{29}u_1+t_{27}u_2+t_{23}u_1u_2-t_{26}u_6-t_{22}u_1u_6-t_{21}u_2u_6-t_{16}u_1u_2u_6)U(I(S)-S)$. If $t_{14}t_{15}t_{16}t_{17}t_{18}\neq 0$, we may assume $t_{20}=t_{23}=t_{24}=0$. In this case, by the action of $\omega_1\omega_2\omega_6$, the element x is conjugate to an element of $U(I(S)-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11},\alpha_{16}\})$. On the other hand, $I(S)-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{16}\}$ is an ideal. Therefore the set $G(I(S)-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{16}\})$ is closed. This shows that the ideal $I(S)-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11}\}$ is equivalent to the ideal $I(S)-\{\alpha_4, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{16}\}$. In general, there exists some element w of W such that $w(S)=\text{Cube}(\alpha_4; \alpha_3, \alpha_1; \alpha_5, \alpha_6; \alpha_2)$ and $w(\text{Side}(S))=\{\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6\}$. Here the ideal I is equivalent to the ideal $I-\{\alpha_0+\beta_1+\gamma_1\}$. The lemma is now proved.

LEMMA 4. *Let I be an ideal of Σ^+ and let $S=\text{Cube}(\alpha_0; \beta_1; \gamma_1, \gamma_2; \delta)$ be an (1, 2)-cube contained in I. Suppose that*

1) *$I-S$ is an ideal,*
2) *$I=w_\alpha(I)$ for all roots $\alpha\in\text{Side}(S)$,*
3) *There exists an element ε of $\mathfrak{B}(I)-\{\alpha_0\}$ such that $w_{\alpha'}(\varepsilon)>\varepsilon$ for some $\alpha'\in\text{Side}(S)$.*

Then the ideal I is equivalent to $I-\{\varepsilon, \alpha_0, \alpha_0+\gamma_1, \alpha_0+\delta, \alpha_0+\beta_1\}$.

PROOF. If α' is β_1 or δ, the lemma follows from Lemma 1. Thus we may assume $w_{\beta_1}(\varepsilon)=w_\delta(\varepsilon)=\varepsilon$. By Lemma 1, we get $I\sim I-\{\alpha_0, \alpha_0+\gamma_1, \alpha_0+\beta_1, \alpha_0+\delta\}$. Since $I-W_{\text{Side}(S)}(\varepsilon)$ is an ideal, the set $I-(S\cup W_{\text{Side}(S)}(\varepsilon))$ is so. Let x be an element of $U(I-\{\alpha_0, \alpha_0+\beta_1, \alpha_0+\gamma_1, \alpha_0+\delta\})$ such that $I(x)=I-\{\alpha_0, \alpha_0+\beta_1, \alpha_0+\gamma_1, \alpha_0+\delta\}$. Then the element x is conjugate to an element of $x_\varepsilon(1)x_{\varepsilon'}(a)x_{\alpha_0+\gamma_1+\gamma_2}(1)x_{\alpha_0+\beta_1+\gamma_1}(1)x_{\alpha_0+\gamma_1+\delta}(1)x_{\alpha_0+\beta_1+\delta}(1)U(I')$, where ε' is the root such that $\varepsilon<\varepsilon'<w_{\gamma_1+\gamma_2}(\varepsilon)$ and where a is an element of the field K and $I'=I-(S\cup W_{\text{Side}(S)}(\varepsilon))$. By the action of the element $\omega_{\gamma_1+\gamma_2}\omega_{\beta_1}\omega_\delta$, the element x is conjugate to an element of the set $U(I-$

$\{\alpha_0, \alpha_0+\beta_1, \alpha_0+\delta, \alpha_0+\gamma_1, \varepsilon\})$. The proof is now finished.

LEMMA 5. *Let I be an ideal of Σ^+ and S be a (2, 3)-cube in I. Suppose that*

1) *$I-S$ is an ideal,*
2) *$I=w_\alpha(I)$ for all $\alpha \in \mathrm{Side}(S)$,*
3) *There exists an element ε of $B(I)-S$ such that $w_{\alpha'}(\varepsilon)>\varepsilon$ for some root $\alpha' \in \mathrm{Side}(S)$.*

Then the ideal I is equivalent to the ideal $I-(\{\alpha \in S \mid ht(\alpha) \leqq ht(\alpha_0)+2\} \cup \{\varepsilon\})$, where α_0 is a unique root in S such that $I(\alpha_0) \supseteqq S$.

PROOF. Since S is a (2, 3)-cube, the root system Σ is of type E_8. By hypothesis, the ideal I must be the ideal $I(\alpha_4, \alpha_{101})$. By Lemmas 1, 2 and 4, we get $I \sim I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{101})$. Let x be an element of U such that $I(x)=I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{101})$. Then the element x is conjugate to an element y in $x_{\alpha_{20}}(1)x_{\alpha_{21}}(1)x_{\alpha_{22}}(1)x_{\alpha_{23}}(1)x_{\alpha_{24}}(1)x_{\alpha_{25}}(1)x_{\alpha_{101}}(1)$ $x_{\alpha_{102}}(a)x_{\alpha_{103}}(b)x_{\alpha_{104}}(c)U(I(\alpha_{28}, \alpha_{105}))$ for some $a, b, c \in K$. By the actions of the unipotent group U, the element y is conjugate to an element z in $yx_{\alpha_{102}}(t)x_{\alpha_{103}}(t^2+2ta)x_{\alpha_{104}}(t^3+3t^2a+3tb)U(I(\alpha_{28}, \alpha_{105}))$. Hence the element z is conjugate to an element of $U(I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{105}))$. This proves the lemma.

LEMMA 6. *Suppose that Σ is of type E_8. Then the ideal $I(\alpha_5)$ is equivalent to the ideal $I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{105})$.*

PROOF. Since the ideal $I(\alpha_5)$ is equivalent to the ideal $I_1=I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$ by the Lemmas 1, 2 and 4, it is sufficient to prove that the ideal I_1 is equivalent to the ideal $I(\alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{105})$. Let x be an element of U such that $I(x)=I_1$. By the actions of B, the element x is conjugate to an element $y=x_{\alpha_{26}}(1)x_{\alpha_{27}}(1)x_{\alpha_{28}}(1)x_{\alpha_{29}}(1)x_{\alpha_{30}}(1)x_{\alpha_{31}}(1)x_{\alpha_{105}}(1)$ $x_{\alpha_{32}}(a)x_{\alpha_{39}}(b)x_{\alpha_{110}}(c)x_{\alpha_{112}}(d)x_{\alpha_{114}}(e)y_1$ for some $y_1 \in U(I(\alpha_{40}))$ and $a, b, c, d, e \in K$. Furthermore, the element y is conjugate to $yx_{\alpha_{32}}(5t)x_{\alpha_{39}}(-4ta-10t^2)x_{\alpha_{110}}$ $(-40t^3-24t^2a-4ta^2+2tb)x_{\alpha_{112}}(-205t^4-164t^3a-44t^2a^2-4ta^3+13t^2b+4tab+3tc)x_{\alpha_{114}}(t^5+t^4a-t^3b-t^2c-t^2ab+tac=td)$ for any $t \in K$. Hence we may assume $e=0$. By the action of $\omega_{\alpha_{103}}\omega_{\alpha_2}\omega_{\alpha_7}\omega_{\alpha_8}X_{\alpha_{103}}$, the element x is conjugate to an element of $U(I(\alpha_{26}, \alpha_{27}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105}))$. Hence the ideal I_1 is equivalent to the ideal $I(\alpha_{26}, \alpha_{27}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$. Let x be an element of U such that $I(x)=I(\alpha_{26}, \alpha_{27}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$. Then the element x is conjugate to an element $z=x_{\alpha_{26}}(1)x_{\alpha_{27}}(1)x_{\alpha_{29}}(1)x_{\alpha_{30}}(1)x_{\alpha_{31}}(1)x_{\alpha_{105}}(1)x_{\alpha_{35}}(a)$ $x_{\alpha_{38}}(b)x_{\alpha_{111}}(c)x_{\alpha_{112}}(d)x_{\alpha_{114}}(e)$ for some $a, b, c, d, e \in K$. If $a \neq 0$, then the element z is conjugate to $x_{\alpha_{26}}(1)x_{\alpha_{27}}(1)x_{\alpha_{29}}(1)x_{\alpha_{30}}(1)x_{\alpha_{31}}(1)x_{\alpha_{105}}(1)x_{\alpha_{35}}(1)x_{\alpha_{38}}(4t+b)x_{\alpha_{111}}(6t^2+3tb+c)x_{\alpha_{112}}(4t^4+3t^2b+2tc+d)x_{\alpha_{114}}(-t^5-t^3b-t^2c-td+e)$ for any

$t \in K$. Hence we may assume $e=0$. By the action of $\omega_{\alpha_{15}}\omega_{\alpha_4}\omega_{\alpha_{102}}\mathfrak{X}_{\alpha_4}\mathfrak{X}_{\alpha_{102}}$, the element z is conjugate to an element of $U(I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105}))$. Hence the ideal I_1 is equivalent to the ideal $I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$. Let z' be an element of U such that $I(z')=I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$. Then the element z' is conjugate to an element $z''=x_{\alpha_{26}}(1)x_{\alpha_{29}}(1)x_{\alpha_{30}}(1)x_{\alpha_{31}}(1)x_{\alpha_{105}}(1)$ $x_{\alpha_{35}}(a)x_{\alpha_{38}}(b)x_{\alpha_{42}}(c)x_{\alpha_{112}}(d)x_{\alpha_{114}}(e)$ for some $a, b, c, d, e \in K$. Furthermore the element z'' is conjugate to $z''x_{\alpha_{42}}(tb)x_{\alpha_{112}}(3t^2b-2tc)x_{\alpha_{114}}(-t^3b+t^2c-td)$ for any $t \in K$. If $b \neq 0$, the element z'' is conjugate to an element of $U(I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{105}))$ by the actions of $\omega_{\alpha_{14}}\omega_{\alpha_3}\omega_{\alpha_{101}}\mathfrak{X}_{\alpha_{14}}$. This shows that the ideal I_1 is equivalent to the ideal $I(\alpha_{26}, \alpha_{29}, \alpha_{30}, \alpha_{105})$. The proof is finished.

§ 3. The case of E_6.

Since the conjugate classes of the simply connected Chevalley group E_6 are determined in [6], we show the Zariski closure of the conjugate classes of unipotent elements. The results are given in table 3.

THEOREM 1. *The Zariski closure of the conjugate classes of unipotent elements in the simply connected Chevalley group E_6 are as in Table 6.*

PROOF. Let x be an element of U such that $I(x)=I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$. Thus the element x is a regular element. Hence the element x is conjugate to the element $x_1=x_{\alpha_1}(1)x_{\alpha_2}(1)x_{\alpha_3}(1)x_{\alpha_4}(1)x_{\alpha_5}(1)x_{\alpha_6}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_6)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6)\sim I(\alpha_1, \alpha_2, \alpha_4, \alpha_5, \alpha_6)\sim I(\alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_6)\sim I(\alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)\sim I(\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_8)$. Hence $G'(I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6))=G(x_1)=G(I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6))-G(I(\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_8))$. Let x be an element of U such that $I(x)=I(\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_8)$. Then the element x is conjugate to an element $x_{\alpha_2}(1)x_{\alpha_4}(1)x_{\alpha_5}(1)x_{\alpha_6}(1)x_{\alpha_8}(1)x_{\alpha_{16}}(a)$ for some $a \in K$. If $a \neq 0$, the element x is conjugate to the element $x_2=x_{\alpha_2}(1)x_{\alpha_4}(1)x_{\alpha_5}(1)$ $x_{\alpha_6}(1)x_{\alpha_8}(1)x_{\alpha_{16}}(1)$. If $a=0$, the element x is conjugate to an element of $U(I(\alpha_2, \alpha_6, \alpha_8, \alpha_{10}, \alpha_{11}))$ via $\omega_{\alpha_5}\omega_{\alpha_6}\omega_{\alpha_{15}}\omega_{\alpha_{16}}$. On the other hand, by Lemma 1, we get following relations; $I(\alpha_2, \alpha_4, \alpha_5, \alpha_6)\sim I(\alpha_2, \alpha_5, \alpha_6, \alpha_{10})\subset I(\alpha_2, \alpha_5, \alpha_6, \alpha_8, \alpha_{10})\sim I(\alpha_2, \alpha_4, \alpha_5, \alpha_8)\sim I(\alpha_4, \alpha_5, \alpha_6, \alpha_8)\sim I(\alpha_2, \alpha_4, \alpha_6, \alpha_8)\sim I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12})$. Hence $G'(I(\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_8))=G(x_2)=G(I(\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_8))-G(I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12}))$. Let x be an element of U such that $I(x)=I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12})$. Then the element x is conjugate to the element $x_3=x_{\alpha_1}(1)x_{\alpha_2}(1)x_{\alpha_{10}}(1)x_{\alpha_{11}}(1)x_{\alpha_{12}}(1)$. On the other hand, by Lemmas 1 and 2, we get $I(\alpha_1, \alpha_2, \alpha_{11}, \alpha_{12})\sim I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11})\sim I(\alpha_8, \alpha_9, \alpha_{11}, \alpha_{12})\subset I(\alpha_1, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12})\sim I(\alpha_2, \alpha_8, \alpha_{10}, \alpha_{11}, \alpha_{12})\sim I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{12})\sim I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}))$.

These show that $G'(I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12})) = G(x_3) = G(I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12})) - G(I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}))$. Let x be an element of U such that $I(x) = I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}))$. Then the element x is conjugate to an element $x_{\alpha_8}(1)x_{\alpha_9}(1)x_{\alpha_{10}}(1)x_{\alpha_{11}}(1)x_{\alpha_{12}}(1)x_{\alpha_{16}}(a)$ for some $a \in K$. If $a=0$, the element x is conjugate to an element of $U(I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}, \alpha_{17}))$ via $\omega_{\alpha_2}\omega_{\alpha_3}\omega_{\alpha_5}$. If $a \neq 0$, the element x is conjugate to the element $x_4 = x_{\alpha_8}(1)x_{\alpha_9}(1)x_{\alpha_{10}}(1)x_{\alpha_{11}}(1)x_{\alpha_{12}}(1)x_{\alpha_{16}}(1)$. On the other hand, by Lemmas 1 and 2, we get $I(\alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}) \sim I(\alpha_{11}, \alpha_{12}, \alpha_{14}, \alpha_{15}) \subset I(\alpha_8, \alpha_{10}, \alpha_{11}, \alpha_{12}) \sim I(\alpha_8, \alpha_{11}, \alpha_{12}, \alpha_{15}) \subset I(\alpha_8, \alpha_9, \alpha_{11}, \alpha_{12}) \sim I(\alpha_8, \alpha_9, \alpha_{12}, \alpha_{16}) \sim I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{12})$ and $I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}) \sim I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{18})$. Hence, these show $G'(I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}) = G(x_4) = G(I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12})) - G(I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}, \alpha_{17})) \cup G(I(\alpha_8, \alpha_9, \alpha_{12}, \alpha_{16})))$. Let x be an element of U such that $I(x) = I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}, \alpha_{17})$. Then the element x is conjugate to the element $x_5 = x_{\alpha_1}(1)x_{\alpha_6}(1)x_{\alpha_{15}}(1)x_{\alpha_{16}}(1)x_{\alpha_{17}}(1)$. On the other hand, by Lemmas 1 and 4, get following relations; $I(\alpha_1, \alpha_6, \alpha_{16}, \alpha_{17}) \sim I(\alpha_6, \alpha_8, \alpha_{16}, \alpha_{17}) \subset I(\alpha_6, \alpha_8, \alpha_{15}, \alpha_{16}, \alpha_{17}) \sim I(\alpha_1, \alpha_{12}, \alpha_{15}, \alpha_{16}, \alpha_{17}) \sim I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17})$, $I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{17}) \sim I(\alpha_6, \alpha_8, \alpha_{15}, \alpha_{17})$ and $I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}) \sim I(\alpha_1, \alpha_{12}, \alpha_{15}, \alpha_{16})$. Hence $G'(I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}, \alpha_{17})) = G(x_5) = G(I(\alpha_1, \alpha_6, \alpha_{15}, \alpha_{16}, \alpha_{17})) - G(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17}))$. Let x be an element of U such that $I(x) = I(\alpha_8, \alpha_9, \alpha_{12}, \alpha_{16})$. Then the element x is conjugate to an element $x_{\alpha_8}(1)x_{\alpha_9}(1)x_{\alpha_{12}}(1)x_{\alpha_{16}}(1)x_{\alpha_{18}}(a)$ for some $a \in K$. If $a=0$, the element x is conjugate to an element of $U(I(\alpha_2, \alpha_{14}, \alpha_{16}, \alpha_{18}))$. If $a \neq 0$, the element x is conjugate to the element $x_6 = x_{\alpha_8}(1)x_{\alpha_9}(1)x_{\alpha_{12}}(1)x_{\alpha_{16}}(1)x_{\alpha_{18}}(1)$. On the other hand, by Lemmas 1 and 3, we get $I(\alpha_{12}, \alpha_{14}, \alpha_{16}, \alpha_{17}) \sim I(\alpha_{12}, \alpha_{16}, \alpha_{17}, \alpha_{20}) \subset I(\alpha_{12}, \alpha_{15}, \alpha_{16}, \alpha_{17}) \sim I(\alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}) \subset I(\alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}) \sim I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{23}) \sim I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{16})$. Hence $G'(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17})) = G(x_6) = G(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17})) - G(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{17}))$. Let x be an element of such that $I(x) = I(\alpha_2, \alpha_{14}, \alpha_{16}, \alpha_{18})$. Then the element x is conjugate to the element $x_7 = x_{\alpha_2}(1)x_{\alpha_{14}}(1)x_{\alpha_{16}}(1)x_{\alpha_{18}}(1)$. By Lemmas 1 and 3, $I(\alpha_2, \alpha_{16}, \alpha_{18}) \sim I(\alpha_2, \alpha_{14}, \alpha_{18}) \sim I(\alpha_2, \alpha_{14}, \alpha_{16}) \sim I(\alpha_9, \alpha_{21}, \alpha_{23}) \subset I(\alpha_9, \alpha_{14}, \alpha_{16}, \alpha_{18}) \sim I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{23})$. Hence $G'(I(\alpha_2, \alpha_{14}, \alpha_{16}, \alpha_{18})) = G(I(\alpha_2, \alpha_{14}, \alpha_{16}, \alpha_{18})) - G(I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{23})) = G(x_7)$. Let x be an element of U such that $I(x) = I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{17})$. Then the element x is conjugate to the element $x_8 = x_{\alpha_{12}}(1)x_{\alpha_{14}}(1)x_{\alpha_{15}}(1)x_{\alpha_{17}}(1)$. On the other hand, by Lemmas 1 and 2, we get $I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{18}) \sim I(\alpha_{12}, \alpha_{15}, \alpha_{17}, \alpha_{21}) \sim I(\alpha_{12}, \alpha_{14}, \alpha_{15}) \sim I(\alpha_{12}, \alpha_{14}, \alpha_{17}) \sim I(\alpha_{14}, \alpha_{16}, \alpha_{24})$. These show $G'(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{17})) = G(x_8) = G(I(\alpha_{12}, \alpha_{14}, \alpha_{15}, \alpha_{17})) - G(I(\alpha_{14}, \alpha_{16}, \alpha_{24}))$. By Lemmas 1 and 2, we get the relation $I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{23}) \sim I(\alpha_{14}, \alpha_{16}, \alpha_{24})$. Let x be an element of such that $I(x) = I(\alpha_{14}, \alpha_{16}, \alpha_{24})$. Then the element x is conjugate to an element $x_{\alpha_{14}}(1)x_{\alpha_{16}}(1)x_{\alpha_{24}}(0)x_{\alpha_{22}}(a)x_{\alpha_{26}}$ for some a, $b \in K$. If $a=0$, the element x is conjugate to an element of $U(I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24}))$. If $a \neq 0$, the element x is conjugate to the element

$x_9=x_{\alpha_{14}}(1)x_{\alpha_{16}}(1)x_{\alpha_{22}}(1)x_{\alpha_{24}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{16}, \alpha_{20}, \alpha_{24})\sim I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24})\supset I(\alpha_{14}, \alpha_{22}, \alpha_{23})\sim I(\alpha_{14}, \alpha_{16})$. Hence, $G'(I(\alpha_{14}, \alpha_{16}, \alpha_{24}))=G(x_9)=G(I(\alpha_{14}, \alpha_{16}, \alpha_{24}))-G(I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24}))$. Let x be an element of U such that $I(x)=I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24})$. Then the element x is conjugate to the element $x_{10}=x_{\alpha_{14}}(1)x_{\alpha_{22}}(1)x_{\alpha_{23}}(1)x_{\alpha_{24}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{14}, \alpha_{22}, \alpha_{23})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23})\subset I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28})\sim I(\alpha_{14}, \alpha_{23}, \alpha_{24}, \alpha_{28})$. Hence $G'(I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24}))=G(I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24}))-(G(I(\alpha_{14}, \alpha_{22}, \alpha_{24}))\cup G(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28}))=G(x_{10})$. Let x be an element of U such that $I(x)=I(\alpha_{14}, \alpha_{22}, \alpha_{24})$. Then the element x is conjugate to the element $x_{11}=x_{\alpha_{14}}(1)x_{\alpha_{22}}(1)x_{\alpha_{24}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{14}, \alpha_{22})\sim I(\alpha_{14}, \alpha_{24}, \alpha_{28})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{24})\sim I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29})$. Hence, these show $G'(I(\alpha_{14}, \alpha_{22}, \alpha_{24}))=G(I(\alpha_{14}, \alpha_{22}, \alpha_{24}))-G(I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}))=G(x_{11})$. Let x be an element of U such that $I(x)=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28})$. Then the element x is conjugate to the element $x_{12}=x_{\alpha_{20}}(1)x_{\alpha_{21}}(1)x_{\alpha_{23}}(1)x_{\alpha_{24}}(1)x_{\alpha_{28}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28})\sim I(\alpha_{20}, \alpha_{23}, \alpha_{24}, \alpha_{28})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{24}, \alpha_{28})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{28})\sim I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29})$. These show $G'(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28}))=G(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28}))-\{G(I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}))\cup G(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}))\}=G(x_{12})$. Let x be an element of U such that $I(x)=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24})$. Then the element x is conjugate to the element $x_{13}=x_{\alpha_{20}}(1)x_{\alpha_{21}}(1)x_{\alpha_{23}}(1)x_{\alpha_{24}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{20}, \alpha_{23}, \alpha_{24})\sim I(\alpha_{21}, \alpha_{23}, \alpha_{24})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{23})\sim I(\alpha_{20}, \alpha_{21}, \alpha_{24})\sim I(\alpha_{26}, \alpha_{27}, \alpha_{35})$. Hence $G'(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}))=G(I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}))-G(I(\alpha_{26}, \alpha_{27}, \alpha_{35}))=G(x_{13})$. Let x be an element of U such that $I(x)=I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29})$. Then the element x is conjugate to the element $x_{14}=x_{\alpha_{26}}(1)x_{\alpha_{27}}(1)x_{\alpha_{28}}(1)x_{\alpha_{29}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{27}, \alpha_{28}, \alpha_{29})\sim I(\alpha_{26}, \alpha_{27}, \alpha_{28})\sim I(\alpha_{26}, \alpha_{27}, \alpha_{29})\sim (I\alpha_{26}, \alpha_{27}, \alpha_{35})$. And we get $I(\alpha_{26}, \alpha_{28}, \alpha_{29})\sim I(\alpha_{26}, \alpha_{35})$ by Lemma 2. Hence $G'(I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}))=G(I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}))-G(I(\alpha_{26}, \alpha_{27}, \alpha_{35}))=G(x_{14})$. Let x be an element of U such that $I(x)=I(\alpha_{26}, \alpha_{27}, \alpha_{35})$. Then the element x is conjugate to the element $x_{15}=x_{\alpha_{26}}(1)x_{\alpha_{27}}(1)x_{\alpha_{35}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{26}, \alpha_{27}, \alpha_{40})\sim I(\alpha_{27}, \alpha_{32}, \alpha_{35})\sim I(\alpha_{37}, \alpha_{38}, \alpha_{40})\subset I(\alpha_{26}, \alpha_{35})$. Hence, these show $G'(I(\alpha_{26}, \alpha_{27}, \alpha_{35}))=G(x_{15})=G(I(\alpha_{26}, \alpha_{27}, \alpha_{35}))-G(I(\alpha_{26}, \alpha_{35}))$. Let x be an element of such that $I(x)=I(\alpha_{26}, \alpha_{35})$. Then the element x is conjugate to the element $x_{16}=x_{\alpha_{26}}(1)x_{\alpha_{35}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{32}, \alpha_{33}, \alpha_{35})\sim I(\alpha_{26}, \alpha_{40})\sim I(\alpha_{37}, \alpha_{38}, \alpha_{40})$. Hence $G'(I(\alpha_{26}, \alpha_{35}))=G(I(\alpha_{26}, \alpha_{35}))-G(I(\alpha_{37}, \alpha_{38}, \alpha_{40}))=G(x_{16})$. Let x be an element of U such that $I(x)=I(\alpha_{37}, \alpha_{38}, \alpha_{40})$. Then the element x is conjugate to the element $x_{17}=x_{\alpha_{37}}(1)x_{\alpha_{38}}(1)x_{\alpha_{40}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{37}, \alpha_{38})\sim I(\alpha_{37}, \alpha_{40})\sim I(\alpha_{38}, \alpha_{40})\sim I(\alpha_{42}, \alpha_{43})$. Hence $G'(I(\alpha_{37}, \alpha_{38}, \alpha_{40}))=G(I(\alpha_{37}, \alpha_{38},$

$\alpha_{40}))-G(I(\alpha_{42}, \alpha_{43}))=G(x_{17})$. Let x be an element of U such that $I(x)=I(\alpha_{42}, \alpha_{43})$. Then the element x is conjugate to the element $x_{18}=x_{\alpha_{42}}(1)x_{\alpha_{43}}(1)$. On the other hand, by Lemma 1, we get $I(\alpha_{42})\sim I(\alpha_{43})\sim I(\alpha_{53})$. Hence $G'(I(\alpha_{42}, \alpha_{43}))=G(I(\alpha_{42}, \alpha_{43}))-G(I(\alpha_{53}))=G(x_{18})$. Let x be an element of $X^*_{\alpha_{53}}$. Then the element x is conjugate to the element $x_{19}=x_{\alpha_{53}}(1)$. Hence $G'(I(\alpha_{53}))=G(x_{19})=G(I(\alpha_{53}))-\{1\}$. These show that the sets $G'(I)$ for above ideals are one classes. The proof is complete.

4. The case of E_7.

The following notations will be used throughout sections 4 and 5;

$$x_i(t)=x_{\alpha_i}(t)\ ,\quad w_i=w_{\alpha_i}\ ,\quad \omega_i=\omega_{\alpha_i}\ ,\quad Z(x)=Z_G(x)/Z_G(x)^\circ\ ,$$
$$W(x)=\{w\in W\,|\,BwB\cap Z_G(x)\neq\varnothing\}\ ,\quad L(x)=Z_G(x)^\circ/\mathrm{Ru}(Z_G(x)^\circ)\ ,$$
$$\overline{Z(x)}=Z_{P|R}(xV_1)/Z_{P/R}(xV_1)^\circ\ ,\quad \text{where } (P, R, V, V_1) \text{ gives a structure of } Z_G(x)\ .$$

If elements x and y are conjugate in G, we shall use the notation $x\underset{c}{\sim}y$. The following three lemmas are often used without reference.

LEMMA 7. *Let x be an element of U. If (P, R, V, V_1) gives a structure of $Z_G(x)$ and if $Z_{P/R}(xV_1)^\circ$ is a reductive group, the following sequence is exact;*

$$0\longrightarrow Z_R(x)/Z_R(x)^\circ\overset{\theta}{\longrightarrow}Z(x)\longrightarrow\overline{Z(x)}\longrightarrow 0\ .$$

PROOF. It is sufficient to show the injectivity of θ. We consider the natural map ϕ, $Z_G(x)/Z_R(x)^\circ\longrightarrow Z_{P/R}(xV_1)$. Since $\mathrm{Ker}\,\phi$ is a finite group, this commutes with the group $Z_G(x)^\circ/Z_R(x)^\circ$. And the group $Z_G(x)^\circ/Z_R(x)^\circ$ is reductive by the assumption. Since the center of reductive group consists of semisimple elements, we get $(Z_R(x)\cap Z_G(x)^\circ)/Z_R(x)^\circ=1$. This shows that θ is injective. The proof is finished.

LEMMA 8. *Let x be an element of U and let w be an element of W such that $BwB\cap Z_G(x)\neq\varnothing$. Then,*

1) $w(\mathfrak{B}(I(x)))\subseteq I(x)$.

2) *Suppose furthermore that any element $u=\prod x_i(u_i)$ of B-orbit $B(x)$ satisfies the relation $f(u)\neq 0$. Here f is a polynomial with respect to the variables u_i. If a set $\{\alpha_{i_1}, \alpha_{i_2}, \cdots, \alpha_{i_s}\}$ satisfies the relation $f(u_{i_1}=u_{i_2}=\cdots=u_{i_s}=0)\equiv 0$, we get*

$$w\{\alpha_{i_1}, \alpha_{i_2}, \cdots, \alpha_{i_s}\}\cap I(x)\neq\varnothing\ .$$

The proof is easily obtained. This lemma is useful to determine the set $W(x)$.

LEMMA 9. *Let V be a connected unipotent group defined over k. Then the number of $V(k)$ is $q^{\dim V}$. Let X be a connected algebraic group defined over k. Then the natural map π: $X(k) \to (X/\mathrm{Ru}(X))(k)$ is surjective.*

PROOF. See [1] 15.7.

Hereafter, we assume that the Chevalley group G is the simply connected Chevalley group of type E_7 over the field K.

LEMMA 10. *Let x be an element of U such that $I(x)=I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)$, then $x \underset{c}{\sim} y_1 = x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_7(1)x_1(1)$. Furthermore, an element in $G(y_1)(k)$ is conjugate in $G(k)$ to one of the following elements*:

$$
\begin{aligned}
&y_1 = x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_7(1)x_1(1)\ , && \\
&y_2 = x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_7(\zeta)x_1(1)\ , && \textit{when } \mathrm{ch}(K)\neq 2\ , \\
&y_3 = x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(\tau)x_6(1)x_7(1)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=3\ , \\
&y_4 = x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(-\tau)x_6(1)x_7(1)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=3\ , \\
&y_5 = x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(\tau)x_6(1)x_7(\zeta)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=3\ , \\
&y_6 = x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(-\tau)x_6(1)x_7(\zeta)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=3\ , \\
&y_7 = x_3(1)x_4(1)x_2(1)x_{15}(\eta)x_5(1)x_6(1)x_7(1)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=2\ , \\
&x_8 = x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_{29}(\eta)x_7(1)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=2\ , \\
&y_9 = x_3(1)x_4(1)x_2(1)x_{15}(\eta)x_5(1)x_6(1)x_{29}(\eta)x_7(1)x_1(1)\ , && \textit{when } \mathrm{ch}(K)=2\ .
\end{aligned}
$$

The group $Z(x)$ is a cyclic group of order 2(6, p).

PROOF. Since the regular unipotent elements of G form a single class [10], we get $x \underset{c}{\sim} y_1$. By T. A. Springer [8] and B. Lou [5], $Z_G(x) = Z(G)\langle x\rangle Z_U(x)^\circ$. From these facts, the lemma is easily verified.

LEMMA 11. 1) *$G'(I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)) = G(y_1) = G(I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)) - G(I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7))$.*

2) *Let $x = \prod x_i(t_i)$ be an element of U such that $I(x) = I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7)$ and $t_{10}t_5 + t_{11}t_3 \neq 0$, then the element x is conjugate to the element $y_{10} = x_1(1)x_3(1)x_{10}(1)x_9(1)x_5(1)x_6(1)x_7(1)$. Furthermore, an element in $G(y_{10})(k)$ is conjugate in $G(k)$ to one of the following elements*:

$$
\begin{aligned}
&y_{10} = x_1(1)x_3(1)x_{10}(1)x_9(1)x_5(1)x_6(1)x_7(1)\ , && \\
&y_{11} = x_1(1)x_3(1)x_{10}(1)x_9(1)x_5(1)x_6(1)x_7(\zeta)\ , && \textit{when } \mathrm{ch}(K)\neq 2\ , \\
&y_{12} = x_1(1)x_3(1)x_{10}(1)x_{28}(\eta)x_9(1)x_5(1)x_6(1)x_7(1)\ , && \textit{when } \mathrm{ch}(K)=2\ .
\end{aligned}
$$

3) $Z_G(y_{10})=Z(G)Z_U(y_{10})$. *If* $\mathrm{ch}(K)\neq 2$, $Z_U(y_{10})$ *is connected. If* $\mathrm{ch}(K)=2$, $Z_U(y_{10})=\langle y_{10}\rangle Z_U(y_{10})^\circ$ *and* $Z(y_{10})$ *is of order* 2. $|Z_{G(k)}(y_i)|=2q^9(i=10, 11, 12)$.

PROOF. By Lemma 1, $I(\alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_7)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)\sim I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7)$. Hence 1) follows from Lemma 10. By the action of the group $B, x\underset{c}{\sim} y_{10}$. By Lemma 8, $W(y_{10})=1$. Hence $Z_G(y_{10})=Z_B(y_{10})=Z(G)Z_U(y_{10})$. Furthermore, $\dim Z_B(y_{10})=\dim B-\dim U(I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7))=9$. We put $P=B$, $R=U(I(\alpha_8, \alpha_{19}, \alpha_{22}, \alpha_{23}, \alpha_{24}))$, $V=U(I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7))$ and $V_1=U(I(\alpha_{20}, \alpha_{21}, \alpha_{29}, \alpha_{30}, \alpha_{31}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{10})$ and $Z_R(y_{10})$ is connected. Hence $Z(y_{10})\cong\overline{Z(y_{10})}$. If an element $u=\prod x_i(u_i)$ stabilizes the set $y_{10}X_{28}V_1$,

$$uy_{10}u^{-1}\equiv y_{10}x_{28}(u_1^2+u_1-2u_{12}) \mod V_1 .$$

By the action of H and above results, we get the lemma.

LEMMA 12. 1) *Let* $I_1=I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7)$ *and* $I_2=I(\alpha_1, \alpha_2, \alpha_3, \alpha_{11}, \alpha_{12}, \alpha_{13})$. *Then* $G'(I_1)=G(y_{10})=G(I_1)-G(I_2)$.

2) *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_2$ *and* $t_2t_{10}-t_3t_9\neq 0$. *Then the element* x *is conjugate to* $y_{13}=x_1(1)x_2(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)$. *Furthermore, an element of* $G(y_{13})(k)$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$\begin{aligned}
&y_{13}=x_1(1)x_2(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1) ,\\
&y_{14}=x_1(1)x_2(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(\zeta) , \qquad \textit{when } \mathrm{ch}(K)\neq 2 ,\\
&y_{15}=x_1(1)x_2(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{19}(\eta)x_{13}(1) , \qquad \textit{when } \mathrm{ch}(K)=2 .
\end{aligned}$$

3) $Z_G(y_{13})=Z(G)Z_U(y_{13})$. *If* $\mathrm{ch}(K)\neq 2$, $Z_U(y_{13})$ *is connected. If* $\mathrm{ch}(K)=2$, $Z_G(y_{13})=\langle y_{13}\rangle Z_U(y_{13})^\circ$ *and* $Z(y_{13})$ *is of order* 2. $|Z_{G(k)}(y_i)|=2q^{11}(i=13, 14, 15)$.

PROOF. By Lemma 1, $I(\alpha_3, \alpha_9, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_9, \alpha_{10}, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_3, \alpha_9, \alpha_{11}, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_7)\sim I(\alpha_1, \alpha_3, \alpha_9, \alpha_5, \alpha_6)\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_{11}, \alpha_{12}, \alpha_{13})$ and $I(\alpha_1, \alpha_3, \alpha_5, \alpha_6, \alpha_7)\sim I(\alpha_1, \alpha_3, \alpha_{11}, \alpha_6, \alpha_7)$. Since an element $u=\prod x_i(u_i)$ of $U(I_1)$ which satisfies two relations $I(u)=I_1$, $u_5u_{10}+u_3u_{11}=0$ is conjugate to some element of $U(I_2)$, the statement 1) is proved. By Lemma 8, $W(x)=\{1\}$. Hence $Z_G(x)=Z_B(x)=Z(G)Z_U(x)$. We put $P=B$, $R=U(I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_7))$, $V=U(I_2)$, $V_1=U(I(\alpha_{26}, \alpha_{40}, \alpha_{19}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$ and $Z_R(x)$ is connected. By calculations, we get that x is conjugate to an element $y=x_1(t_1)x_2(t_2)x_3(t_3)$

$x_9(a)x_{11}(t_{11})x_{12}(t_{12})x_{13}(t_{13})x_{29}(b)$ for some $a, b \in K$. Suppose $\mathrm{ch}(K)=2$. Then y is conjugate in G to $y'=y_{13}x_{29}(b')$ for some $b' \in K$. Since $y' \underset{c}{\sim} y_{13}x_{29}(c^2+c)$ for any c, the proof of the lemma is immediate.

LEMMA 13. *Let* $I_3=I(\alpha_1, \alpha_9, \alpha_{10}, \alpha_{12}, \alpha_7)$ *and* $I_4=I(\alpha_1, \alpha_3, \alpha_9, \alpha_{11}, \alpha_{12}, \alpha_{13})$. *Then* $G'(I_2)=G(y_{13})=G(I_2)-\{G(I_3)\cup G(I_4)\}$.

2) *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_3$, $t_{10}t_{17}-t_9t_{16}\neq 0$ *and* $(t_{22}t_1t_{10}-t_1t_{15}t_{16})+t_8(t_{10}t_{17}-t_9t_{16})\neq 0$. *Then the element* x *is conjugate to* $y_{16}=x_1(1)x_9(-1)x_{10}(1)x_{17}(1)x_{22}(1)x_{12}(1)x_7(1)$. *Furthermore, an element of* $G(y_{16})(k)$ *is conjugate in* $G(k)$ *to one of the following elements*:

$y_{16}=x_1(1)x_9(-1)x_{10}(1)x_{17}(1)x_{22}(1)x_{12}(1)x_7(1)$,

$y_{17}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{18}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_7(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{19}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_7(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{20}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{22}(\eta)x_{12}(1)x_7(1)$, *when* $\mathrm{ch}(K)=2$.

3) $Z_G(y_{16})\subset B\langle w_2, w_3\rangle B$, $Z_G(y_{16})^\circ\subset U$ *and* $Z(y_{16})\cong Z_2\times Z_{(2,p-1)}$. *Furthermore*, $|Z_{G(k)}(y_i)|=2(2, p-1)q^{13}(i=16, 17, 18, 19, 20)$.

PROOF. By Lemmas 1 and 2, $I(\alpha_1, \alpha_2, \alpha_3, \alpha_{11}, \alpha_{12})\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_{11}, \alpha_{13})\sim I(\alpha_1, \alpha_2, \alpha_3, \alpha_{12}, \alpha_{13})\sim I(\alpha_1, \alpha_3, \alpha_9, \alpha_{18}, \alpha_{19})\subset I(\alpha_1, \alpha_3, \alpha_9, \alpha_{11}, \alpha_{12}, \alpha_{13})$ and $I(\alpha_2, \alpha_3, \alpha_{11}, \alpha_{12}, \alpha_{18})\sim I(\alpha_1, \alpha_2, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_{13})\sim I(\alpha_1, \alpha_9, \alpha_{10}, \alpha_{12}, \alpha_7)$. By Lemmas 1, 2 and 12, an element u of U such that $I(u)=I_2$ and $u\notin G(y_{13})$ is conjugate to an element of $U(I_3)$. This shows 1). By the action of B, $x\underset{c}{\sim}y_{16}$. By Lemma 8, $W(y_{16})=\langle w_2w_3\rangle$. Let $P=B\langle w_2, w_3\rangle B$, $R=U(I(\alpha_1, \alpha_4, \alpha_6, \alpha_7))$, $V=U(I(\alpha_1, \alpha_4, \alpha_{12}, \alpha_7))$, $V_1=U(I(\alpha_{14}, \alpha_{18}, \alpha_{28}, \alpha_{13}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{16})$ and $Z_R(y_{16})$ is connected. Hence $\dim Z_G(y_{16})=\dim Z_U(y_{16})=13$ and $\overline{Z_G(y_{16})}=Z(G)\langle\bar u\rangle$. Here, $\bar u=h_1(-1)h_2(-1)x_2(1)x_3(-1)x_5(1)\omega_2\omega_3x_2(-1)x_3(-1)V_1$. On the other hand, by calculations, $B(y_i)(17\leqq i\leqq 20)$ are open in $U(I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_7))$. Furthermore, $I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_7)$ is equivalent to I_3 and $y_{16}\underset{c}{\sim}x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(1)$ via $h_1(-1)\omega_2\omega_3$. Now 2) and 3) are clear.

LEMMA 14. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_4$, *then* $x\underset{c}{\sim}y_{21}=x_1(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)$. *Furthermore, an element of* $G(y_{21})(k)$ *is conjugate in* $G(k)$ *to one of the following elements*:

$y_{21}=x_1(1)x_3(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)$,

$y_{22}=x_1(1)x_3(1)x_9(1)x_{11}(1)x_{28}(\eta)x_{12}(1)x_{13}(1)$, *when* $\mathrm{ch}(K)=2$,

$y_{23}=x_1(1)x_3(1)x_9(1)x_{11}(1)x_{32}(\tau)x_{12}(1)x_{13}(1)$, *when* $\mathrm{ch}(K)=3$,

$$y_{24}=x_1(1)x_8(1)x_9(1)x_{11}(1)x_{32}(-\tau)x_{12}(1)x_{13}(1)\ , \qquad \textit{when}\ \mathrm{ch}(K)=3\ .$$

$$Z_G(y_{21})\subsetneqq B\langle w_2w_5w_7\rangle B\ ,\quad L(y_{21})=A_1\ ,\quad Z_G(y_{21})=\langle y_{21}\rangle Z_G(y_{21})^\circ\ ,$$

$Z(y_{21})\cong Z_{(6,p)}$, $|Z_{G(k)}(y_i)|=(6,\ p)(q^2-1)q^{11}(i=21,\ 22,\ 23,\ 24)$.

PROOF. By Lemma 8, $W(x)\subsetneqq\langle w_2w_5w_7\rangle$. Let $P=B\langle w_2,\ w_5,\ w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=U(I(\alpha_8,\ \alpha_{10},\ \alpha_{18}))$. Then $(P,\ R,\ V,\ V_1)$ gives a scructure of $Z_G(x)$ and $x\underset{c}{\sim}y_{21}$. Since $Z_R(y_{21})/Z_R(y_{21})^\circ$ is of order $(6,\ p)$, the proof is obvious.

LEMMA 15. *Let* $I_5=I(\alpha_8,\ \alpha_9,\ \alpha_{11},\ \alpha_{12},\ \alpha_{13})$, *then* $G'(I_4)=G(y_{21})=G(I_4)-G(I_5)$. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_5$ *and* $t_{11}t_{15}-t_9t_{16}\neq 0$, *then* $x\underset{c}{\sim}y_{25}=x_8(1)x_9(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)$. *Furthermore, an element of* $G(y_{25})(k)$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{25}=x_8(1)x_9(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)\ ,$$
$$y_{26}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_{13}(1)\ , \qquad \textit{when}\ \mathrm{ch}(K)\neq 2\ ,$$
$$y_{27}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{15}(1)x_{22}(\eta)x_{12}(1)x_{13}(1)\ , \qquad \textit{when}\ \mathrm{ch}(K)=2\ .$$

$Z_G(y_{25})\subset B\langle w_2w_5w_7\rangle B$, $Z_G(y_{25})^\circ\subset B$, $Z(y_{25})\cong Z_2$, $L(y_{25})\cong T_1$ (*one dimensional torus*), $|Z_{G(k)}(y_{25})|=2(q-1)q^{14}$, $|Z_{G(k)}(y_i)|=2(q+1)q^{14}(i=26,\ 27)$.

PROOF. By Lemma 8, we get $W(x)\subsetneqq\langle w_2w_5w_7\rangle$. Thus we put $P=B\langle w_2,\ w_5,\ w_7\rangle B$, $R=U(I(\alpha_1,\ \alpha_4,\ \alpha_6))$, $V=U(I(\alpha_8,\ \alpha_4,\ \alpha_6))$ and $V_1=U(I(\alpha_{14},\ \alpha_{18},\ \alpha_{28}))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(x)$ and $Z_R(x)$ is connected. By the action of B, $x\underset{c}{\sim}y_{25}$. The element y_{25} is conjugate to the element $y'_{25}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{15}(1)x_{12}(1)x_{13}(1)$ via $\omega_2\omega_5\omega_7x_2(-1)x_5(-1)x_7(-1)$. Now the proof is easy.

LEMMA 16. *Let* $I_6=I(\alpha_1,\ \alpha_{15},\ \alpha_{16},\ \alpha_{17},\ \alpha_6,\ \alpha_7)$, *then* $G'(I_3)=G(y_{16})=G(I_3)-\{G(I_5)\cup G(I_6)\}$. *Let* x *be an element of* U *such that* $I(x)=I_6$, *then* $x\underset{c}{\sim}y_{28}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)$. *Furthermore, a* k-*rational point in* $G(y_{28})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{28}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)\ ,$$
$$y_{29}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(\zeta)\ , \qquad \textit{when}\ \mathrm{ch}(K)\neq 2\ ,$$
$$y_{30}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{40}(\eta)x_6(1)x_7(1)\ , \qquad \textit{when}\ \mathrm{ch}(K)=2\ .$$

$Z_G(y_{28})\subset B\langle w_4\rangle B$, $Z(y_{28})\cong Z_2$, $L(y_{28})\cong A_1$, $|Z_{G(k)}(y_i)|=2(q^2-1)q^{13}(i=28,\ 29,\ 30)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{28}$. By Lemma 8, $W(y_{28})=\langle w_4\rangle$.

Thus we put $P=B\langle w_4\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I_6)$, $V_1=U(I(\alpha_8, \alpha_{22}, \alpha_{12}, \alpha_{13}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{28})$. By calculations, $Z_R(y_{28})/Z_R(y_{28})^\circ \cong Z_{(2,p)}$ and $\overline{Z(y_{28})}\cong Z(G)$. Now the proof is easy.

LEMMA 17. *Let* $I_7=I(\alpha_1, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{13})$, *then* $G'(I_5)=G(y_{25})=G(I_5)-G(I_7)$. *Let* $x=\prod x_i(t_i)$ *be an element of such that* $I(x)=I_7$ *and* $t_1(t_{13}t_{17}t_{23}-t_{13}t_{16}t_{24}+t_{13}t_{18}t_{22}+t_{15}t_{18}t_{19})+2t_8t_{13}t_{17}t_{18}\neq 0$, *then* $x \underset{c}{\sim} y_{35}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{23}(1)x_{13}(1)$. *Furthermore a k-rational point in* $G(y_{35})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$\begin{aligned}
&y_{31}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{35}(1)x_7(1)\,, && \textit{when } \mathrm{ch}(K)\neq 2\,,\\
&y_{32}=x_{14}(1)x_{15}(1)x_{16}(\zeta)x_{17}(1)x_{12}(1)x_{35}(1)x_7(1)\,, && \textit{when } \mathrm{ch}(K)\neq 2\,,\\
&y_{33}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{35}(\zeta)x_7(1)\,, && \textit{when } \mathrm{ch}(K)\neq 2\,,\\
&y_{34}=x_{14}(1)x_{15}(1)x_{16}(\zeta)x_{17}(1)x_{12}(1)x_{35}(\zeta)x_7(1)\,, && \textit{when } \mathrm{ch}(K)\neq 2\,,\\
&y_{35}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{23}(1)x_{13}(1)\,, && \textit{when } \mathrm{ch}(K)=2\,.
\end{aligned}$$

$Z_G(y_{35}) = Z_H(y_{35})Z_U(y_{35})^\circ$, $Z(y_{35}) \cong Z_{(2,p-1)}\times Z_{(2,p-1)}$, $L(y_{35}) = 1$, $|Z_{G(k)}(y_i)|=(2, p-1)^2q^{17}(i=31, 32, 33, 34, 35)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{35}$. By Lemma 8, $W(y_{35})=\{1\}$. By calculations, $Z_U(y_{35})$ is connected. Hence $Z(y_{35})\cong\overline{Z(y_{35})}$. Now the proof is easy.

LEMMA 18. $G'(I_6)=G(y_{28})=G(I_6)-G(I_7)$. *Let* $I_8=I(\alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{12}, \alpha_{13})$ *and let* x *be an element of* U *such that* $I(x)=I_8$, *then* $x\underset{c}{\sim}y_{36}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{13}(1)$. *Furthermore, a k-rational point in* $G(y_{36})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$\begin{aligned}
&y_{36}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{13}(1)\,,\\
&y_{37}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{23}(1)x_{13}(1)x_{36}(\eta)\,, && \textit{when } \mathrm{ch}(K)=2\,.
\end{aligned}$$

$Z_G(y_{36})\subset B\langle w_8w_2w_5w_7\rangle B$, $Z(y_{36})\cong Z_{(2,p)}$. *If* $\mathrm{ch}(K)\neq 2$, $L(y_{36})=A_1$. *If* $\mathrm{ch}(K)=2$, $L(y_{36})=T_1$ *(one dimensional torus). If* $\mathrm{ch}(K)\neq 2$, $|Z_{G(k)}(y_{36})|=(q^2-1)q^{17}$. *If* $\mathrm{ch}(K)=2$, $|Z_{G(k)}(y_{36})|=2(q-1)q^{18}$ *and* $|Z_{G(k)}(y_{37})|=2(q+1)q^{18}$.

PROOF. By Lemma 8, we get $W(x)\subseteq\langle w_8w_2w_5w_7\rangle$. Thus we put $P=B\langle w_1, w_2, w_3, w_5, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=\mathrm{Ru}(P)$, $V_1=U(I(\alpha_{18}, \alpha_{23}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Suppose $\mathrm{ch}(K)\neq 2$. Then $x\underset{c}{\sim}y_{36}$ by the action of B. Suppose $\mathrm{ch}(K)=2$, the element x is conjugate to an element $y_{36}x_{23}(a)x_{36}(b)$ for some a, $b\in K$. Furthermore, by the action of B, $x\underset{c}{\sim}y_{36}x_{23}(a)x_{36}(b+d+d^2a)$ for any $d\in K$. Therefore, $x\underset{c}{\sim}y_{36}$. Now

the proof is easy.

LEMMA 19. *Let* $I_9=I(\alpha_1, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{19})$ *and let* x *be an element of* U *such that* $I(x)=I_9$, *then* $x \underset{c}{\sim} y_{38}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)$. *Furthermore, a k-rational point in* $G(y_{38})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{38}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)\ ,$$
$$y_{39}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(\zeta)x_{18}(1)x_{19}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$y_{40}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{49}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(y_{38})\subset B\langle w_4w_7\rangle B$, $Z(y_{38})\cong Z_2$, $L(y_{38})=A_1$, $|Z_{G(k)}(y_i)|=2(q^2-1)q^{17}$ $(i=38, 39, 40)$.

PROOF. By the action of B, $x \underset{c}{\sim} y_{38}$. By Lemma 8, get get $W(y_{38})\subsetneqq \langle w_4w_7\rangle$. Thus we put $P=B\langle w_4, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_1, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{12})$, $V_1=U(I(\alpha_8, \alpha_{22}, \alpha_{23}, \alpha_{24}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{36})$. Since $\overline{Z(y_{38})}\cong Z_{(2,p-1)}$ and $Z_R(y_{38})/Z_R(y_{38})^\circ\cong Z_{(2,p)}$, we get $Z(y_{38})\cong Z_2$. Now the proof is easy.

LEMMA 20. *Let* $I_{10}=I(\alpha_1, \alpha_{15}, \alpha_{17}, \alpha_{23}, \alpha_7)$, *then* $G'(I_7)=G(y_{35})=G(I_7)-\{G(I_8)\cup G(I_9)\cup G(I_{10})\}$. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_{10}$ *and* $t_{24}t_7+t_{13}t_{17}\neq 0$, *then* $x \underset{c}{\sim} y_{41}=x_1(1)x_{15}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)$. *Furthermore, a k-rational element in* $G(y_{41})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{41}=x_1(1)x_{15}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)\ ,$$
$$y_{42}=x_1(1)x_{15}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(\zeta)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$y_{43}=x_1(1)x_{15}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)x_{45}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(y_{41})\subset B\langle w_4\rangle B$, $Z(y_{41})\cong Z_2$, $L(y_{41})=A_1$, $|Z_{G(k)}(y_i)|=2(q^2-1)q^{17}(i=41, 42, 43)$.

PROOF. By the action of B, $x \underset{c}{\sim} y_{41}$. By Lemma 8, $W(y_1)=\langle w_4\rangle$. Thus we put $P=B\langle w_4\rangle B$, $R=U(I(\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_7))$, $V=U(I_{10})$, $V_1=U(I(\alpha_8, \alpha_{22}, \alpha_{19}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{41})$. Since $Z_R(y_{41})/Z_R(y_{41})^\circ\cong Z_{(2,p)}$ and $\overline{Z(y_{41})}\cong Z(G)$, we get $Z(y_{41})\cong Z_2$. Now the proof is easy.

LEMMA 21. *Let* $I_{11}=I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{18}, \alpha_{19})$, *then* $G'(I_8)=G(y_{36})=G(I_8)-G(_{11})$. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_{11}$ *and* $f_1=t_{14}t_{17}t_{23}-t_{14}t_{18}t_{22}+t_{15}t_{18}t_{21}\neq 0$.

1) *Suppose* $\mathrm{ch}(K)=2$. *If* $f_2=t_{15}t_{17}t_{27}-t_{15}t_{21}t_{24}-t_{15}t_{18}t_{26}+t_{18}t_{20}t_{22}-t_{14}t_{17}t_{29}+$

$t_{14}t_{22}t_{24}-t_{17}t_{20}t_{23}\neq 0$, *then* $x \underset{c}{\sim} y_{44}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{27}(1)$. *Furthermore, a k-rational point in* $G(y_{44})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{44}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{27}(1)\ ,$$
$$y_{45}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{27}(1)x_{33}(\eta)\ ,$$
$$y_{46}=x_{1}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{26}(1)x_{33}(\tau)\ .$$

$Z_G(y_{44})\subset B\langle w_1w_2w_6\rangle B$, $Z(y_{44})\cong S_3$ (*symmetric group of degree* 3), $|Z_{G(k)}(y_{44})|=6q^{21}$, $|Z_{G(k)}(y_{45})|=2q^{21}$, $|Z_{G(k)}(y_{46})|=3q^{21}$.

2) *Suppose* $\mathrm{ch}(K)\neq 2$. *If* $f_3=4(t_{15}t_{17}t_{33}-t_{21}t_{24}t_{26}-t_{17}t_{20}t_{29}+t_{20}t_{22}t_{24})f_1+f_2^2\neq 0$, *then* $x \underset{c}{\sim} y_{47}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{33}(1)$. *Furthermore, a k-rational point in* $G(y_{47})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{47}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{33}(1)\ ,$$
$$y_{48}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(\zeta)x_{23}(1)x_{33}(1)\ ,$$
$$y_{49}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)x_{33}(\zeta)\ ,$$
$$y_{50}=x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(\zeta)x_{23}(1)x_{33}(\zeta)\ ,$$
$$y_{46}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{26}(1)x_{33}(\tau)\ ,$$
$$y_{51}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(\zeta)x_{26}(1)x_{33}(\tau)\ .$$

$Z_G(y_{47})\subset B\langle w_1w_2w_6\rangle B$, $Z(y_{47})\cong Z_2\times S_3$, $|Z_{G(k)}(y_{47})|=|Z_{G(k)}(y_{48})|=12q^{21}$, $|Z_{G(k)}(y_{49})|=|Z_{G(k)}(y_{40})|=4q^{21}$, $|Z_{G(k)}(y_{46})|=|Z_{G(k)}(y_{51})|=6q^{21}$.

PROOF. By the action of the group H, we may assume that $t_{14}=t_{15}=t_{17}=t_{18}=t_{19}=1$. By the action of $\prod x_i(u_i)$, the element x is conjugate to $x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{20}(t_{20}+u_1+u_2)x_{21}(t_{21}-u_5)x_{22}(t_{22}+u_3-u_5)x_{23}(t_{23}+u_3)x_{24}(t_{24}+u_2-u_6)x_{26}(t_{26}+t_{22}u_1+t_{21}u_2-t_{20}u_5-u_1u_5-u_2u_5+u_8)x_{27}(t_{27}+t_{23}u_1-t_{21}u_6+u_5u_6+u_8-u_{12})x_{29}(t_{29}+t_{23}u_2+t_{24}u_3+u_2u_3-t_{22}u_6-u_3u_6+u_5u_6-u_{12})x_{33}(t_{33}+t_{29}u_1+t_{27}u_2+t_{23}u_1u_2-t_{26}u_6-t_{22}u_1u_6-t_{21}u_2u_6+t_{20}u_5u_6+u_1u_5u_6+u_2u_5u_6+t_{24}u_8+u_2u_8-u_6u_8-t_{20}u_{12}-u_1u_{12}-u_2u_{12})g$ for some $g\in U(I(\alpha_{25},\alpha_{28}))$. Thus we take $u_5=t_{21}$, $u_3=t_{21}-t_{22}$, $u_1=-t_{20}-u_2$, $u_6=t_{24}+u_2$. Then the element x is conjugate to $x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(f_1)x_{26}(t_{26}-t_{20}t_{22}-t_{22}u_2+t_{21}u_2+u_8)x_{27}(t_{27}-t_{23}t_{20}-t_{23}u_2+u_8-u_{12})x_{29}(t_{29}+f_1u_2+t_{21}t_{24}-t_{22}t_{24}-u_{12})x_{33}(t_{33}-t_{20}t_{29}-t_{24}t_{26}+t_{20}t_{22}t_{24}+u_2(-t_{29}+t_{27}-t_{20}t_{23}-t_{26}+t_{22}t_{24}+t_{20}t_{22}-t_{21}t_{24})+u_2^2(-t_{23}+t_{22}-t_{21}))g$. Furthermore we take $u_8=-t_{26}+t_{20}t_{22}+t_{22}u_2-t_{21}u_2$, $u_{12}=t_{29}+f_1u_2+t_{21}t_{24}-t_{22}t_{24}$. Then the element x is conjugate to $x_{14}(1)x_{15}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(f_1)x_{27}(f_2-2f_1u_2)x_{33}(t_{33}-t_{20}t_{29}-t_{24}t_{26}+t_{20}t_{22}t_{24}+f_2u_2+f_1u_2^2)g$. By assumptions, we may assume $f_1=1$. By the action of $U(I(\alpha_4,\alpha_7))$, we can take $g=1$. This shows $x \underset{c}{\sim} y_{44}$ (resp. $x \underset{c}{\sim} y_{47}$) in the case $\mathrm{ch}(K)=2$ (resp. $\mathrm{ch}(K)\neq 2$). On the other hand, by Lemma 8, we get $W(x)\subseteqq\langle w_1w_2w_6\rangle$. Thus we put $P=B\langle w_1, w_2, w_3, w_5, w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=\mathrm{Ru}(P)$, $V_1=U(I(\alpha_{25}, \alpha_{28}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Since $Z_R(x)/Z_R(x)^\circ$ is connected, we get $Z(x)\cong\overline{Z(x)}$. From above facts, we can prove the lemma.

LEMMA 22. *Let* $I_{12}=I(\alpha_1, \alpha_{15}, \alpha_{16}, \alpha_{18}, \alpha_{19})$, *then* $G'(I_9)=G(y_{38})=G(I_9)-\{G(I_{11})\cup G(I_{12})\}$. *Let* x *be an element of* U *such that* $I(x)=I_{12}$, *then* $x\underset{c}{\sim}y_{52}=x_1(1)x_{15}(1)x_{16}(1)x_{18}(1)x_{19}(1)$. *Furthermore a k-rational point in* $G(y_{52})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$$y_{52}=x_1(1)x_{15}(1)x_{16}(1)x_{18}(1)x_{19}(1)\ ,$$
$$y_{53}=x_1(1)x_{15}(1)x_{16}(1)x_{18}(1)x_{19}(1)x_{49}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(y_{52})\subset B\langle w_4w_7, w_{17}\rangle B$, $Z(y_{52})\cong Z_{(2,p)}$, $L(y_{52})=2A_1$, $|Z_{G(k)}(y_i)|=(2, p)(q^2-1)_2q^{17}$ $(i=52, 53)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{52}$. By Lemma 8, we get $W(y_{52})\subseteqq\langle w_4w_7, w_{17}\rangle$. Thus we put $P=B\langle w_2, w_4, w_5, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=U(I(\alpha_8, \alpha_{23}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{52})$ and $\overline{Z(y_{52})}=1$. Since $Z_R(y_{52})/Z_R(y_{52})^\circ$ is of order $(2, p)$, we get $Z(y_{52})\cong Z_{(2,p)}$. Now the proof is easy.

LEMMA 23. *Let* $I_{13}=I(\alpha_8, \alpha_{15}, \alpha_{16}, \alpha_{24}, \alpha_{25})$, *then* $G'(I_{10})=G(y_{41})=G(I_{10})-\{G(I_{11})\cup G(I_{12})\}$ *and* $G'(I_{12})=G(y_{52})=G(I_{12})-G(I_{13})$.

PROOF. Let x be an element of U such that $I(x)=I_{10}$ and $x\in G(y_{41})$, then x is conjugate to an element of $U(I_{12})$. By Lemma 1, we can prove the lemma.

LEMMA 24. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I(\alpha_8, \alpha_{15}, \alpha_{16}, \alpha_{24}, \alpha_{25})$. *Suppose that the element* x *satisfies the following conditions*:

i) *If* $\mathrm{ch}(K)=2$, *then* $f_1=t_{20}t_{16}+t_{21}t_{15}+t_8t_{28}-t_{14}t_{22}\neq 0$.

ii) *If* $\mathrm{ch}(K)\neq 2$, *then* $f_2=4(t_{32}t_8+t_{20}t_{21}-t_{14}t_{26})t_{15}t_{16}-f_1^2\neq 0$. *Then the element* x *is conjugate to* $y_{54}=x_8(1)x_{15}(1)x_{16}(1)x_{20}(1)x_{24}(1)x_{25}(1)$. *Furthermore,*

1) *Suppose* $\mathrm{ch}(K)=2$. *A k-rational point in* $G(y_{54})$ *is conjugate to* y_{54} *or* $y_{55}=x_8(1)x_{15}(1)x_{16}(1)x_{20}(1)x_{32}(\eta)x_{24}(1)x_{25}(1)$.

2) *Suppose* $\mathrm{ch}(K)\neq 2$. *A k-rational point in* $G(y_{54})$ *is conjugate in* $G(k)$ *to* y_{56} *or* y_{57}, *where*

$$y_{56}=x_8(1)x_{15}(1)x_{16}(1)x_{32}(1)x_{24}(1)x_{25}(1)\ ,$$
$$y_{57}=x_8(1)x_{15}(1)x_{16}(1)x_{32}(\zeta)x_{24}(1)x_{25}(1)\ .$$

In the both cases, $Z_G(x) \subseteqq B\langle w_2w_5w_7\rangle B$, $Z(x) \cong Z_2$, $L(x)=A_1$, $|Z_{G(k)}(y_i)| = 2(q^2-1)q^{21}(i=54, 55, 56, 57)$.

Proof. By the action of B, $x \underset{c}{\sim} y_{54}$. By Lemma 8, $W(x) \subseteqq \langle w_2w_5w_7\rangle$. Thus we put $P=B\langle w_2, w_5, w_7\rangle B$, $R=U(I(\alpha_3, \alpha_6))$, $V=U(\alpha_8, \alpha_{10}, \alpha_{18}))$, $V_1=U(I(\alpha_{23}, \alpha_{37}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Since $Z_R(x)/Z_R(x)^\circ$ is connected, $Z(x) \cong \overline{Z(x)}$. Now the proof is easy.

Lemma 25. *Let* $I_{14}=I(\alpha_{13}, \alpha_{15}, \alpha_{17}, \alpha_{21}, \alpha_{23})$, *then* $G'(I_{11})=G(y_{44})=G(I_{11})-\{G(I_{13}) \cup G(I_{14})\}$. *Let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_{14}$ *and* $t_{17}(t_{20}t_{23}-t_{27}t_{15})+t_{15}t_{21}t_{24} \neq 0$. *Then* $x \underset{c}{\sim} y_{58}=x_{15}(1)x_{20}(1)x_{21}(1)x_{23}(1)x_{13}(1)x_{17}(1)$. *Furthermore a k-rational point in* $G(y_{58})$ *is conjugate in* $G(k)$ *to* y_{58} *or* $y_{59}=x_{15}(1)x_{20}(\zeta)x_{21}(1)x_{23}(1)x_{13}(1)x_{17}(1)$. $Z_G(y_{58}) \subset B\langle w_4\rangle B$, $Z(y_{58}) \cong Z_{(2,p-1)}$, $L(y_{58})=A_1$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)q^{21}$ $(i=58, 59)$.

Proof. By the action of B, $x \underset{c}{\sim} y_{58}$. By Lemma 8, we get $W(y_{58}) \subseteqq \langle w_4\rangle$. Furthermore, by easy computations, $Z_G(y_{58})=Z(G)LZ_R(y_{58})$, where $L=\langle X_4, X_{-4}\rangle$, $R=\mathrm{Ru}(B\langle w_4\rangle B)$. From this facts, we get the lemma.

Lemma 26. *Let* $I_{15}=I(\alpha_{12}, \alpha_{13}, \alpha_{20}, \alpha_{21}, \alpha_{28})$, $I_{16}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28}, \alpha_7)$, $I_{17}=I(\alpha_{14}, \alpha_{19}, \alpha_{22}, \alpha_{23}, \alpha_{24})$. *Then* $G'(I_{14})=G(y_{58})=G(I_{14})-\{G(I_{15}) \cup G(I_{16}) \cup G(I_{17})\}$ *and* $G'(I_{13})=G(y_{54})=G(I_{13})-\{G(I_{15}) \cup G(I_{17})\}$. *Let x be an element of U such that* $I(x)=I(\alpha_{12}, \alpha_{13}, \alpha_{20}, \alpha_{21}, \alpha_{28})$, *then* $x \underset{c}{\sim} y_{60}=x_{28}(1)x_{12}(1)x_{13}(1)x_{20}(1)x_{21}(1)$. $Z_G(y_{60}) \subset B\langle w_2w_5w_7, w_3\rangle B$, $Z(y_{60})=1$, $L(y_{60})=2A_1$, $|Z_{G(k)}(y_{60})|=(q^2-1)^2q^{21}$. *Let y be an element of U such that* $I(y)=I_{16}$, *then* $y \underset{c}{\sim} y_{61}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_{28}(1)x_7(1)$. *Furthermore, a k-rational point in* $G(y_{61})$ *is conjugate in* $G(k)$ *to* y_{61} *or* $y_{62}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_{28}(1)x_7(\zeta)$. $Z_G(y_{61}) \subset B\langle w_2w_3w_5\rangle B$, $Z(y_{61}) \cong Z_{(2,p-1)}$, $L(y_{61})=A_1$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)q^{23}(i=61, 62)$.

Let $z=\prod x_i(u_i)$ *be an element of* U *such that* $I(z)=I_{17}$ *and* $u_{19}u_{28}+u_{22}u_{25} \neq 0$, *then* $z \underset{c}{\sim} y_{63}=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{19}(1)$. *Furthermore a k-rational point in* $G(y_{63})$ *is conjugate in* $G(k)$ *to* y_{63} *or* $y_{64}=x_{14}(1)x_{22}(1)x_{23}(\zeta)x_{24}(1)x_{25}(1)x_{19}(1)$. $Z_G(y_{63}) \subset B\langle w_{15}w_{13}\rangle B$, $Z(y_{63}) \cong Z_{(2,p-1)}$, $L(y_{63})=A_1|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)q^{23}(i=63, 64)$.

Proof. By the action of B, $z \underset{c}{\sim} y_{63}$. By Lemma 8, we get $W(y_{63}) \subseteqq \langle w_{15}w_{13}\rangle$. Thus if we put $P=B\langle w_2, w_3, w_4, w_6, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=U(I(\alpha_{21}, \alpha_{40}))$, then (P, R, V, V_1) gives a structure of $Z_G(y_{63})$ and $Z_R(y_{63})/Z_R(y_{63})^\circ$ is connected. Now the proof is easy.

Lemma 27. *Let* $I_{18}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_7)$, $I_{19}=I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25})$ *and* $I_{20}=I(\alpha_8, \alpha_{24}, \alpha_{25}, \alpha_{28})$. *Then* $G'(I_{16})=G(y_{61})=G(I_{16})-\{G(I_{18}) \cup G(I_{19})\}$,

$G'(I_{15})=G(y_{60})=G(I_{15})-G(I_{19})$, $G'(I_{17})=G(y_{63})=G(I_{17})-\{G(I_{19})\cup G(I_{20})\}$.

Let x be an element of U such that $I(x)=I_{18}$, then $x \underset{c}{\sim} y_{65}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_7(1)$. Furthermore, a k-rational point in $G(y_{65})$ is conjugate in $G(k)$ to y_{65} or $y_{66}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_7(\zeta)$. $Z_G(y_{65})\subset B\langle w_2w_3w_5, w_4\rangle B$, $Z(y_{65})\cong Z(G)$, $L(y_{65})=G_2$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)(q^6-1)q^{23}(i=65, 66)$.

Let y be an element of U such that $I(x)=I_{19}$, then $x \underset{c}{\sim} y_{67}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)$. Furthermore, a k-rational point in $G(y_{67})$ is conjugate in $G(k)$ to y_{67}. $Z_G(y_{67})\subset B\langle w_8w_2w_6w_{19}\rangle B$, $Z(y_{67})=1$, $L(y_{67})=A_1$, $|Z_{G(k)}(y_{67})|=(q^2-1)q^{25}$.

Let $z=\prod x_i(u_i)$ be an element of U such that $I(x)=I_{20}$ and $t_{24}t_{30}-t_{25}t_{29}\neq 0$, then $x \underset{c}{\sim} y_{68}=x_8(1)x_{28}(1)x_{24}(1)x_{25}(1)x_{30}(1)$. Furthermore, a k-rational point in $G(y_{68})$ is conjugate in $G(k)$ to one of the following elements:

$$y_{68}=x_8(1)x_{28}(1)x_{24}(1)x_{25}(1)x_{30}(1)\ ,$$
$$y_{69}=x_8(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$y_{70}=x_8(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{29}(1)x_{36}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(y_{68})\subset B\langle w_2w_7, w_5\rangle B$, $Z(y_{68})\cong Z_2$, $L(y_{68})=T_1+A_1$, $|Z_{G(k)}(y_{68})|=2(q-1)(q^2-1)q^{24}$, $|Z_{G(k)}(y_i)|=2(q+1)(q^2-1)q^{24}(i=69, 70)$.

PROOF. By the action of B, $x \underset{c}{\sim} y_{65}$. By Lemma 8, $W(y_{65})\subseteqq\langle w_4, w_2w_3w_5\rangle$. Thus we put $P=B\langle w_2, w_3, w_4, w_5\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{65})$. By the action of B, $y \underset{c}{\sim} y_{67}$. By Lemma 8, $W(y_{67})\subseteqq\langle w_8w_2w_6w_{19}\rangle$. Thus if we put $P=B\langle w_1, w_2, w_3, w_5, w_6, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$, then (P, R, V, V_1) gives a structure of $Z_G(y_{67})$. By the action of B, $z \underset{c}{\sim} y_{68}$. By Lemma 8, $W(y_{68})\subseteqq\langle w_2w_7, w_5\rangle$. Thus we put $P=B\langle w_2, w_5, w_7\rangle B$, $R=U(I(\alpha_1, \alpha_4, \alpha_6))$, $V=U(I(\alpha_8, \alpha_{18}, \alpha_{28}))$, $V_1=U(I(\alpha_{14}, \alpha_{35}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{68})$. Now the proof is easy.

LEMMA 28. *Let $I_{21}=I(\alpha_{20}, \alpha_{21}, \alpha_{24}, \alpha_{28}, \alpha_{30})$, $I_{22}=I(\alpha_{20}, \alpha_{21}, \alpha_{24}, \alpha_{30})$, $I_{23}=I(\alpha_1, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31})$. Then $G'(I_{18})=G(y_{65})=G(I_{18})-G(I_{22})$, $G'(I_{19})=G(y_{67})=G(I_{19})-G(I_{21})$ and $G'(I_{20})=G(y_{68})=G(I_{20})-\{G(I_{21})\cup G(I_{23})\}$.*

Let x be an element of U such that $I(x)=I_{22}$, then $x \underset{c}{\sim} y_{71}=x_{20}(1)x_{21}(1)x_{24}(1)x_{30}(1)$. Furthermore a k-rational point in $G(y_{71})$ is conjugate in $G(k)$ to one of the following elements:

$$y_{71}=x_{20}(1)x_{21}(1)x_{24}(1)x_{30}(1)\ ,$$
$$y_{72}=x_{20}(1)x_{21}(1)x_{28}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$y_{73}=x_{20}(1)x_{21}(1)x_{28}(1)x_{24}(1)x_{25}(1)x_{29}(1)x_{36}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(y_{71})\subset B\langle w_4,\ w_2w_3w_5w_7\rangle B,\ Z(y_{71})\cong Z_2,\ L(y_{71})=T_1+A_2,\ |Z_{G(k)}(y_{71})|=2(q-1)(q^2-1)(q^3-1)q^{27},\ |Z_{G(k)}(y_i)|=2(q+1)(q^2-1)(q^3+1)q^{27}(i=72,\ 73)$.

Let y be an element of U such that $I(y)=I_{21}$, then $y\underset{c}{\sim}y_{74}=x_{20}(1)x_{21}(1)x_{24}(1)x_{28}(1)x_{30}(1)$. Furthermore a k-rational point in $G(y_{74})$ is conjugate in $G(k)$ to one of the following elements:

$$y_{74}=x_{20}(1)x_{21}(1)x_{24}(1)x_{28}(1)x_{30}(1)\ ,$$
$$y_{75}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)\neq 2\ ,$$
$$y_{76}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{29}(1)x_{36}(\eta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)=2\ .$$

$Z_G(y_{74})\subset B\langle w_2w_3w_5w_7\rangle B,\quad Z(y_{74})\cong Z_2,\quad L(y_{74})=2T_1,\quad |Z_{G(k)}(y_{74})|=2(q-1)^2q^{27},\ |Z_{G(k)}(y_i)|=2(q+1)^2q^{27}(i=75,\ 76)$.

Let z be an element of U such that $I(z)=I_{23}$, then $z\underset{c}{\sim}y_{77}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)$. Furthermore, a k-rational point in $G(y_{77})$ is conjugate in $G(k)$ to one of the following elements:

$$y_{77}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)\ ,$$
$$y_{78}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(\zeta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)\neq 2\ ,$$
$$y_{79}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{53}(\eta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)=2\ .$$

$Z_G(y_{77})\subset B\langle w_4w_6,\ w_5\rangle B,\quad Z(y_{77})\cong Z_2,\quad L(y_{77})=B_2,\quad |Z_{G(k)}(y_i)|=2(q^2-1)(q^4-1)q^{25}\ (i=77,\ 78,\ 79)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{71}$. By Lemma 8, $W(y_{71})\subseteq\langle w_4,\ w_2w_3w_5w_7\rangle$. Thus we put $P=B\langle w_2,\ w_3,\ w_4,\ w_5,\ w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{71})$. By the action of B, $y\underset{c}{\sim}y_{74}$. By Lemma 8, $W(y_{74})\subseteq\langle w_2w_3w_5w_7\rangle$. Thus we put $P=B\langle w_2,\ w_3,\ w_5,\ w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{14},\ \alpha_{18},\ \alpha_{23}))$, $V_1=U(I(\alpha_{27},\ \alpha_{32},\ \alpha_{35}))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{74})$. By the action of B, $z\underset{c}{\sim}y_{77}$. By Lemma 8, $W(y_{77})\subseteq\langle w_4w_6,\ w_5\rangle$. Thus we put $P=B\langle w_4,\ w_5,\ w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_1,\ \alpha_{15},\ \alpha_{30},\ \alpha_{31}))$, $V_1=U(I(\alpha_8,\ \alpha_{36}))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{77})$. Now the proof is easy.

LEMMA 29. *Let $I_{24}=I(\alpha_{27},\ \alpha_{29},\ \alpha_{30},\ \alpha_{31},\ \alpha_{32})$, $I_{25}=I(\alpha_{26},\ \alpha_{27},\ \alpha_{28},\ \alpha_{29},\ \alpha_{30},\ \alpha_{31})$, $I_{26}=I(\alpha_1,\ \alpha_{28},\ \alpha_{29},\ \alpha_{30})$. Then $G'(I_{22})=G(y_{71})=G(I_{22})-G(I_{24})$, $G'(I_{21})=G(y_{74})=G(I_{21})-\{G(I_{22})\cup G(I_{25})\}$, $G'(I_{23})=G(y_{77})=G(I_{23})-\{G(I_{25})\cup G(I_{26})\}$.*

Let x be an element of U such that $I(x)=I_{24}$.

1) *Suppose* $\mathrm{ch}(K)\neq 2$. *Then $x\underset{c}{\sim}y_{80}=x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{32}(1)$. Furthermore, a k-rational point in $G(y_{80})$ is conjugate in $G(k)$ to y_{80} or $y_{81}=x_{27}(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{39}(-\zeta)$. $Z_G(y_{80})\subset B\langle w_5,\ w_8w_2w_7\rangle B$, $Z(y_{80})\cong Z_2$, $L(y_{80})=T_1+A_1$, $|Z_{G(k)}(y_{80})|=2(q-1)(q^2-1)q^{32}$, $|Z_{G(k)}(y_{81})|=2(q+1)(q^2-1)q^{32}$.*

2) *Suppose* $\mathrm{ch}(K)=2$. *Then* x *is conjugate to* y_{80} *or* $y_{82}=x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{32}(1)x_{37}(1)$. $Z_G(y_{82})\subset B\langle w_5\rangle B$, $Z_G(y_{80})\subset B\langle w_5,\ w_8w_2w_7\rangle B$, $Z(y_{80})=Z(y_{82})=1$, $L(y_{80})=2A_1$, $L(y_{82})=A_1$ $|Z_{G(k)}(y_{80})|=(q^2-1)^2q^{33}$, $|Z_{G(k)}(y_{82})|=(q^2-1)q^{33}$.

Let y *be an element of* U *such that* $I(x)=I_{25}$. *Then* $x\underset{c}{\sim}y_{83}=x_{29}(1)x_{27}(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)$. *Furthermore a* k*-rational point in* $G(y_{83})$ *is conjugate in* $G(k)$ *to* y_{83} *or* $y_{84}=x_{26}(1)x_{27}(1)x_{28}(1)x_{29}(\zeta)x_{30}(1)x_{31}(1)$. $Z_G(y_{83})\subset B\langle w_{20}w_{10}w_{13}\rangle B$, $Z(y_{83})\cong Z_{(2,p-1)}$, $L(y_{83})=A_1$, $|Z_{G(k)}(y_i)|=(2,\ p-1)(q^2-1)q^{31}(i=83,\ 84)$.

Let z *be an element of* U *such that* $I(x)=I_{26}$. *Then* $z\underset{c}{\sim}y_{85}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)$. *Furthermore a* k*-rational point in* $G(y_{85})$ *is conjugate in* $G(k)$ *to* y_{85} *or* $y_{86}=y_{85}x_{53}(\eta)$. $Z_G(y_{85})\subset B\langle w_2w_7,\ w_4w_6,\ w_5\rangle B$, $Z(y_{85})\cong Z_{(2,p)}$, $L(y_{85})=C_3$, $|Z_{G(k)}(y_j)|=(2,\ p)(q^2-1)(q^4-1)(q^6-1)q^{25}(i=85,\ 86)$.

PROOF. By the action of B, the element x is conjugate to $y_{80}x_{37}(a)$ for some $a\in K$. By Lemma 8, $W(x)\subseteqq\langle w_5,\ w_8w_2w_7\rangle$. Thus we put $P=B\langle w_1,\ w_2,\ w_3,\ w_5,\ w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{18},\ \alpha_{28}))$, $V_1=U(I(\alpha_{35}))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{82})$. If $\mathrm{ch}(K)\neq 2$, then $y_{82}\underset{c}{\sim}y_{80}$. By direct calculations, $Z_{P/R}(y_{82}V_1)$ is generated by $\mathfrak{X}_5$, $\mathfrak{X}_{-5}$, $x_1(a)x_2(a)x_7(a)x_8(a^2)V_1$ and $Z_{P/R}(y_{80}V_1)$ is generated by $\mathfrak{X}_5$, $\mathfrak{X}_{-5}$, $x_1(a)x_2(a)x_7(a)x_8(a^2)$, $w_8w_2w_7$. This shows the first assertion.

By the action of B, $y\underset{c}{\sim}y_{83}$. By Lemma 8, $W(y_{83})\subseteqq\langle w_{20}w_{10}w_{13}\rangle$. Thus we put $P=B\langle w_1,\ w_2,\ w_3,\ w_4,\ w_6,\ w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{83})$.

By the action of B, $z\underset{c}{\sim}y_{85}$. By Lemma 8, $W(y_{85})\subseteqq\langle w_2w_7,\ w_4w_6,\ w_5\rangle$. Thus we put $P=B\langle w_2,\ w_4,\ w_5,\ w_6,\ w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then $(P,\ R,\ V,\ V_1)$ gives a structure of $Z_G(y_{85})$. Now the proof is easy.

LEMMA 30. *Let* $I_{27}=I(\alpha_{28},\ \alpha_{30},\ \alpha_{31},\ \alpha_{33})$, $I_{28}=I(\alpha_{28},\ \alpha_{30},\ \alpha_{33})$. *Then* $G'(I_{25})=G(y_{83})=G(I_{25})-G(I_{24})$, $G(I_{24})=G(y_{80})\cup G(y_{82})\cup G(I_{27})$. $G(y_{82})$ *is open in* $G(I_{24})$. $G'(I_{26})=G(y_{85})=G(I_{26})-G(I_{28})$.

Let $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=I_{27}$ *and* $t_{30}t_{32}-t_{28}t_{34}\neq 0$. *Then* $x\underset{c}{\sim}y_{87}=x_{28}(1)x_{30}(1)x_{31}(1)x_{33}(1)x_{34}(1)$. *Furthermore a* k*-rational point in* $G(y_{87})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$y_{87}=x_{28}(1)x_{30}(1)x_{31}(1)x_{33}(1)x_{34}(1)$,

$y_{88}=x_{28}(1)x_{30}(1)x_{31}(\zeta)x_{33}(1)x_{34}(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{89}=x_{26}(1)x_{28}(1)x_{29}(1)x_{38}(\zeta)x_{34}(1)x_{31}(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{90}=x_{26}(1)x_{28}(1)x_{29}(1)x_{38}(\zeta)x_{34}(1)x_{31}(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{91}=x_{26}(1)x_{28}(1)x_{29}(1)x_{32}(1)x_{38}(\eta)x_{34}(1)x_{31}(1)$, *when* $\mathrm{ch}(K)=2$.

$Z_G(y_{87})\subset B\langle w_1, w_4, w_5, w_6\rangle B$, $Z(y_{87})\cong Z_2\times Z_{(2,p-1)}$, $L(y_{87})=2A_1$, $|Z_{G(k)}(y_i)|=2(2, p-1)(q^2-1)^2q^{33}(i=87, 88)$, $|Z_{G(k)}(y_j)|=2(2, p-1)(q^4-1)q^{33}(j=89, 90, 91)$.

Let $y=\prod x_i(u_i)$ be an element of U such that $I(y)=I_{28}$ and $u_{30}u_{32}-u_{28}u_{34}\neq 0$. Then $y\underset{c}{\sim}y_{92}=x_{28}(1)x_{30}(1)x_{33}(1)x_{34}(1)$. Furthermore a k-rational point in $G(y_{92})$ is conjugate in $G(k)$ to one of the following elements:

$y_{92}=x_{28}(1)x_{30}(1)x_{33}(1)x_{34}(1)$,

$y_{93}=x_{26}(1)x_{27}(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{41}(-1)x_{44}(\tau)$,

$y_{94}=x_{26}(1)x_{27}(1)x_{28}(1)x_{36}(1)x_{44}(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{95}=x_{27}(1)x_{29}(1)x_{30}(1)x_{36}(1)x_{39}(\eta)x_{32}(1)$, *when* $\mathrm{ch}(K)=2$.

$Z_G(y_{92})\subset B\langle w_1, w_2w_7, w_4w_6, w_5\rangle B, Z(y_{92})\cong S_3, L(y_{92})=3A_1, |Z_{G(k)}(y_{92})|=6(q^2-1)^3q^{33}$, $|Z_{G(k)}(y_{93})|=3(q^6-1)q^{33}$, $|Z_{G(k)}(y_i)|=2(q^2-1)(q^4-1)q^{33}(i=94, 95)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{87}$. By Lemma 8, $W(y_{87})\subseteqq\langle w_1w_4w_6, w_5\rangle$. Thus we put $P=B\langle w_1, w_4, w_5, w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{15}, \alpha_{30}, \alpha_{31}))$, $V_1=U(I(\alpha_{36}, \alpha_{37}))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{87})$.

By the action of B, $y\underset{c}{\sim}y_{92}$. By Lemma 8, $W(y_{92})\subseteqq\langle w_1, w_2w_7, w_4w_6, w_5\rangle$. Thus we put $P=B\langle w_1, w_2, w_4, w_5, w_6, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(y_{92})$. Now the proof is easy.

LEMMA 31. *Let $I_{29}=I(\alpha_{30}, \alpha_{31}, \alpha_{32}, \alpha_{33}, \alpha_{40})$, $I_{30}=I(\alpha_{26}, \alpha_{27}, \alpha_{40}, \alpha_{41})$, $I_{31}=I(\alpha_{30}, \alpha_{31}, \alpha_{32}, \alpha_{33})$. Then $G'(I_{27})=G(y_{87})=G(I_{27})-\{G(I_{28})\cup G(I_{29})\}$, $G'(I_{28})=G(y_{92})=G(I_{28})-G(I_{30})$, $G'(I_{29})=G(y_{96})=G(I_{29})-\{G(I_{30})\cup G(I_{31})\}$.*

If $\mathrm{ch}(K)=2$, then the closure $\overline{G(y_{80})}$ of $G(y_{80})$ is the disjoint union of $G(y_{80})$ and $G(I_{30})$.

Let x be an element of U such that $I(x)=I_{29}$. Then $x\underset{c}{\sim}y_{96}=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(1)x_{40}(1)$. Furthermore a k-rational point in $G(y_{96})$ is conjugate in $G(k)$ to y_{96} or $y_{97}=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(\zeta)x_{40}(1)$. $Z_G(y_{96})\subset B\langle w_{15}, w_4w_6\rangle B$, $Z(y_{96})\cong Z_{(2,p-1)}$, $L(y_{96})=2A_1$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)^2q^{35}(i=96, 97)$.

Let y be an element of U such that $I(y)=I_{30}$. Then $y\underset{c}{\sim}y_{98}=x_{26}(1)x_{27}(1)x_{40}(1)x_{41}(1)$. $Z_G(y_{98})\subset B\langle w_2w_6, w_3, w_{19}\rangle B$, $Z(y_{98})=1$, $L(y_{98})=3A_1$, $|Z_{G(k)}(y_{98})|=(q^2-1)^3q^{35}$.

Let z be an element of U such that $I(z)=I_{31}$. Then $z\underset{c}{\sim}y_{99}=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(1)$. Furthermore a k-rational point in $G(y_{99})$ is conjugate in $G(k)$ to y_{99} or $y_{100}=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(\zeta)$. $Z_G(y_{99})\subset B\langle w_4w_6, w_5, w_{15}\rangle B$, $Z(y_{99})\cong Z_{(2,p-1)}$, $L(y_{99})=B_3$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)(q^4-1)(q^6-1)q^{35}(i=99, 100)$.

PROOF. By the action of B, $x\underset{c}{\sim}y_{96}$, $y\underset{c}{\sim}y_{98}$, $z\underset{c}{\sim}y_{99}$. By Lemma 8,

$W(y_{96})\subseteq\langle w_4w_6, w_{15}\rangle$, $W(y_{98})\subseteq\langle w_2w_6, w_3, w_{19}\rangle$, $W(y_{99})\subseteq\langle w_4w_6, w_5, w_{15}\rangle$. If we put $P=B\langle w_2, w_3, w_4, w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{19}, \alpha_{21}, \alpha_{40}))$, $V_1=U(I(\alpha_{19}, \alpha_{21}, \alpha_{40}))$, then (P, R, V, V_1) gives a structure of $Z_G(y_{96})$. If we put $P=B\langle w_2, w_3, w_5, w_6, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{14}, \alpha_{23}))$, $V_1=U(I(\alpha_{32}))$, then (P, R, V, V_1) gives a structure of $Z_G(y_{98})$. If we put $P=B\langle w_2, w_3, w_4, w_5, w_6\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$, then (P, R, V, V_1) gives a structure of $Z_G(y_{99})$.

We consider the Zariski closure $\overline{G(y_{80})}$. By general theory, $\overline{G(y_{80})}=G(\overline{B(y_{80})})$. Furthermore, $\overline{B(y_{80})}=\{v=\prod x_i(v_i)\in U(I_{24})|f(v)=0\}$, where $f(v)=v_{27}v_{32}v_{36}+v_{30}v_{32}v_{33}+v_{27}v_{31}v_{37}+v_{29}v_{32}v_{34}$. Let $u=\prod x_i(u_i)$ be an element of $\overline{B(y_{80})}-B(y_{80})$. Then $u_{27}u_{29}u_{30}u_{31}u_{32}=0$ and $f(u)=0$. Suppose $u_{27}=0$. Then $u_{27}(u_{30}u_{33}+u_{29}u_{34})=0$. If $u_{27}=u_{32}=0$, the element u is conjugate to an element of $U(I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40}))$. If $u_{27}=u_{30}u_{33}+u_{29}u_{34}=0$, the element u is conjugate to an element of $U(I_{30})$. Suppose $u_{29}=0$. Then we get $u_{27}u_{31}u_{37}+u_{27}u_{32}u_{36}+u_{20}u_{32}u_{33}=0$. Therefore the element u is conjugate to an element of $U(I(\alpha_{30}, \alpha_{31}, \alpha_{33}, \alpha_{35}, \alpha_{37}))$. By Lemma 1, we get $u\in G(I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40}))$. Suppose $u_{30}=0$. Then the element u is conjugate to an element of $U(I(\alpha_{29}, \alpha_{31}, \alpha_{34}, \alpha_{37}))$. By Lemma 1, $u\in G(I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40}))$. Suppose $u_{31}=0$. If $u_{31}=u_{32}=0$, by Lemma 1, we get $u\in G(I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40}))$. If $u_{31}=u_{27}u_{36}+u_{30}u_{33}+u_{29}u_{34}=0$, the element u is conjugate to an element of $U(I(\alpha_{28}, \alpha_{33}, \alpha_{34}, \alpha_{36}))$. Suppose $u_{32}=0$ and $u_{27}u_{29}u_{30}u_{31}\neq 0$. Then $u_{37}=0$. By Lemma 1, we get $u\in G(I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40}))$. Since $I_{30}\sim I(\alpha_{27}, \alpha_{32}, \alpha_{36}, \alpha_{40})\supset I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40})$, we get $\overline{G(y_{80})}\subseteq G(y_{80})\cup G(I_{30})$. The opposite inclusion is clear. Now the proof is easy.

LEMMA 32. *Let* $I_{32}=I(\alpha_{20}, \alpha_{21}, \alpha_{49})$, $I_{33}=I(\alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{38}, \alpha_{40})$, $I_{34}=I(\alpha_{34}, \alpha_{36}, \alpha_{38}, \alpha_{40})$. *Then* $G'(I_{30})=G(y_{98})=G(I_{30})-(G(I_{32})\cup G(I_{33}))$, $G'(I_{31})=G(y_{99})=G(I_{31})-\{G(I_{32})\cup G(I_{34})\}$.

Let x be an element of U such that $I(x)=I_{32}$. *Then* $x\underset{c}{\sim}y_{101}=x_{20}(1)x_{21}(1)x_{49}(1)$. $Z_G(y_{101})\subset B\langle w_2w_5, w_4, w_3, w_7\rangle B$, $Z(y_{101})=1$, $L(y_{101})=A_1+B_3$, $|Z_{G(k)}(y_{101})|=(q^2-1)^2(q^4-1)(q^6-1)q^{35}$.

Let y be an element of U such that $I(x)=I_{33}$. *Then* $y\underset{c}{\sim}y_{102}=x_{34}(1)x_{36}(1)x_{37}(1)x_{38}(1)x_{40}(1)$. $Z_G(y_{102})\subset B\langle w_1w_2w_5, w_1w_4w_7w_{17}\rangle B$, $Z(y_{102})=1$, $L(y_{102})=2A_1$, $|Z_{G(k)}(y_{102})|=(q^2-1)^2q^{39}$.

Let z be an element of U such that $I(x)=I_{34}$. *Then* $z\underset{c}{\sim}y_{103}=x_{34}(1)x_{36}(1)x_{38}(1)x_{40}(1)$. $Z_G(y_{103})\subset B\langle w_1w_2w_5, w_{10}, w_9w_{11}w_7\rangle B$, $L(y_{103})=A_1+G_2$, $|Z_{G(k)}(y_{103})|=(q^2-1)^2(q^6-1)q^{39}$.

The proof is easy.

LEMMA 33. *Let* $I_{35}=I(\alpha_{37}, \alpha_{38}, \alpha_{39}, \alpha_{40}, \alpha_{41})$, $I_{36}=I(\alpha_{42}, \alpha_{43}, \alpha_{44}, \alpha_{45})$. *Then* $G'(I_{33})=G(y_{102})=G(I_{33})-\{G(I_{35})\cup G(I_{34})\}$, $G'(I_{32})=G(y_{101})=G(I_{32})-G(I_{36})$, $G'(I_{35})=G(y_{104})=G(I_{35})-G(I_{36})$, $G'(I_{34})=G(y_{103})=G(I_{34})-G(I_{36})$.

Let x *be an element of* U *such that* $I(x)=I_{35}$. *Then* $x\underset{c}{\sim}y_{104}=x_{37}(1)x_{38}(1)x_{39}(1)x_{40}(1)x_{41}(1)$. *Furthermore a k-rational point in* $G(y_{104})$ *is conjugate in* $G(k)$ *to* y_{104} *or* $y_{105}=x_{37}(\zeta)x_{38}(1)x_{39}(1)x_{40}(1)x_{41}(1)$. $Z_G(y_{104})\subset B\langle w_3w_6, w_1w_{11}w_7\rangle B$, $Z(y_{104})\cong Z_{(2,p-1)}$, $L(y_{104})=G_2$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)(q^6-1)q^{41}$ $(i=104, 105)$.

Let y *be an element of* U *such that* $I(y)=I_{36}$. *Then* $y\underset{c}{\sim}y_{106}=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)$. $Z_G(y_{106})\subset B\langle w_2, w_8w_5w_7, w_{12}w_{13}\rangle B$, $Z(y_{106})=1$, $L(y_{106})=3A_1$, $|Z_{G(k)}(y_{106})|=(q^2-1)^3q^{45}$.

The proof is easy.

LEMMA 34. *Let* $I_{37}=I(\alpha_{44}, \alpha_{46}, \alpha_{49})$, $I_{38}=I(\alpha_{44}, \alpha_{46})$, $I_{39}=I(\alpha_{47}, \alpha_{48}, \alpha_{49}, \alpha_{53})$. *Then* $G'(I_{36})=G(y_{106})=G(I_{36})-G(I_{37})$, $G'(I_{37})=G(y_{107})-G(I_{37})-\{G(I_{38})\cup G(I_{39})\}$.

Let x *be an element of* U *such that* $I(x)=I_{37}$. *Then* $x\underset{c}{\sim}y_{107}=x_{44}(1)x_{46}(1)x_{49}(1)$. *Furthermore a k-rational point in* $G(y_{107})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$y_{107}=x_{44}(1)x_{46}(1)x_{49}(1)$,

$y_{108}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)x_{49}(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{109}=x_{42}(1)x_{43}(1)x_{44}(1)x_{46}(1)x_{51}(\eta)x_{49}(1)$, *when* $\mathrm{ch}(K)=2$.

$Z_G(y_{107})\subset B\langle w_3w_5w_7, w_2, w_{10}, w_{11}\rangle B$, $Z(y_{107})\cong Z_2$, $L(y_{107})=T_1+A_3$, $|Z_{G(k)}(y_{107})|=2(q-1)(q^2-1)(q^3-1)(q^4-1)q^{47}$, $|Z_{G(k)}(y_i)|=2(q+1)(q^2-1)(q^3-1)(q^4-1)q^{47}(i=108, 109)$.

Let y *be an element of* U *such that* $I(y)=I_{38}$. *Then* $y\underset{c}{\sim}y_{110}=x_{44}(1)x_{46}(1)$. *Furthermore a k-rational point in* $G(y_{110})$ *is conjugate in* $G(k)$ *to one of the following elements*:

$y_{110}=x_{44}(1)x_{46}(1)$,

$y_{111}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$,

$y_{112}=x_{42}(1)x_{43}(1)x_{44}(1)x_{46}(1)x_{51}(\eta)$, *when* $\mathrm{ch}(K)=2$.

$Z_G(y_{110})\subset B\langle w_6, w_{11}, w_2, w_{10}, w_{19}, w_3w_5w_7\rangle B$, $Z(y_{110})\cong Z_2$, $L(y_{110})=A_5$, $|Z_{G(k)}(y_{110})|=2(q^2-1)(q^3-1)(q^4-1)(q^5-1)(q^6-1)q^{47}$, $|Z_{G(k)}(y_i)|=2(q^2-1)(q^3+1)(q^4-1)(q^5+1)(q^6-1)q^{47}(i=111, 112)$.

Let z *be an element of* U *such that* $I(z)=I_{39}$. *Then* $z\underset{c}{\sim}y_{113}=x_{47}(1)x_{48}(1)x_{49}(1)x_{53}(1)$. *Furthermore a k-rational point in* $G(y_{113})$ *is conjugate in*

$G(k)$ *to* y_{113} *or* $y_{114}=x_{47}(\zeta)x_{48}(1)x_{49}(1)x_{53}(1)$. $Z_G(y_{113})\cap B\langle w_1w_6, w_3w_5, w_4\rangle B$, $Z(y_{113})\cong Z_{(2,p-1)}$, $L(y_{113})=C_3$. $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)(q^4-1)(q^6-1)q^{51}(i=113, 114)$.

The proof is easy.

LEMMA 35. *Let* $I_{40}=I(\alpha_{47}, \alpha_{48}, \alpha_{49})$, $I_{41}=I(\alpha_{53}, \alpha_{54}, \alpha_{55})$, $I_{42}=I(\alpha_{58}, \alpha_{59})$, $I_{43}=I(\alpha_{63})$. *Then* $G'(I_{38})=G(y_{110})=G(I_{38})-G(I_{41})$, $G'(I_{39})=G(y_{113})=G(I_{39})-\{G(I_{40})\cup G(I_{41})\}$.

Let x *be an element of* U *such that* $I(x)=I_{40}$. *Then* $x\underset{c}{\sim}y_{115}=x_{47}(1)x_{48}(1)x_{49}(1)$. *Furthermore a k-rational point in* $G(y_{115})$ *is conjugate in* $G(k)$ *to* y_{115} *or* $y_{116}=x_{47}(\zeta)x_{48}(1)x_{49}(1)$. $Z_G(y_{115})\subset B\langle w_1w_6, w_3w_5, w_4, w_2\rangle B$, $Z(y_{115})\cong Z_{(2,p-1)}$, $L(y_{115})=F_4$, $|Z_{G(k)}(y_i)|=(2, p-1)(q^2-1)(q^6-1)(q^8-1)(q^{12}-1)q^{51}$ $(i=115, 116)$.

Let y *be an element of* U *such that* $I(y)=I_{41}$. *Then* $y\underset{c}{\sim}y_{117}=x_{53}(1)x_{54}(1)x_{55}(1)$. $Z_G(y_{117})\subset B\langle w_2w_7, w_4w_6, w_1, w_5\rangle B$, $Z(y_{117})=1$, $L(y_{117})=C_3+A_1$, $|Z_{G(k)}(y_{117})|=(q^2-1)^2(q^4-1)(q^6-1)q^{55}$.

Let z *be an element of* U *such that* $I(z)=I_{42}$. *Then* $z\underset{c}{\sim}y_{118}=x_{58}(1)x_{59}(1)$. $Z_G(y_{118})\subset B\langle w_2w_5, w_4, w_3, w_1, w_7\rangle B$, $Z(y_{118})=1$, $L(y_{118})=B_4+A_1$, $|Z_{G(k)}(y_{118})|=(q^2-1)^2(q^4-1)(q^6-1)(q^8-1)q^{59}$.

The proof is easy.

LEMMA 36. $G'(I_{40})=G(y_{115})=G(I_{40})-G(I_{42})$, $G'(I_{41})=G(y_{117})=G(I_{41})-G(I_{42})$, $G'(I_{42})=G(y_{118})=G(I_{42})-G(I_{43})$, $G'(I_{43})=G(y_{119})=G(I_{43})-\{1\}$.

Let x *be an element of* U *such that* $I(x)=I_{43}$. *Then* $x\underset{c}{\sim}y_{119}=x_{63}(1)$. $Z_G(y_{119})\subset B\langle w_2, w_3, w_4, w_5, w_6, w_7\rangle B$, $Z(y_{119})=1$, $L(y_{119})=D_6$, $|Z_{G(k)}(y_{119})|=(q^2-1)(q^4-1)(q^6-1)^2(q^8-1)(q^{10}-1)q^{63}$.

The proof is easy.

By the series of lemmas, we proved that

THEOREM 2. *Let* G *be a simply connected semisimple algebraic group of type* E_7 *split over finite field* k. *Then the conjugate classes of unipotent elements of* G *are as in Tables* 2 *and* 4. *The inclusion relations of the Zariski closures of the conjugate classes are given in Table* 7. *The structures of* $Z_G(x)$ *are given in Table* 9.

§ 5. The case of E_8.

In this section, let G be the simply connected Chevalley group of type E_8 over the field K.

LEMMA 37. *Let* x *be a regular unipotent element, then* $x\underset{c}{\sim}z_1=x_1(1)$

$x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_1)$ *is conjugate in* $G(k)$ *to one of the following element:*

$$z_1=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_6(1)x_7(1)x_{101}(1)\,,$$
$$z_2=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(\tau)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=3\,,$$
$$z_3=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{22}(-\tau)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=3\,,$$
$$z_4=x_1(1)x_3(1)x_4(1)x_2(1)x_{15}(\eta)x_5(1)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_5=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{28}(\eta)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_6=x_1(1)x_3(1)x_4(1)x_2(1)x_{15}(\eta)x_5(1)x_{28}(\eta)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_7=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{32}(\mu)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=5\,,$$
$$z_8=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{32}(2\mu)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=5\,,$$
$$z_9=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{32}(3\mu)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=5\,,$$
$$z_{10}=x_1(1)x_3(1)x_4(1)x_2(1)x_5(1)x_{32}(-\mu)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=5\,.$$

$Z_G(z_1)=\langle z_1\rangle Z_U(z_1)^\circ$, $Z(z_1)\cong Z_r$, $(r=(60,\, p^2))$, $|Z_{G(k)}(z_i)|=rq^8$, $(1\leqq i\leqq 10)$.

The lemma is a direct consequence of B. Lou [5].

LEMMA 38. *Let* $J_1=I(\alpha_1,\ \alpha_2,\ \alpha_{10},\ \alpha_5,\ \alpha_6,\ \alpha_7,\ \alpha_{101})$. *Then* $G'(\Sigma^+)=G(z_1)=G(\Sigma^+)-G(J_1)$.

Let $x=\prod x_i(t_i)$ *be an element of U such that* $I(x)=J_1$ *and* $t_5t_9+t_2t_{11}\neq 0$. *Then* $x\underset{c}{\sim}z_{11}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{11})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{11}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{101}(1)\,,$$
$$z_{12}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{25}(\eta)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_{13}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{34}(\eta)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_{14}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{25}(\eta)x_{34}(\eta)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=2\,,$$
$$z_{15}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_{32}(\tau)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=3\,,$$
$$z_{16}=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_{32}(-\tau)x_6(1)x_7(1)x_{101}(1)\,,\quad \textit{when } \mathrm{ch}(K)=3\,.$$

$Z_G(z_{11})=\langle z_{11}\rangle Z_U(z_{11})^\circ$, $Z(z_{11})\cong Z_{(12,p^2)}$, $|Z_{G(k)}(z_i)|=(12,\, p^2)q^{10}$.

PROOF. By Lemma 8, we get $W(x)=\{1\}$. By the action of B, $x \underset{c}{\sim} y(a, b, c)=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_{32}(a)x_6(1)x_7(1)x_{25}(b)x_{34}(c)x_{101}(1)$ for some $a, b, c \in K$. Furthermore, if $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b, c)\}$, then $uyu^{-1}=y(a+u_1u_{15}+2u_{15}-3u_{22}, b+u_1^2+u_1-2u_{15}, c-u_1^3-u_1^2u_{15}+u_{15}^2-u_{15}+u_1u_{15}+u_1+2u_1u_{22}-2u_{29})$. Now the proof is easy.

LEMMA 39. *Let $J_2=I(\alpha_1, \alpha_2, \alpha_3, \alpha_{11}, \alpha_{13}, \alpha_{101})$ and let $x=\prod x_i(t_i)$ be an element of U such that $I(x)=J_2$ and $t_3t_9-t_2t_{10}\neq 0$. Then $x \underset{c}{\sim} z_{17}=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)x_{101}(1)$. Furthermore a k-rational point in $G(z_{17})$ is conjugate in $G(k)$ to one of the following elements*:

$$z_{17}=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)x_{101}(1)\ ,$$

$$z_{18}=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{35}(\eta)x_{13}(1)x_{101}(1)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$

$$z_{19}=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)x_{101}(1)x_{108}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$

$$z_{20}=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{35}(\eta)x_{13}(1)x_{101}(1)x_{108}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$

$Z_G(z_{17})=\langle z_{17}\rangle Z_D(z_{17})^\circ$, $Z(z_{17})\cong Z_{(2,p)}{}^2$, $|Z_{G(k)}(z_i)|=(2, p)^2q^{12}$, $G'(J_1)=G(z_{11})=G(J_1)-G(J_2)$.

PROOF. By Lemma 8, $W(x)=\{1\}$. By the action of B, x is conjugate to $y(a, b)=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{35}(a)x_{13}(1)x_{101}(1)x_{108}(b)$ for some $a, b \in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b)|a, b \in K\}$, then $uyu^{-1}=y(a+u_3^2+u_3-2u_8, b+(u_3^2+u_3-2u_8)^2+(u_3^2+u_3-2u_8)a+2(u_{34}+u_{22}u_3)-u_8^2-u_8)$. Now the proof is easy.

LEMMA 40. *Let $J_3=I(\alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_7, \alpha_{101})$ and let $x=\prod x_i(t_i)$ be an element of U such that $I(x)=J_3$. Furthermore we assume that x satisfies the following conditions*:

i) *if* $\mathrm{ch}(K)=2$, $f_1(x)=t_9t_{16}-t_{11}t_{15}-t_{10}t_{17}\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_2(x)=4(t_{22}t_9-t_{15}t_{17})t_{10}t_{11}+f_1(x)^2\neq 0$.

Then the element x is conjugate to the element $z_{21}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(1)x_{101}(1)$. Furthermore a k-rational point in $G(z_{21})$ is conjugate in $G(k)$ to one of the following elements:

$$z_{21}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(1)x_{101}(1)\ ,$$

$$z_{22}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_7(1)x_{101}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$

$$z_{23}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{22}(\eta)x_{12}(1)x_7(1)x_{101}(1)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$

$z_{24}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{35}(\eta)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{25}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{22}(\eta)x_{12}(1)x_{35}(\eta)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{26}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(1)x_{41}(\tau)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$,

$z_{27}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{26}(1)x_{12}(1)x_7(1)x_{41}(-\tau)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$,

$z_{28}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_7(1)x_{41}(\tau)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$,

$z_{29}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_7(1)x_{41}(-\tau)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$.

$Z_G(z_{21})\subset B$, $Z(z_{21})\cong Z_2\times Z_{(6,p)}$, $|Z_{G(k)}(z_i)|=2(6,\ p)q^{14}(21\leqq i\leqq 29)$.

PROOF. By Lemma 8, $W(x)=\{1\}$. By the action of B, x is conjugate to $y(a,\ b,\ c,\ d)=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(a)x_{22}(b)x_{12}(1)x_{35}(c)x_7(1)x_{41}(d)x_{101}(1)$ for some $a,\ b,\ c,\ d\in K$. $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a,\ b,\ c,\ d)|\ a,\ b,\ c,\ d\in K\}$, $uyu^{-1}=y(a-2u_2,\ b+u_2a-u_2^2,\ c+u_8^2+u_8-2u_{20},\ d-3u_{31}+2u_{20}+u_8u_{20}+u_8c)$. From above facts, we get the lemma.

LEMMA 41. *Let* $J_4=I(\alpha_1,\ \alpha_{15},\ \alpha_{16},\ \alpha_{17},\ \alpha_6,\ \alpha_7,\ \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_4$. *Then* $x\underset{c}{\sim}z_{30}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{30})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$z_{30}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)x_{101}(1)$,

$z_{31}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{40}(\eta)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{32}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{53}(\eta)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{33}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{40}(\eta)x_{53}(\eta)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{34}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{43}(\tau)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$,

$z_{35}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{43}(-\tau)x_7(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=3$.

$Z_G(z_{30})\subset B\langle w_4\rangle B$, $Z(z_{30})\cong Z_{(12,p^2)}$, $L(z_{30})=A_1$, $|Z_{G(k)}(z_i)|=(12,\ p^2)(q^2-1)q^{14}(30\leqq i\leqq 35)$.

Let $J_5=I(\alpha_8,\ \alpha_9,\ \alpha_{10},\ \alpha_{11},\ \alpha_{12},\ \alpha_{13},\ \alpha_{102})$ *and let* $y=\prod x_i(t_i)$ *be an element of* U *such that* $I(y)=J_5$. *Furthermore we assume that* y *satisfies the following conditions*;

i) *if* $\mathrm{ch}(K)=2$, $f_1(y)=t_9t_{16}-t_{11}t_{15}-t_{10}t_{17}\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_2(y)=4(t_{22}t_9-t_{15}t_{17})t_{10}t_{11}+f_1(y)^2\neq 0$. *Then* $y\underset{c}{\sim}z_{36}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{36})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{36}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)x_{102}(1)\ ,$$
$$z_{37}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{22}(\zeta)x_{12}(1)x_{13}(1)x_{102}(1)\ ,\qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$z_{38}=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{22}(\eta)x_{12}(1)x_{13}(1)x_{102}(1)\ ,\qquad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(z_{36})\subset B$, $Z(z_{36})\cong Z_2$, $|Z_{G(k)}(z_i)|=2q^{16}(i=36,\ 37,\ 38)$ $G'(J_3)=G(z_{21})=G(J_3)-\{G(J_4)\cup G(J_5)\}$.

PROOF. By the action of B, $x\underset{c}{\sim}y(a,\ b,\ c)=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_{40}(a)x_{43}(b)x_{53}(c)x_7(1)x_{101}(1)$ for some $a,\ b,\ c\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a,\ b,\ c)|a,\ b,\ c\in K\}$, then $uy(a,\ b,\ c)u^{-1}=y(a+u_1^2+u_1-2u_{21},\ b-3u_{27}+2u_{21}+3u_1u_{21}-u_1^3-u_1^2,\ c-3u_{27}+2u_{34}-2u_1u_{27}+u_1u_{21}+u_1^2a+u_1a-2u_{21}a+u_{21}^2+u_{21})$. Hence $x\underset{c}{\sim}z_{30}=y(0,\ 0,\ 0)$. By Lemma 8, $W(z_{30})=\langle w_4\rangle$.

By the action of B, $y\underset{c}{\sim}y(a,\ b)=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(a)x_{22}(b)x_{12}(1)x_{13}(1)x_{102}(1)$ for some $a,\ b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a,\ b)|a,\ b\in K\}$, then $uy(a,\ b)u^{-1}=y(a-2u_2,\ b+u_2a-u_2^2)$. Therefore $x\underset{c}{\sim}z_{36}=y(1,\ 0)$. By Lemma 8, $W(z_{36})=\{1\}$. Now the proof is easy.

LEMMA 42. *Let* $J_6=I(\alpha_7,\ \alpha_8,\ \alpha_{15},\ \alpha_{16},\ \alpha_{17},\ \alpha_{18},\ \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_6$ *and* $f(x)=(t_{29}t_{18}-t_{23}t_{24})t_7^2+(t_{13}t_{18}t_{22}-t_{13}t_{16}t_{24}+t_{15}t_{18}t_{19}-t_{13}t_{17}t_{23})t_7-t_{13}^2t_{16}t_{17}\neq 0$. *Then* $x\underset{c}{\sim}z_{39}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{39})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{39}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_7(1)x_{101}(1)\ ,$$
$$z_{40}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_7(1)x_{19}(1)x_{41}(\eta)x_{101}(1)\ ,\qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{41}=x_8(1)x_{15}(1)x_{16}(\zeta)x_{17}(1)x_{18}(1)x_7(1)x_{19}(1)x_{101}(1)\ ,\qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{39})\subset B$, $Z(z_{39})\cong Z_2$, $|Z_{G(k)}(z_i)|=2q^{18}(i=39,\ 40,\ 41)$.

PROOF. By Lemma 8, $W(x)=\{1\}$. By the action of B, $x\underset{c}{\sim}y(a)=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_7(1)x_{19}(1)x_{41}(a)x_{101}(1)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a)|a\in K\}$, $uy(a)u^{-1}=y(-u_8^2-u_8+2u_{27}+a)$. This shows the lemma.

LEMMA 43. *Let* $J_7=I(\alpha_1,\ \alpha_{15},\ \alpha_{16},\ \alpha_{17},\ \alpha_{18},\ \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be* *an element of* U *such that* $I(x)=J_7$ *and* $t_{23}t_{17}-t_{16}t_{24}\neq 0$. *Then* $x \underset{c}{\sim} z_{42}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{24}(1)x_{13}(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{42})$ *is conjugate in* $G(k)$ *to* z_{42} *or* $z_{43}=x_1(1)x_{15}(1)x_{10}(1)x_{17}(1)x_{24}(1)x_{13}(1)x_{49}(\eta)x_{101}(1)$. $Z_G(z_{42})\subset B\langle w_4\rangle B$, $Z(z_{42})\cong Z_{(2,p)}$, $L(z_{42})=A_1$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)q^{18}$ $(i=42, 43)$.

PROOF. By the action of B, $x \underset{c}{\sim} y(a)=z_{42}x_{49}(a)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a)|a\in K\}$, then $uy(a)u^{-1}=y(u_1^2+u_1-2u_{30}+a)$. Now the proof is easy.

LEMMA 44. *Let* $J_8=I(\alpha_8, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{13}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_8$. *Furthermore we suppose that* x *satisfies the conditions;*

i) *if* $\mathrm{ch}(K)=2$, $f_1(x)=t_{13}t_{18}t_{22}-t_{13}t_{16}t_{24}+t_{15}t_{18}t_{19}-t_{13}t_{17}t_{23}\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_2(x)=4f_3(x)t_{13}t_{16}t_{17}+f_1(x)^2t_{102}\neq 0$, *where* $f_3(x)=t_{13}t_{18}t_{29}t_{102}-t_{13}t_{23}t_{24}t_{102}+t_{13}t_{15}t_{18}t_{104}-t_{15}t_{18}t_{19}t_{103}$. *Then* $x \underset{c}{\sim} z_{44}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{13}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{44})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$z_{44}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{13}(1)x_{102}(1)$,

$z_{45}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{29}(\zeta)x_{13}(1)x_{102}(1)$, *when* $\mathrm{ch}(K)\neq 2$,

$z_{46}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{29}(\eta)x_{13}(1)x_{102}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{47}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{38}(\eta)x_{13}(1)x_{101}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{48}=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{29}(\eta)x_{38}(\eta)x_{13}(1)x_{102}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{49}=z_{44}x_{111}(\eta)$, *when* $\mathrm{ch}(K)=2$.

$$Z_G(z_{44})\subset B,\ Z(z_{44})\cong\begin{cases} Z_2, & \textit{if } \mathrm{ch}(K)\neq 2, \\ D_8(\textit{dihedral group of order } 8), & \textit{if } \mathrm{ch}(K)=2 . \end{cases}$$

$|Z_{G(k)}(z_i)|=2(2, p)^2q^{20}(i=44, 49)$, $|Z_{G(k)}(z_i)|=4q^{20}(i=46, 47, 48)$, $|Z_{G(k)}(z_{45})|=2q^{20}$.

PROOF. By the action of B, $x \underset{c}{\sim} y(a, b, c)=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{29}(a)x_{38}(b)x_{13}(1)x_{102}(1)x_{111}(c)$ for some $a, b, c\in K$. If $\mathrm{ch}(K)\neq 2$, the proof is easy. Thus we assume $\mathrm{ch}(K)=2$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b, c)|a, b, c\in K\}$, $uy(a, b, c)u^{-1}=y(u_2^2+u_2+a, u_1^2+u_1+b, u_1^2a+u_2(u_1^2+u_1+b)+u_8^2+u_8+c)$. Now the proof is easy.

LEMMA 45. *$G'(J_4)=G(z_{30})=G(J_4)-G(J_6)$, $G'(J_5)=G(z_{36})=G(J_5)-G(J_6)$, $G'(J_6)=G(z_{39})=G(J_6)-\{G(J_7)\cup G(J_8)\}$.*

This lemma follows from Lemmas 1 and 4.

LEMMA 46. *Let $J_9=I(\alpha_{14}, \alpha_{15}, \alpha_{17}, \alpha_{18}, \alpha_{19}, \alpha_{101})$ and let $x=\prod x_i(t_i)$ be an element of U such that $I(x)=J_9$ and $f_1(x)=t_{15}t_{18}t_{21}-t_{14}t_{18}t_{22}+t_{14}t_{17}t_{23}\neq 0$. Furthermore we suppose that x satisfies the following conditions;*

i) *if* $\mathrm{ch}(K)=2$, $f_2(x)=t_{15}t_{18}t_{26}+t_{14}t_{17}t_{29}-t_{15}t_{17}t_{27}+t_{17}t_{20}t_{23}-t_{18}t_{20}t_{22}+t_{15}t_{21}t_{24}-t_{14}t_{22}t_{24}\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_4(x)=4f_1(x)f_3(x)+f_2(x)^2\neq 0$, *where* $f_3(x)=t_{15}t_{17}t_{33}-t_{15}t_{24}t_{26}-t_{17}t_{20}t_{29}+t_{20}t_{22}t_{24}$.

Then $x\underset{c}{\sim}z_{50}=x_{14}(1)x_{15}(1)x_{21}(1)x_{17}(1)x_{18}(1)x_{29}(1)x_{19}(1)x_{101}(1)$. Furthermore a k-rational point in $G(z_{50})$ is conjugate in $G(k)$ to one of the following elements;

$$z_{50}=x_{14}(1)x_{15}(1)x_{21}(1)x_{17}(1)x_{18}(1)x_{29}(1)x_{19}(1)x_{101}(1)\ ,$$
$$z_{51}=x_{14}(1)x_{15}(1)x_{21}(1)x_{17}(1)x_{18}(1)x_{29}(1)x_{19}(1)x_{101}(1)x_{108}(\eta)\ ,\quad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{52}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{27}(-1)x_{33}(\tau)x_{101}(1)\ ,$$
$$z_{53}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{27}(-1)x_{33}(\tau)x_{101}(1)x_{108}(\eta)\ ,\quad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{54}=x_{14}(1)x_{15}(1)x_{17}(1)x_{21}(1)x_{18}(1)x_{33}(\zeta)x_{19}(1)x_{101}(1)\ ,\quad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$z_{55}=x_{14}(1)x_{15}(1)x_{17}(1)x_{21}(1)x_{18}(1)x_{29}(1)x_{33}(\eta)x_{19}(1)x_{101}(1)\ ,\quad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{56}=z_{55}x_{108}(\eta)\ ,\quad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(z_{50})\subset B\langle w_1w_2w_6\rangle B$, $Z(z_{50})\cong S_3\times Z_{(2,p)}$, $|Z_{G(k)}(z_i)|=6(2, p)q^{22}$ $(i=50, 51)$, $|Z_{G(k)}(z_i)|=3(2, p)q^{22}(i=52, 53)$, $|Z_{G(k)}(z_i)|=2(2, p)q^{22}(i=54, 55, 56)$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_1w_2w_6\rangle$. Thus we put $P=B\langle w_1, w_2, w_6\rangle B$, $R=U(I(\alpha_4, \alpha_7, \alpha_{101}))$, $V=U(I(\alpha_{10}, \alpha_{11}, \alpha_{19}, \alpha_{101}))$, $V_1=U(I(\alpha_{25}, \alpha_{28}, \alpha_{102}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Suppose $\mathrm{ch}(K)\neq 2$. Since $Z_R(x)$ is connected, the proof is easy. Suppose $\mathrm{ch}(K)=2$. By the action of B, $x\underset{c}{\sim}y(a, b)=x_{14}(1)x_{15}(1)x_{17}(1)x_{21}(1)x_{18}(1)x_{29}(1)x_{33}(a)x_{19}(1)x_{101}(1)x_{108}(b)$ for some $a, b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b)|a, b\in K\}$, $uy(a, b)u^{-1}=y(u_1^2+u_1+a, u_{14}^2+u_{14}+b)$. Hence $x\underset{c}{\sim}z_{50}$ and $Z_B(z_{50})/Z_B(z_{50})^\circ$ is isomorphic to $Z_2\times Z_2$. On the other hand, $Z_G(z_{50})\cap B\langle w_1w_2w_6\rangle B=\{\omega_1\omega_2\omega_6, g\omega_1\omega_2\omega_6\}Z_B(z_{50})$ for some $g\in B$. Therefore, $Z(z_{50})\cong S_3\times Z_2$. The proof of the rest is easy.

LEMMA 46′. *Let* $J_{10}=I(\alpha_1, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{19}, \alpha_{103})$ *and let* x *be an element of* U *such that* $I(x)=J_{10}$. *Then* $x \underset{c}{\sim} z_{57}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{103}(1)$. *Furthermore, a k-rational point in* $G(z_{57})$ *is conjugate in* $G(k)$ *to* z_{57} *or* $z_{58}=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{49}(\eta)x_{103}(1)$. $Z_G(z_{57})\subset B\langle w_4w_7\rangle B$, $Z(z_{57})\cong Z_{(2,p)}$, $L(z_{57})=A_1$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)q^{20}$ $(i=57, 58)$. $G'(J_7)=G(z_{42})=G(J_7)-G(J_9)$, $G'(J_8)=G(z_{44})=G(J_8)-\{G(J_9)\cup G(J_{10})\}$.

PROOF. By Lemma 8, we get $W(x)\subsetneqq\langle w_4w_7\rangle$. By the action of B, $x \underset{c}{\sim} y(a)=z_{57}x_{49}(a)$ for some $a\in K$. If $\mathrm{ch}(K)\neq 2$, $x\underset{c}{\sim} y(0)=z_{57}$ and $Z_U(z_{57})$ is connected. Suppose $\mathrm{ch}(K)=2$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{57}\mathfrak{X}_{49}$, $uy(a)u^{-1}=y(u_1^2+u_1+a)$. Hence $x\underset{c}{\sim} z_{57}$. The last assertion of this lemma follows from Lemmas 1, 2, 3 and 4.

LEMMA 47. *Let* $J_{11}=I(\alpha_{20}, \alpha_{21}, \alpha_{17}, \alpha_{23}, \alpha_7, \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{11}$ *and* $t_7t_{24}+t_{13}t_{17}\neq 0$. *Then* $x\underset{c}{\sim} z_{59}=x_{20}(1)x_{21}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{59})$ *is conjugate in* $G(k)$ *to* z_{59} *or* $z_{60}=x_{20}(1)x_{21}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)x_{49}(\eta)x_{101}(1)$. $Z_G(z_{59})\subset B\langle w_4\rangle B$, $Z(z_{59})\cong Z_{(2,p)}$, $L(z_{59})=A_1$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)q^{22}$ $(i=59, 60)$.

PROOF. By the action of B, $x\underset{c}{\sim} y(a)=z_{59}x_{49}(a)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{59}\mathfrak{X}_{49}$, then $uy(a)u^{-1}=y(u_7^2+u_7-2u_{30}+a)$. On the other hand, by Lemma 8, we get $W(z_{59})\subsetneqq\langle w_4\rangle$. Now the proof is clear.

LEMMA 48. *Let* $J_{12}=I(\alpha_{14}, \alpha_{18}, \alpha_{22}, \alpha_{13}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{12}$ *and* $f_1(x)f_2(x)f_3(x)\neq 0$, *where* $f_1(x)=t_{14}t_{24}-t_{18}t_{20}$, $f_2(x)=t_{21}t_{13}-t_{14}t_{19}$, $f_3(x)=-t_{13}t_{18}t_{22}t_{33}t_{102}+f_1(x)(t_{13}t_{22}t_{104}-t_{19}t_{22}t_{103}+t_{19}t_{29}t_{102})+t_{13}t_{29}t_{102}(t_{18}t_{26}-t_{21}t_{24})$. *Then* $x\underset{c}{\sim} z_{61}=x_{14}(1)x_{20}(1)x_{21}(1)x_{22}(1)x_{18}(1)x_{33}(1)x_{13}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{61})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{61}=x_{14}(1)x_{20}(1)x_{21}(1)x_{22}(1)x_{18}(1)x_{33}(1)x_{13}(1)x_{102}(1)\ ,$$
$$z_{62}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{27}(-1)x_{33}(\tau)x_{19}(1)x_{103}(1)\ ,$$
$$x_{63}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{23}(1)x_{33}(\zeta)x_{19}(1)x_{103}(1)\ ,$$
$$\textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$z_{64}=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{23}(1)x_{29}(1)x_{33}(\eta)x_{19}(1)x_{103}(1)\ ,$$
$$\textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(z_{61})\subset B\langle w_1, w_2, w_3, w_5, w_6, w_{101}\rangle B$, $Z(z_{61})\cong S_3$, $|Z_{G(k)}(z_{61})|=6q^{24}$, $|Z_{G(k)}(z_{62})|=$

$3q^{24}$, $|Z_{G(k)}(z_i)|=2q^{24}(i=63, 64)$.

$$G'(J_9)=G(z_{50})=G(J_9)-\{G(J_{11})\cup G(J_{12})\}\ ,$$

$G'(J_{10})=G(z_{57})=G(J_{10})-G(J_{12})$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\{1, w_1w_2w_5, w_2w_3w_6w_{101}, w_1w_3w_2w_5 w_6w_{101}, w_3w_1w_2w_6w_5w_{101}, w_8w_{12}w_{101}\}$. Thus we put $P=B\langle w_1, w_2, w_3, w_5, w_6, w_{101}\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Now the proof is easy.

LEMMA 49. *Let* $J_{13}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_{28}, \alpha_7, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{13}$. *Then* $x\underset{c}{\sim}z_{65}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1)$. *Fnrthermore a k-rational point in* $G(z_{65})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$\begin{aligned}
&z_{65}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1)\ , \\
&x_{66}=z_{65}x_{49}(\eta)\ , && \textit{when } \mathrm{ch}(K)=2\ , \\
&z_{67}=z_{65}x_{59}(\tau)\ , && \textit{when } \mathrm{ch}(K)=3\ , \\
&z_{68}=z_{65}x_{59}(-\tau)\ , && \textit{when } \mathrm{ch}(K)=3\ .
\end{aligned}$$

$Z_G(z_{65})\subset B\langle w_2w_3w_5\rangle B$, $Z(z_{65})\cong Z_{(6,p)}$, $L(z_{65})=A_1$, $|Z_{G(k)}(z_i)|=(6, p)(q^2-1)q^{24}$.

PROOF. By the action of B, $x\underset{c}{\sim}y(a, b)=z_{65}x_{49}(a)x_{59}(b)$ for some $a, b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{65}\mathfrak{X}_{49}\mathfrak{X}_{59}$, $uy(a, b)u^{-1}=y(a+u_7^2+u_7-2u_{102}, b-u_7^3-u_7^2-3u_{102}+3u_{49}-3u_7a)$. Hence $x\underset{c}{\sim}z_{65}$. By Lemma 8, $W(z_{65})=\langle w_2w_3w_5\rangle$. Now the proof is easy.

LEMMA 50. *Let* $J_{14}=I(\alpha_{14}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{19}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{14}$ *and* $t_{19}t_{28}+t_{22}t_{25}\neq 0$.

1) *Suppose* $\mathrm{ch}(K)\neq 2$. *Then* $x\underset{c}{\sim}z_{69}=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{25}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{69})$ *is conjugate in* $G(k)$ *to* z_{69} *or* $z_{70}=x_{20}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{29}(1)x_{19}(1)x_{41}(-\zeta)x_{103}(1)$. $Z_G(z_{69})\subset B\langle w_{15}w_{13}\rangle B$, $Z(z_{69})\cong Z_2$, $L(z_{69})=T_1$, $|Z_{G(k)}(z_{69})|=2(q-1)q^{25}$, $|Z_{G(k)}(z_{70})|=2(q+1)q^{25}$.

2) *Suppose* $\mathrm{ch}(K)=2$. *Then* x *is conjugate to* z_{69} *or* $z_{71}=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{25}(1)x_{35}(1)x_{102}(1)$. *A k-rational point in* $G(z_{69})$ *is conjugate in* $G(k)$ *to* z_{69} *or* $z_{72}=z_{69}x_{53}(\eta)$.
$Z_G(z_{69})\subset B\langle w_{15}w_{13}\rangle B$, $Z(z_{19})\cong Z_2$, $L(z_{69})=A_1$, $|Z_{G(k)}(z_i)|=2(q^2-1)q^{26}(i=69, 72)$.
$Z_G(z_{71})\subset B$, $Z(z_{71})=1$, $|Z_{G(k)}(z_{71})|=q^{26}$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_{15}w_{13}\rangle$. By the action of B, $x\underset{c}{\sim}y(a, b)=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{35}(a)x_{53}(b)x_{19}(1)x_{25}(1)x_{102}(1)$ for some $a, b\in K$.

If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b)|a, b\in K\}$, $uy(a, b)u^{-1}=y(a+2u_7, b+u_{14}^2+u_{14}-u_{26}(a+2u_7)-2u_{38})$. Hence x is conjugate to z_{69} or z_{71}. Now the proof is easy.

LEMMA 51. $G'(J_{11})=G(z_{59})=G(J_{11})-\{G(J_{13})\cup G(J_{14})\}$, $G'(J_{12})=G(z_{61})=G(J_{12})-G(J_{14})$.

The proof is easy.

LEMMA 52. *Let* $J_{15}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{24}, \alpha_7, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{15}$. *Then* $x\underset{c}{\sim}z_{73}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{73})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{73}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1)\ ,$$
$$z_{74}=z_{73}x_{49}(\eta)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{75}=z_{73}x_{59}(\tau)\ , \qquad \textit{when } \mathrm{ch}(K)=3\ ,$$
$$z_{76}=z_{73}x_{59}(-\tau)\ , \qquad \textit{when } \mathrm{ch}(K)=3\ .$$

$Z_G(z_{73})\subset B\langle w_2w_3w_5, w_4\rangle B$, $Z(z_{73})\cong Z_{(6,p)}$, $L(z_{73})=G_2$, $|Z_{G(k)}(z_i)|=(6, p)(q^2-1)(q^6-1)q^{24}(i=73, 74, 75, 76)$.

The proof is similar to that of Lemma 49.

LEMMA 53. *Let* $J_{16}=I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{16}$.

1) *Suppose* $\mathrm{ch}(K)=2$ *and* x *satisfies the condition* $f_1(x)=(t_{24}t_{30}-t_{23}t_{31}-t_{25}t_{29})t_{102}-t_{22}t_{25}t_{103}\neq 0$. *Then* $x\underset{c}{\sim}z_{77}=x_{20}(1)x_{21}(1)x_{22}(2)x_{23}(1)x_{24}(1)x_{25}(1)x_{102}(1)x_{103}(1)$. *Furthermore a k-rational point in* $G(z_{77})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{77}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{102}(1)x_{103}(1)\ ,$$
$$z_{78}=z_{77}x_{104}(\eta)\ ,$$
$$z_{79}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{101}(1)x_{103}(1)x_{104}(\tau)\ .$$

$Z_G(z_{77})\subset B\langle w_2w_6w_8w_{19}\rangle B$, $Z(z_{77})\cong S_3$, $|Z_{G(k)}(z_{77})|=6q^{28}$, $|Z_{G(k)}(z_{79})|=3q^{28}$, $|Z_{G(k)}(z_{78})|=2q^{28}$.

2) *Suppose that* $\mathrm{ch}(K)=3$ *and* x *satisfies the condition* $f_3(x)=f_1(x)^2t_{21}+4f_2(x)t_{25}\neq 0$, *where* $f_3(x)=f_1(x)(t_{21}t_{22}t_{103}+t_{22}t_{27}t_{102}-t_{23}t_{26}t_{102})+t_{20}t_{22}t_{23}t_{25}t_{102}t_{104}+(t_{21}t_{24}t_{36}+t_{22}t_{25}t_{33}-t_{22}t_{24}t_{34}-t_{21}t_{29}t_{31}+t_{22}t_{27}t_{31}+t_{24}t_{26}t_{30}-t_{23}t_{28}t_{31}-t_{25}t_{26}t_{29})t_{23}t_{102}^2$. *Then* $x\underset{c}{\sim}z_{80}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{102}(1)x_{104}(1)$. *Furthermore a k-rational point in* $G(z_{80})$ *is conjugate in* $G(k)$ *to* z_{80} *or* $z_{81}=z_{80}x_{104}(\zeta-1)$

$Z_G(z_{80})\subset B$, $Z(z_{80})\cong Z_2$, $|Z_{G(k)}(z_i)|=2q^{28}$ $(i=80, 81)$.

3) *Suppose* $\mathrm{ch}(K)\neq 2, 3$. *If* x *satisfies the condition* $f_3(x)\neq 0$, *then* $x \underset{c}{\sim} z_{80}$. *Furthermore a k-rational point in* $G(z_{80})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{82}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{102}(1)x_{104}(-3)\ ,$$
$$z_{83}=z_{82}x_{104}(3-3\zeta)\ ,$$
$$z_{84}=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{101}(1)x_{103}(-1/3)x_{104}(\tau)\ .$$

$Z_G(z_{82})\subset B\langle w_8w_2w_6w_{19}\rangle B$, $Z(z_{82})\cong S_3$, $|Z_{G(k)}(z_{82})|=6q^{28}$, $|Z_{G(k)}(z_{84})|=3q^{28}$, $|Z_{G(k)}(z_{83})|=2q^{28}$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_8w_2w_6w_{19}\rangle$. Suppose $\mathrm{ch}(K)=2$. By the action of B, $x\underset{c}{\sim}y(a)=z_{77}x_{104}(a)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{77}\mathfrak{X}_{104}$, $uy(a)u^{-1}=y(u_2^2+u_2+a)$. Hence $x\underset{c}{\sim}z_{77}$. Thus we put $P=B\langle w_1, w_2, w_3, w_5, w_6, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{77})$. Suppose $\mathrm{ch}(K)=3$. By the action of B, $x\underset{c}{\sim}z_{80}$. By calculations, $Z_G(z_{80})\subset B$. Suppose $\mathrm{ch}(K)\neq 2, 3$. By the action of B, $x\underset{c}{\sim}z_{77}$. Let $y(a, b, c)=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{101}(1)x_{102}(a)x_{103}(b)x_{104}(c)$ and let $Y=\{y(a, b, c)|a, b, c\in K\}$. If $u=\prod x_i(u_i)$ stabilizes the set Y, then $uy(a, b, c)u^{-1}=y(a+u_2, b+2u_2a+u_2^2, c+3u_2^2a+u_2^3+3u_2b)$. Now the proof is easy.

LEMMA 54. *Let* $J_{17}=I(\alpha_8, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{28}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{17}$. *Furthermore we assume that* x *satisfies the following conditions;*

i) *if* $\mathrm{ch}(K)=2$, $f_1(x)=t_{30}t_{24}-t_{29}t_{25}-t_{23}t_{31}\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_2(x)=4(t_{24}t_{36}-t_{29}t_{31})t_{23}t_{25}+f_1(x)^2\neq 0$.

Then $x\underset{c}{\sim}z_{85}=x_8(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{85})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{85}=x_8(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{102}(1)\ ,$$
$$z_{86}=z_{85}x_{36}(\eta)\ ,\qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{87}=z_{85}x_{36}(\zeta-1)\ ,\qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{85})\subset B\langle w_5\rangle B$, $Z(z_{85})\cong Z_2$, $L(z_{85})=A_1$, $|Z_{G(k)}(z_i)|=2(q^2-1)q^{26}(i=85, 86, 87)$.

The proof is easy.

LEMMA 55. $G'(J_{13})=G(z_{65})=G(J_{13})-\{G(J_{15})\cup G(J_{16})\}$. *If* $\mathrm{ch}(K)\neq 2$, $G'(J_{14})=G(z_{69})=G(J_{14})-\{G(J_{16})\cup G(J_{17})\}$.

PROOF. Let $y=\prod x_i(u_i)$ be an element of U such that $I(y)=J_{15}$ and $u_{19}u_{28}+u_{22}u_{25}=0$. Then y is conjugate to an element of $G(J_{17})$. Now the lemma follows from Lemmas 1, 2 and 5.

LEMMA 56. *Let* $J_{18}=I(\alpha_{20}, \alpha_{21}, \alpha_{28}, \alpha_{23}, \alpha_{24}, \alpha_{19}, \alpha_{103})$ *and let* x *be an element of* U *such that* $I(x)=J_{18}$.

1) *Suppose* $\mathrm{ch}(K)\neq 3$. *Then* $x \underset{c}{\sim} z_{88}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{103}(1)$. $Z_G(z_{88})\subset B\langle w_2w_3w_5w_{101}\rangle B$, $Z(z_{88})=1$, $L(z_{88})=A_1$, $|Z_{G(k)}(z_{88})|=(q^2-1)q^{28}$.

2) *Suppose* $\mathrm{ch}(K)=3$. *Then* x *is conjugate to* z_{88} *or* $z_{89}=z_{88}x_{40}(1)$. $Z_G(z_{88})\subset B\langle w_2w_3w_5w_{101}\rangle B$, $Z(z_{89})=\subset B$, $Z(z_{88})=Z(z_{89})=1$, $L(z_{88})=A_1$, $|Z_{G(k)}(z_{88})|=(q^2-1)q^{30}$, $|Z_{G(k)}(z_{89})|=q^{30}$.

PROOF. By the action of B, $x\underset{c}{\sim} y(a, b)=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{35}(a)x_{40}(b)x_{19}(1)x_{103}(1)$ for some $a, b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $Y=\{y(a, b)|a, b\in K\}$, $uy(a, b)u^{-1}=y(a-3u_1, b-2u_2a+3(u_8+u_1u_2))$. From this facts, we get the lemma.

LEMMA 57. *Let* $J_{19}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{28}, \alpha_{31}, \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{19}$ *and* $t_{30}t_{101}+t_{23}t_{102}\neq 0$. *Then* $x\underset{c}{\sim} z_{90}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1)$. *Furthermore a* k*-rational point in* $G(z_{90})$ *is conjugate to one of the following elements;*

$$z_{90}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1)\ ,$$
$$z_{91}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{36}(\eta)x_{102}(1)\ ,\quad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{92}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)x_{102}(1)\ ,\quad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{90})\subset B\langle w_2w_3w_5w_7\rangle B$, $Z(z_{90})\cong Z_2$, $L(z_{90})=T_1$, $|Z_{G(k)}(z_{90})|=2(q-1)q^{29}$, $|Z_{G(k)}(z_i)|=2(q+1)q^{29} (i=91, 92)$.

PROOF. By Lemma 8, we get $W(x)=\langle w_2w_3w_5w_7\rangle$. Thus we put $P=B\langle w_2, w_3, w_5, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{14}, \alpha_{28}, \alpha_{18}, \alpha_{101}))$, $V_1=U(I(\alpha_{27}, \alpha_{32}, \alpha_{35}, \alpha_{103}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Now the proof is easy.

LEMMA 58. *Let* $J_{20}=I(\alpha_1, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{20}$. *Then* $x\underset{c}{\sim} z_{93}=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{101}(1)$. *Furthermore a* k*-rational point in* $G(z_{93})$ *is conjugate in* $G(k)$ *to* z_{93} *or* $z_{94}=z_{93}x_{53}(\eta)$. $Z_G(z_{93})\subset B\langle w_4w_6, w_5\rangle B$, $Z(z_{93})\cong Z_{(2,p)}$, $L(z_{93})=B_2$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)(q^4-1)q^{26}$ $(i=93, 94)$.

PROOF. By Lemma 8, we get $W(x)=\langle w_4w_6, w_5\rangle$. Thus we put $P=$

$B\langle w_4, w_5, w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_1, \alpha_{15}, \alpha_{30}, x_{31}, \alpha_{101}))$, $V_1=U(I(\alpha_8, \alpha_{36}, \alpha_{102}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. On the other hand, by the action of B, we get $x \underset{c}{\sim} z_{93}x_{53}(a)=y(a)$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{93}\mathfrak{X}_{53}$, $uy(a)u^{-1}=y(u_1^2+u_1+a)$ in the case $\mathrm{ch}(K)=2$. Now the proof is easy.

LEMMA 59. *If* $\mathrm{ch}(K)=2$, *then the Zariski closure* $\overline{G(z_{69})}=G(z_{69})\cup G(J_{18})\cup G(J_{20})$ *and* $G'(J_{16})=G(z_{77})=G(J_{16})-\{G(J_{19})\cup G(J_{18})\}$. *If* $\mathrm{ch}(K)\neq 2$, *then* $G'(J_{16})=G(z_{80})=G(J_{16})-\{G(J_{18})\cup G(J_{19})\}$. $G'(J_{17})=G(z_{85})=G(J_{17})-\{G(J_{19})\cup G(J_{20})\}$.

PROOF. By calculations, we get $\overline{B(z_{69})}=\{y=\prod x_i(u_i) | I(y)\subseteqq J_{14}, f(y)=0\}$, where $f(y)=(u_{25}u_{29}+u_{24}u_{30}+u_{23}u_{31}+u_{29}u_{35})u_{103}+u_{102}(u_{19}u_{28}+u_{22}u_{25})$. Let y be an element of $\overline{B(z_{69})}-B(z_{69})$. If $I(y)=J_{14}$, then $u_{22}u_{25}+u_{19}u_{28}=0$. Hence y is in $G(J_{20})$. By calculations, we get the following results;

if $u_{14}=0$, y is in $G(J_{18})$,
if $u_{22}=0$, y is in $G(J_{18})$,
if $u_{23}=0$, y is in $G(I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{25}, \alpha_{104}))$,
if $u_{24}=9$, y is in $G(I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{25}, \alpha_{104}))$,
if $u_{19}=0$, then y is in $G(J_{18})$,
if $u_{102}=0$, y is in $G(I(\alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{25}, \alpha_{104}))$.

Therefore we get $\overline{G(z_{69})}\subseteqq G(z_{69})\cup G(J_{18})\cup G(J_{20})$. The opposite inclusion is obvious. The proof of the rest is easy.

LEMMA 60. *Let* $J_{21}=I(\alpha_{20}, \alpha_{21}, \alpha_{25}, \alpha_{28}, \alpha_{29}, \alpha_{103})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $t_{25}f_{26}t_{103}+t_{20}t_{25}t_{104}-t_{21}t_{31}t_{103}\neq 0$. *Then* $x \underset{c}{\sim} z_{95} = x_{20}(1)x_{21}(1)x_{28}(1)x_{29}(1)x_{25}(1)x_{31}(1)x_{103}(1)$. *Furthermore a k-rational point in* $G(z_{95})$ *is conjugate to one of the following elements*;

$$z_{95}=x_{20}(1)x_{21}(1)x_{28}(1)x_{29}(1)x_{25}(1)x_{31}(1)x_{103}(1),$$
$$z_{96}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)x_{104}(1), \quad \text{when } \mathrm{ch}(K)\neq 2,$$
$$z_{97}=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{36}(\eta)x_{104}(1), \quad \text{when } \mathrm{ch}(K)=2.$$

$Z_G(z_{95})\subset B\langle w_2w_3w_5w_7\rangle B$, $Z(z_{95})\cong Z_2$, $L(z_{95})=T_1$, $|Z_{G(k)}(z_{95})|=2(q-1)q^{31}$, $|Z_{G(k)}(z_i)|=2(q+1)q^{31}(i=96, 97)$.

PROOF. By Lemma 8, we get $W(x)=\langle w_2w_3w_5w_7\rangle$. Thus we put $P=B\langle w_2, w_3, w_5, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{14}, \alpha_{18}, \alpha_{28}, \alpha_{103}))$, $V_1=U(I(\alpha_{27}, \alpha_{32},$

$\alpha_{35}, \alpha_{105}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. Now the proof is easy.

LEMMA 61. *Let* $J_{22}=I(\alpha_{20}, \alpha_{21}, \alpha_{23}, \alpha_{31}, \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{22}$ *and* $t_{23}t_{102}+t_{30}t_{101}\neq 0$. *Then* $x \underset{c}{\sim} z_{98}= x_{20}(1)x_{21}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{98})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{98}=x_{20}(1)x_{21}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1)\ ,$$
$$z_{99}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{36}(\zeta)x_{102}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ ,$$
$$z_{100}=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{36}(\eta)x_{102}(1)\ , \quad \textit{when } \mathrm{ch}(K)=2\ .$$

$Z_G(z_{98})\subset B\langle w_2w_3w_5w_7, w_4\rangle B$, $Z(z_{98})\cong Z_2$, $L(z_{98})=A_2$, $|Z_{G(k)}(z_{98})|=2(q^2-1)(q^3-1)q^{29}$, $|Z_{G(k)}(z_i)|+2(q^2-1)(q^3+1)q^{29}(i=99, 100)$.

The proof is similar to that of Lemma 60.

LEMMA 62. *Let* $J_{23}=I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{102})$ *and let x be an element of U such that* $I(x)=J_{23}$.

1) *Suppose* $\mathrm{ch}(K)\neq 2$. *Then* $x \underset{c}{\sim} z_{101}=x_{26}(1)x_{28}(1)x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)$ $x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{101})$ *is conjugate in* $G(k)$ *to* z_{101} *or* $z_{102}=z_{101}x_{103}(2\zeta)x_{101}(1)x_{102}(-1)$ $Z_G(z_{101})\subset B\langle w_{20}w_{10}w_{13}\rangle B$, $Z(z_{101})\cong Z_2$, $L(z_{101})=T_1$, $|Z_{G(k)}(z_{101})|=2(q-1)q^{33}$, $|Z_{G(k)}(z_{102})|=2(q+1)q^{33}$.

2) *Suppose* $\mathrm{ch}(K)=2$. *Then x is conjugatc to* z_{101} *or* $z_{103}=z_{101}x_{103}(1)$. *Furthermore a k-rational point in* $G(z_{101})$ *is conjugate in* $G(k)$ *to* z_{101} *or* $z_{104}=z_{101}x_{122}(\eta)$. $Z_G(z_{101})\subset B\langle w_{20}w_{10}w_{13}\rangle B$, $Z_G(z_{103})\subset B$, $Z(z_{101})\cong Z_2$, $Z(z_{104})=1$, $L(z_{101})=A_1$, $|Z_{G(k)}(z_{101})|=2(q^2-1)q^{34}$, $|Z_{G(k)}(z_{104})|=2(q^2-1)q^{34}$, $|Z_{G(k)}(z_{103})|=q^{34}$.

PROOF. By Lemma 8, $W(x)\subseteqq\langle w_{20}w_{10}w_{13}\rangle$. Thus we put $P=B\langle w_1, w_2, w_3, w_4, w_6, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{103})$. On the other hand, by the action of B, $x\underset{c}{\sim} y(a, b)=z_{101}x_{103}(a)x_{122}(b)$ for some $a, b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{101}\mathfrak{X}_{103}\mathfrak{X}_{122}$, $uy(a, b)u^{-1}=y(a, u_{97}^2+u_{27}+u_{21}^2a+u_{21}a+u_{101}a+b)$ in the case $\mathrm{ch}(K)=2$. Now the proof is easy.

LEMMA 63. $G'(J_{19})=G(z_{90})=G(J_{19})-G(J_{22})$, $G(J_{18})=G(z_{86})\cup G(z_{89})\cup G(J_{21})$, $G'(J_{15})=G(z_{73})=G(J_{15})-G(J_{22})$, $G'(J_{20})=G(z_{93})=G(J_{20})-G(J_{23})$, $G'(J_{21})=G(z_{95})= G(J_{21})-G(J_{23})$.

PROOF. This lemma is derived from the following two results;

i) $I(\alpha_{20}, \alpha_{21}, \alpha_{28}, \alpha_{29}, \alpha_{19}, \alpha_{108})\sim J_{21}$,

ii) $I(\alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{101})\sim J_{23}$.

LEMMA 64. *Let* $J_{24}=I(\alpha_{20}, \alpha_{21}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$ *and let* x *be an element of* U *such that* $I(x)=J_{24}$. *Then* $x \underset{c}{\sim} z_{105}=x_{20}(1)x_{21}(1)x_{28}(1)x_{29}(1)x_{31}(1)x_{105}(1)$. $Z_G(z_{105}) \subset B\langle w_2w_3w_5w_{102}\rangle B$, $Z(z_{105})=1$, $L(z_{105})=A_1$, $|Z_{G(k)}(z_{105})|=(q^2-1)q^{34}$.

The proof is easy.

LEMMA 65. *Let* $J_{25}=I(\alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{25}$ *and* $f_1(x)=(t_{28}t_{39}-t_{31}t_{37}-t_{32}t_{36})t_{27}t_{28}-(t_{30}t_{32}-t_{34}t_{28})(t_{29}t_{32}-t_{28}t_{33})\neq 0$. *Then* $x \underset{c}{\sim} z_{106}=x_{28}(1)x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{39}(1)x_{102}(1)$. *Furthermore a k-rational point in* $G(z_{106})$ *is conjugate to* z_{106} *or* $z_{107}=z_{106}x_{39}(\zeta-1)$. $Z_G(z_{106})\subset B\langle w_5\rangle B$, $Z(z_{106})\cong Z_{(2,p-1)}$, $L(z_{106})=A_1$, $|Z_{G(k)}(z_i)|=(2, p-1)(q^2-1)q^{34}(i=106, 107)$.

The proof is easy.

LEMMA 66. *Let* $J_{26}=I(\alpha_{28}, \alpha_{29}, \alpha_{31}, \alpha_{34}, \alpha_{101})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{26}$ *and* $t_{28}t_{33}-t_{29}t_{32}\neq 0$. *Then* $x\underset{c}{\sim} z_{108}=x_{28}(1)x_{29}(1)x_{33}(1)x_{31}(1)x_{34}(1)x_{101}(1)$. *Furthermore a k-rational point in* $G(z_{108})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$
\begin{aligned}
&z_{108}=x_{28}(1)x_{29}(1)x_{33}(1)x_{31}(1)x_{34}(1)x_{101}(1), \\
&z_{109}=x_{26}(1)x_{28}(1)x_{29}(1)x_{38}(\zeta)x_{31}(1)x_{34}(1)x_{101}(1)\,, && \text{when } \mathrm{ch}(K)\neq 2\,, \\
&z_{110}=z_{108}x_{121}(\eta)\,, && \text{when } \mathrm{ch}(K)=2\,, \\
&z_{111}=x_{26}(1)x_{28}(1)x_{29}(1)x_{33}(1)x_{38}(\eta)x_{31}(1)x_{34}(1)x_{101}(1)\,, && \text{when } \mathrm{ch}(K)=2\,, \\
&z_{112}=z_{111}x_{121}(\eta)\,, && \text{when } \mathrm{ch}(K)=2\,.
\end{aligned}
$$

$Z_G(z_{108})\subset B\langle w_4w_6, w_5\rangle B$, $Z(z_{108})\simeq Z_2\times Z_{(2,p)}$, $L(z_{108})=2A_1$, $|Z_{G(k)}(z_i)|=2(2, p)(q^2-1)^2q^{34}(i=108, 110)$ $|Z_{G(k)}(z_i)|=2(2, p)(q^4-1)q^{34}(i=109, 111, 112)$.

PROOF. By the action of B, $x\underset{c}{\sim} y(a)=z_{108}x_{121}(a)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{108}\mathfrak{X}_{121}$, then $uy(a)u^{-1}=y(a-u_{28}^2-u_{28}+2u_{108})$. Hence $x\underset{c}{\sim} z_{108}$. Thus we put $P=B\langle w_4, w_5, w_6\rangle B$, $R=U(I(\alpha_2, \alpha_3, \alpha_7, \alpha_{101}))$, $V=U(I(\alpha_{15}, \alpha_{31}, \alpha_{34}, \alpha_{101}))$, $V_1=U(I(\alpha_{36}, \alpha_{37}, \alpha_{102}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{108})$. Now the proof is clear.

LEMMA 67. *Let* $J_{27}=I(\alpha_{20}, \alpha_{21}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{105})$ *and let* x *be an element of* U *such that* $I(x)=J_{27}$. *Then* $x\underset{c}{\sim} z_{113}=x_{20}(1)x_{21}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{105}(1)$. *Furthermore a k-rational point in* $G(z_{113})$ *is conjugate in* $G(k)$ *to* z_{113} *or* $z_{114}=z_{113}x_{49}(1)x_{119}(\eta)$. $Z_G(z_{113})\subset B\langle w_2w_3w_5w_{102}, w_{28}\rangle B$, $Z(z_{113})\cong Z_{(2,p)}$,

$$L(z_{113})=\begin{cases}A_1+T_1 & \textit{if}\ \ \mathrm{ch}(K)=2,\\ 2A_1 & \textit{if}\ \ \mathrm{ch}(K)\neq 2.\end{cases}$$

$$Z_{G(k)}(z_{113})=\begin{cases}(q^2-1)^2q^{34} & \textit{if}\ \ \mathrm{ch}(K)\neq 2\ ,\\ 2(q-1)(q^2-1)q^{35} & \textit{if}\ \ \mathrm{ch}(K)=2\ .\end{cases}$$

$Z_{G(k)}(z_{114})=2(q+1)(q^2-1)q^{35}$ *if* $\mathrm{ch}(K)=2$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_2w_3w_5w_{102},\ w_{28}\rangle$. If $\mathrm{ch}(K)\neq 2$, the proof is easy. Thus we assume $\mathrm{ch}(K)=2$. By the action of B, $x\underset{c}{\sim}y(a,\ b)=z_{113}x_{49}(a)x_{120}(b)$ for some $a,\ b\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{113}\mathfrak{X}_{49}\mathfrak{X}_{120}$, $uy(a,\ b)u^{-1}=y(a,\ b+u_2+u_2^2a)$. From this fact, we get the lemma.

LEMMA 68. *$G'(J_{22})=G(z_{98})=G(J_{22})-G(J_{25})$, $G'(J_{24})=G(z_{105})=G(J_{24})-G(J_{27})$, $G'(J_{25})=G(z_{106})=G(J_{25})-\{G(J_{26})\cup G(J_{27})\}$, $G(J_{23})=G(z_{101})\cup G(z_{103})\cup G(J_{24})\cup G(J_{25})$.*

If $\mathrm{ch}(K)=3$, *the Zariski closure* $\overline{G(z_{88})}=G(_{88})\cup G(J_{24})$.

PROOF. By calculations, we get $\overline{B(z_{88})}\subseteqq\{x=\prod x_i(t_i)\mid f_1(x)=0,\ f_2(x)=0\}$, where $f_1(x)=(t_{21}t_{25}t_{29}-t_{21}t_{24}t_{30}+t_{21}t_{23}t_{31}+t_{23}t_{25}t_{26}-t_{22}t_{25}t_{27})t_{103}+t_{20}t_{23}t_{25}t_{104}$, $f_2(x)=(t_{20}t_{40}t_{103}-t_{24}t_{37}t_{103}-t_{22}t_{38}t_{103}-t_{20}t_{24}t_{107}+t_{20}t_{23}t_{106})t_{22}t_{25}-t_{21}t_{22}t_{24}t_{41}t_{103}+(-t_{23}t_{31}+t_{30}t_{24}+t_{25}t_{29})(t_{22}t_{32}-t_{26}t_{28})t_{103}-t_{21}t_{24}t_{28}t_{36}t_{103}-(t_{31}t_{21}-t_{25}t_{26})t_{24}t_{35}t_{103}-t_{20}t_{24}t_{25}t_{35}t_{104}-(t_{30}t_{24}-t_{31}t_{23})t_{20}t_{22}t_{105}-(t_{36}t_{24}t_{21}t_{103}-t_{29}t_{31}t_{21}t_{103})t_{28}-(t_{26}t_{103}-t_{20}t_{104})t_{25}t_{28}t_{29}+(t_{21}t_{31}t_{29}+t_{22}t_{25}t_{33})t_{28}t_{103}+(t_{23}t_{31}-t_{30}t_{24})t_{20}t_{28}t_{104}$. Let $y\in\overline{B(z_{88})}-B(z_{88})$. Suppose $u_{103}=0$. Then $u_{20}u_{23}u_{25}u_{104}=0$ and $u_{20}((-u_{24}u_{107}+u_{23}u_{106})u_{22}u_{25}+u_{24}u_{25}u_{35}u_{104}+u_{22}u_{105}(u_{24}u_{30}-u_{31}u_{23})-u_{28}(u_{25}u_{29}u_{104}+u_{23}u_{31}u_{104}+u_{24}u_{30}u_{104}))=0$. If $u_{20}=0$, y is in $G(J_{25})$. If $u_{25}=0$, $(u_{28}u_{104}+u_{22}u_{105})(u_{24}u_{30}-u_{31}u_{23})=0$. Hence y is in $G(J_{25})$. If $u_{104}=0$, y is in $G(J_{25})$. By similar way, we get $\overline{B(z_{88})}\subseteqq B(z_{88})\cup G(J_{25})$. The opposite inclusion is obvious. The rest of the lemma follows from Lemmas 1–6.

LEMMA 69. *Let $J_{28}=I(\alpha_{30},\ \alpha_{31},\ \alpha_{32},\ \alpha_{33},\ \alpha_{40},\ \alpha_{101})$ and let x be an element of U such that $I(x)=J_{28}$. Then $x\underset{c}{\sim}z_{115}=x_{32}(1)x_{33}(1)x_{40}(1)x_{30}(1)x_{31}(1)x_{101}(1)$. Furthermore a k-rational point in $G(z_{115})$ is conjugate in $G(k)$ to z_{115} or $z_{116}=z_{115}x_{121}(\eta)$. $Z_G(z_{115})\subset B\langle w_4w_6,\ w_{15}\rangle B$, $Z(z_{115})\cong Z_{(2,p)}$, $L(z_{115})=2A_1$, $|Z_{G(k)}(z_i)|=(2,\ p)(q^2-1)^2q^{36}(i=115,\ 116)$.*

The proof is easy.

LEMMA 70. *Let $J_{29}=I(\alpha_{26},\ \alpha_{29},\ \alpha_{30},\ \alpha_{105})$ and let $x=\prod x_i(t_i)$ be an element of U such that $I(x)=J_{29}$, $f_1(x)f_2(x)f_3(x)\neq 0$, where $f_1(x)=t_{26}t_{35}-t_{29}t_{32}$,*

$f_2(x)=(t_{26}t_{38}-t_{32}t_{33})t_{30}-f_1(x)t_{34}$ *and* $f_3(x)=(t_{26}t_{44}-t_{32}t_{39})t_{29}t_{30}t_{105}+t_{30}t_{32}t_{33}(t_{36}t_{105}-t_{107}t_{30})-t_{26}t_{30}t_{36}t_{38}t_{105}+t_{26}t_{30}t_{30}t_{38}t_{107}-t_{26}t_{34}t_{41}t_{29}t_{105}-t_{26}t_{34}t_{35}(t_{36}t_{105}-t_{30}t_{107})$. *Furthermore we assume that x satisfies the following conditions;*

i) *if* $\mathrm{ch}(K)=2,\ f_4(x)\neq 0,$

ii) *if* $\mathrm{ch}(K)\neq 2,\ f_5(x)\neq 0,$

where $f_4(x)=t_{30}t_{105}(t_{107}t_{30}-t_{36}t_{105})(t_{26}t_{42}-t_{33}t_{37})-f_3(x)t_{106}+t_{30}t_{105}t_{109}((t_{32}t_{33}-t_{26}t_{38})t_{30}+t_{26}t_{34}t_{35})+t_{26}t_{29}t_{30}t_{105}(t_{47}t_{105}+t_{30}t_{112}-t_{34}t_{111})-t_{29}t_{30}t_{105}(t_{30}t_{32}t_{110}+t_{37}t_{39}t_{105}+t_{34}t_{37}t_{107})-(t_{30}t_{108}-t_{34}t_{106})((t_{29}t_{41}-t_{35}t_{36})t_{26}t_{105}+(t_{26}t_{35}-t_{29}t_{32})t_{30}t_{107})$, $f_5(x)=4f_6(x)f_3(x)t_{105}-f_4(x)^2$, $f_6(x)=t_{26}t_{29}t_{30}(t_{30}t_{105}t_{114}-t_{30}t_{108}t_{111}+t_{34}t_{111}t_{106})+t_{26}t_{30}t_{35}t_{109}(t_{30}t_{108}-t_{34}t_{106})+t_{30}t_{30}t_{33}t_{37}t_{105}t_{109}+t_{29}t_{30}t_{37}(t_{30}t_{107}t_{108}-t_{34}t_{106}t_{107}+t_{39}t_{105}t_{106})+t_{30}t_{36}t_{106}(t_{26}t_{42}t_{105}-t_{26}t_{35}t_{108}-t_{33}t_{37}t_{105})+t_{26}t_{34}t_{35}t_{36}t_{106}{}^2+t_{26}t_{29}t_{41}t_{106}(t_{30}t_{108}-t_{34}t_{106})-t_{26}t_{29}t_{30}t_{47}t_{105}t_{106}$.

Then $x\underset{c}{\sim}z_{117}=x_{26}(1)x_{29}(1)x_{35}(1)x_{38}(1)x_{30}(1)x_{44}(1)x_{47}(1)x_{105}(1)$. $Z_G(z_{117})\subset B\langle w_1, w_2, w_3, w_4, w_6, w_7, w_{101}\rangle B$, $Z(z_{117})\cong S_5$, $Z_G(z_{117})^\circ\subset U$, $\dim Z_G(z_{117})=40$.

PROOF. Lemma 8, we get $W(x)\subseteqq X$, where

$$X=\left|\begin{array}{l}1,\ w_6w_1,\ w_7w_9,\ w_6w_7w_1w_9,\ w_7w_6w_1w_9,\ w_{13}w_1w_9,\ w_{101}w_3w_{14},\ w_{101}w_6w_3w_{14}\ ,\\ w_{101}w_7w_3w_9w_{14},\ w_7w_{101}w_{14}w_9w_8,\ w_{101}w_6w_7w_{10}w_1w_3w_9\ ,\\ w_7w_{101}w_6w_9w_3w_1w_{10},\ w_{101}w_7w_6w_8w_1w_4w_3w_2w_4,\ w_6w_7w_{101}w_9w_{10}w_1w_3\ ,\\ w_{101}w_{13}w_8w_9w_{10},\ w_{13}w_{101}w_{10}w_9w_8,\ w_{102}w_4w_{15},\ w_{102}w_{14}w_{15}\ ,\\ w_{13}w_{102}w_{10}w_{20},\ w_{102}w_6w_{15}w_1w_{10},\ w_6w_{102}w_{10}w_1w_{15},\ w_{102}w_6w_{15}w_4w_1\ ,\\ w_6w_{102}w_1w_4w_{15},\ w_7w_{101}w_{13}w_{10}w_{20},\ w_{13}w_{101}w_7w_{10}w_{20}\ ,\\ w_7w_{101}w_{13}w_2w_4w_3w_1,\ w_{13}w_{101}w_7w_1w_3w_4w_2,\ w_{103}w_{10}w_{20},\ w_{103}w_4w_{20}\ ,\\ w_7w_{103}w_{10}w_{20},\ w_7w_{103}w_2w_8,\ w_7w_{103}w_2w_4w_8,\ w_7w_{103}w_8w_4w_2\ .\end{array}\right|$$

Thus we put $P=B\langle w_1, w_2, w_3, w_4, w_6, w_7, w_{101}\rangle B$, $R=\mathrm{Ru}(P)$, $V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{117})$. On the other hand, by the action of B, $x\underset{c}{\sim}z_{117}$. Thus we compute the order of $Z_{P/R}(z_{117}V_1)$. That is 120. On the other hand, $Z_{P/R}(z_{117}V_1)$ contains the group L generated by $\bar{g}_1, \bar{g}_2, \bar{g}_3, \bar{g}_4$. Here

$$g_1=h_1(-1)h_2(-1)h_{104}(-1)x_3(1)x_{14}(-1)x_{20}(-1)x_{101}(1)\ ,$$
$$g_2=h_{109}(-1)x_3(1)x_{14}(-1)x_{101}(1)x_7(-1)x_9(-1)\ ,$$
$$\omega_{14}\omega_3\omega_{101}x_7(1)x_9(1)x_3(1)x_{14}(-1)x_{101}(1)\ ,$$
$$g_3=h_{27}(-1)x_{14}(-1)x_6(-1)x_1(-1)x_{-7}(1)x_{-9}(1)\ ,$$
$$g_4=h_2(-1)h_3(-1)h_5(-1)x_9(1)x_4(1)x_{-6}(-1)x_{-1}(-1)\ .$$

Since L is isomorphic to S_5, we get the lemma.

LEMMA 71. *$G'(J_{26})=G(z_{108})=G(J_{26})-\{G(J_{28})\cup G(J_{29})\}$, $G'(J_{27})=G(z_{113})=G(J_{27})-G(J_{29})$.*

If $\mathrm{ch}(K)=2$, *then* $\overline{G(z_{101})}=G(z_{101})\cup G(J_{28})\cup G(J_{29})$.

PROOF. By calculations, we get $\overline{G(z_{101})}=\{y=\prod x_i(u_i)|I(y)\subseteqq J_{24}, f(y)=0\}$, where $f(y)=u_{30}u_{102}^2(u_{29}u_{32}+u_{26}u_{35})^2+u_{28}u_{29}u_{34}u_{102}^2(u_{26}u_{35}+u_{29}u_{32})+u_{28}u_{29}u_{33}u_{102}^2(u_{20}u_{32}+u_{28}u_{34})+u_{26}u_{28}u_{29}u_{30}u_{38}u_{102}^2+u_{27}u_{28}u_{29}u_{102}^2(u_{26}u_{41}+u_{39}u_{28}+u_{31}u_{37})+u_{30}u_{26}u_{28}u_{103}^2$. Let y be an element of $\overline{G(z_{101})}-G(z_{101})$. Then $u_{26}u_{27}u_{28}u_{29}u_{30}u_{31}u_{102}=0$. Suppose $u_{26}=0$. Then $u_{29}u_{102}((u_{28}u_{33}+u_{29}u_{32})(u_{30}u_{32}+u_{34}u_{28})+u_{27}u_{28}(u_{39}u_{28}+u_{31}u_{37}))=0$. If $u_{29}u_{102}=0$, then y is in $G(J_{29})$. If $u_{29}u_{102}\neq 0$, y is conjugate to an element of $U(I(\alpha_{27}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{37}, \alpha_{101}))$. Hence y is in $G(J_{28})$. Suppose $u_{102}=0$. Then $u_{28}u_{26}u_{103}=0$. Hence y is in $G(J_{29})$. Suppose $u_{28}=0$. Then $u_{30}u_{102}(u_{29}u_{32}+u_{26}u_{35})=0$. Furthermore, if $u_{30}=0$, y is in $G(J_{29})$. If $u_{29}u_{32}+u_{26}u_{35}=0$, y is in $G(J_{28})$. Suppose $u_{29}=0$. Then $u_{30}u_{26}(u_{28}u_{103}+u_{35}u_{102})=0$. Hence y is in $G(J_{29})$. Suppose $u_{30}=0$. Then $u_{102}(u_{27}u_{28}u_{29}(u_{26}u_{41}+u_{28}u_{39}+u_{31}u_{37})+u_{28}u_{29}u_{33}u_{28}u_{34}+u_{28}u_{29}u_{34}(u_{26}u_{35}+u_{29}u_{32}))=0$. If $u_{102}\neq 0$, y is in $G(J_{28})$. Suppose $u_{27}u_{31}=0$. Then y is in $G(J_{29})$. Hence $\overline{G(z_{101})}\subseteqq G(z_{101})\cup G(J_{28})\cup G(J_{29})$. The opposite inclusion is clear. The rest of the proof is easy.

LEMMA 72. *Let $J_{30}=I(\alpha_{29}, \alpha_{31}, \alpha_{32}, \alpha_{34}, \alpha_{105})$ and let $x=\prod x_i(t_i)$ be an element of U such that $I(x)=J_{30}$ and $t_{37}t_{105}+t_{32}t_{106}\neq 0$. Furthermore suppose that x satisfies the conditions;*

i) *if* $\mathrm{ch}(K)=2$, $f_2(x)\neq 0$,

ii) *if* $\mathrm{ch}(K)\neq 2$, $f_3(x)\neq 0$,

where $f_2(x)=(t_{34}t_{107}-t_{31}t_{108}+t_{39}t_{105})t_{29}+t_{33}(t_{31}t_{106}-t_{36}t_{105})$ $f_3(x)=4((t_{29}t_{110}-t_{33}t_{109})t_{31}-(t_{29}t_{39}-t_{33}t_{36})t_{107})t_{29}t_{34}t_{105}-f_2(x)^2$. *Then* $x\underset{c}{\sim}z_{118}=x_{32}(1)x_{37}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{105}(1)x_{107}(1)$. *A k-rational point in $G(z_{118})$ is conjugate in $G(k)$ to one of the following elements;*

$z_{118}=x_{32}(1)x_{37}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{105}(1)x_{107}(1)$,

$z_{119}=z_{118}x_{110}(\eta)$, *when* $\mathrm{ch}(K)=2$,

$z_{120}=x_{32}(1)x_{37}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{105}(1)x_{110}(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$,

$z_{121}=x_{37}(1)x_{27}(1)x_{31}(1)x_{29}(1)x_{30}(1)x_{105}(1)x_{108}(-1)x_{110}(\tau)$.

$Z_G(z_{118})\subset B\langle w_5, w_2w_8w_{102}\rangle B$, $Z(z_{118})\cong S_3$, $L(z_{118})=A_1$, $|Z_{G(k)}(z_{118})|=6(q^2-1)q^{40}$, $|Z_{G(k)}(z_i)|=2(q^2-1)q^{40}$ $(i=119, 120)$, $|Z_{G(k)}(z_{121})|=3(q^2-1)q^{40}$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_5, w_2w_8w_{102}\rangle$. Thus we put $P=B\langle w_1, w_2, w_3, w_\kappa, w_7, w_{101}\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{28}, \alpha_{18}))$, $V_1=U(I(\alpha_{35}))$. Then (P, R, V, V_1) gives a structure of $Z_G(x)$. By the action of B, we get $x\underset{c}{\sim}z_{118}$. Now the proof is easy.

LEMMA 73. *Let* $J_{31}=I(\alpha_{27}, \alpha_{32}, \alpha_{36}, \alpha_{40}, \alpha_{105})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{31}$ *and* $(t_{32}t_{106}+t_{37}t_{105})(t_{33}t_{105}-t_{27}t_{107})\neq 0$. *Then* $x\underset{c}{\sim}z_{122}=x_{32}(1)x_{37}(1)x_{27}(1)x_{33}(1)x_{40}(1)x_{36}(1)x_{105}(1)$. *A k-rational point in* $G(z_{122})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{122}=x_{32}(1)x_{37}(1)x_{27}(1)x_{33}(1)x_{40}(1)x_{36}(1)x_{105}(1)\ ,$$
$$z_{123}=x_{37}(1)x_{33}(1)x_{40}(1)x_{30}(1)x_{31}(1)x_{105}(1)x_{107}(1)x_{109}(\eta)\ ,\quad \text{when } \mathrm{ch}(K)=2\ ,$$
$$z_{124}=x_{37}(1)x_{33}(1)x_{40}(1)x_{30}(1)x_{31}(1)x_{105}(1)x_{109}(\zeta)\ ,\quad \text{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{122})\subset B\langle w_9w_{10}w_6, w_2w_3w_{101}\rangle B$, $Z(z_{122})\cong Z_2$, $L(z_{122})=A_1$, $|Z_{G(k)}(z_i)|=2(q^2-1)q^{42}$ $(i=122, 123, 124)$.

PROOF. By Lemma 8, we get $W(x)\subset\langle w_2w_3w_{101}, w_9w_{10}w_6\rangle$. By the action of B, we get $x\underset{c}{\sim}z_{122}$. Thus we put $P=B\langle w_2, w_3, w_4, w_6, w_{101}\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{19}, \alpha_{21}, \alpha_{40}))$, $V_1=U(I(\alpha_{34}, \alpha_{43}, \alpha_{45}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{122})$. Now the proof is easy.

LEMMA 74. *Let* $J_{32}=I(\alpha_{29}, \alpha_{31}, \alpha_{32}, \alpha_{34}, \alpha_{106})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{32}$ *and* $(t_{31}t_{108}-t_{34}t_{107})t_{29}-t_{31}t_{33}t_{106}\neq 0$. *Then* $x\underset{c}{\sim}z_{125}=x_{32}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{106}(1)x_{108}(1)$. *A k-rational point in* $G(z_{125})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{125}=x_{32}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{106}(1)x_{108}(1)\ ,$$
$$z_{126}=x_{26}(1)x_{28}(1)x_{29}(1)x_{38}(\zeta)x_{34}(1)x_{107}(1)x_{106}(1)\ ,\quad \text{when } \mathrm{ch}(K)\neq 2\ ,$$
$$z_{127}=x_{26}(1)x_{28}(1)x_{29}(1)x_{35}(1)x_{38}(\eta)x_{34}(1)x_{106}(1)x_{107}(1)\ ,\quad \text{when } \mathrm{ch}(K)=2\ .$$

$Z_G(z_{125})\subset B\langle w_1w_4w_6w_{101}, w_5\rangle B$, $Z(z_{125})\cong Z_2$, $L(z_{125})=2A_1$, $|Z_{G(k)}(z_{125})|=2(q^2-1)^2q^{40}$ *and* $|Z_{G(k)}(z_i)|=2(q^4-1)q^{40}(i=126, 127)$.

LEMMA 75. *Let* $J_{33}=I(\alpha_{30}, \alpha_{31}, \alpha_{32}, \alpha_{33}, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{33}$. *Then* $x\underset{c}{\sim}z_{128}=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(1)x_{101}(1)$. *A k-rational point in* $G(z_{128})$ *is conjugate in* $G(k)$ *to* z_{128} *or* $z_{129}=z_{128}x_{121}(\eta)$. $Z_G(z_{128})\subset B\langle w_4w_6, w_5, w_{15}\rangle B$, $Z(z_{128})\cong Z_{(2,p)}$, $L(z_{128})=B_3$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)(q^4-1)(q^6-1)q^{36}(i=128, 129)$.

PROOF. By the action of B, $x\underset{c}{\sim}y(a)=z_{128}x_{121}(a)$ for some $a\in K$. If $u=\prod x_i(t_i)$ stabilizes the set $z_{128}\mathfrak{X}_{121}$, then $uy(a)u^{-1}=y(a+u_{30}+u_{30}^2)$ in the case $\mathrm{ch}(K)=2$. By Lemma 8, we get $W(x)\subseteq\langle w_4w_6, w_5, w_{15}\rangle$. Thus we put $P=B\langle w_4, w_5, w_6, w_2, w_3\rangle B$, $R=\mathrm{Ru}(P)$, $V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{128})$. Now the proof is easy.

LEMMA 76. $G'(J_{28})=G(z_{115})-\{G(J_{31})\cup G(J_{33})\}$, $G'(J_{29})=G(z_{117})=G(J_{29})-$

$G(J_{30})$, $G'(J_{30})=G(z_{118})=G(J_{30})-\{G(J_{31})\cup G(J_{32})\}$.

The proof is easy.

LEMMA 77. *Let* $J_{34}=I(\alpha_{33}, \alpha_{34}, \alpha_{36}, \alpha_{37}, \alpha_{40}, \alpha_{105})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{34}$ *and* $t_{36}t_{38}-t_{33}t_{41}\neq 0$. *Then* $x\underset{c}{\sim} z_{130}= x_{37}(1)x_{33}(1)x_{40}(1)x_{34}(1)x_{36}(1)x_{41}(1)x_{105}(1)$. $Z_G(z_{130})\subset B\langle w_1w_{17}w_7\rangle B$, $Z(z_{130})=1$, $L(z_{130})=A_1$, $|Z_{G(k)}(z_{130})|=(q^2-1)q^{44}$.

The proof is easy.

LEMMA 78. *Let* $J_{35}=I(\alpha_{26}, \alpha_{27}, \alpha_{40}, \alpha_{41}, \alpha_{106}, \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{35}$. *Then* $x\underset{c}{\sim} z_{131}=x_{26}(1)x_{27}(1)x_{40}(1)x_{41}(1)x_{106}(1)x_{107}(1)$. $Z_G(z_{131})\subset B\langle w_2w_3w_6, w_{19}\rangle B$, $Z(z_{131})=1$, $L(z_{131})=2A_1$, $|Z_{G(k)}(z_{131})|=(q^2-1)^2q^{42}$.

The proof is easy.

LEMMA 79. *Let* $J_{36}=I(\alpha_{27}, \alpha_{32}, \alpha_{36}, \alpha_{105})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{36}$ *and* $(t_{32}t_{106}+t_{37}t_{105})(t_{33}t_{105}-t_{27}t_{107})\neq 0$. *Then* $x\underset{c}{\sim} z_{132}=x_{32}(1)x_{27}(1)x_{37}(1)x_{33}(1)x_{36}(1)x_{105}(1)$. *A k-rational point in* $G(z_{132})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{132}=x_{32}(1)x_{27}(1)x_{37}(1)x_{33}(1)x_{36}(1)x_{105}(1)\ ,$$
$$z_{133}=x_{37}(1)x_{33}(1)x_{30}(1)x_{31}(1)x_{105}(1)x_{107}(1)x_{109}(\eta)\ ,\qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{134}=x_{37}(1)x_{33}(1)x_{30}(1)x_{31}(1)x_{105}(1)x_{109}(\zeta)\ ,\qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{132})\subset B\langle w_2w_3w_{101}, w_9w_{10}w_6, w_5\rangle B$, $Z(z_{132})\cong Z_2$, $L(z_{132})=G_2$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^6-1)q^{42}(i=132, 133, 134)$.

The proof is similar to that of Lemma 73.

LEMMA 80. $G'(J_{32})=G(z_{125})=G(J_{32})-\{G(J_{34})\cup G(J_{35})\}$, $G'(J_{31})=G(z_{122})=G(J_{31})-\{G(J_{34})\cup G(J_{35})\cup G(J_{36})\}$, $G'(J_{33})=G(z_{128})=G(J_{33})-G(J_{36})$.

The proof is easy.

LEMMA 81. *Let* $J_{37}=I(\alpha_{37}, \alpha_{38}, \alpha_{39}, \alpha_{40}, \alpha_{41}, \alpha_{104})$ *and let* x *be an element of* U *such that* $I(x)=J_{37}$.

1) *Suppose* $\mathrm{ch}(K)\neq 2$. *Then* $x\underset{c}{\sim} z_{135}=x_{37}(1)x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{104}(1)$. *A k-rational point in* $G(z_{135})$ *is conjugate to* z_{136} *or* $z_{136}=x_{37}(1)x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{48}(\zeta)x_{103}(1)$. $Z_G(z_{135})\subset B\langle w_3w_6, w_{21}w_{25}, w_1w_7w_{11}\rangle B$, $Z(z_{135})\cong Z_2$, $L(z_{135})=A_2$, $|Z_{G(k)}(z_{135})|=2(q^2-1)(q^3-1)q^{45}$, $|Z_{G(k)}(z_{136})|=2(q^2-1)(q^3+1)q^{45}$.

2) *Suppose* $\mathrm{ch}(K)=2$. *Then* x *is conjugate to* z_{135} *or* $z_{137}=z_{135}x_{108}(1)$.

A k-rational point in $G(z_{135})$ *is conjugate in* $G(k)$ *to* z_{135} *or* $z_{138}=z_{135}x_{140}(\eta)$. $Z_G(z_{135})\subset B\langle w_3w_6,\ w_{21}w_{25},\ w_1w_7w_{11}\rangle B$, $Z(z_{135})\cong Z_2$, $L(z_{135})=G_2$, $Z_G(z_{137})\subset B\langle w_3w_6\rangle B$, $Z(z_{137})=1$, $L(z_{137})=A_1$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^6-1)q^{48}(i=135,\ 138)$, $|Z_{G(k)}(z_{137})|=(q^2-1)q^{48}$.

PROOF. By Lemma 8, we get $W(x)\subsetneqq\langle w_3w_6,\ w_{21}w_{25},\ w_1w_7w_{11}\rangle$. By the action of B, $z\underset{c}{\sim}y(a,b,c,d)=z_{135}x_{105}(a)x_{106}(b)x_{108}(c)x_{140}(d)$ for some $a, b, c, d\in K$. Suppose $\mathrm{ch}(K)\neq 2$. Then $x\underset{c}{\sim}z_{135}$. Suppose $\mathrm{ch}(K)=2$. By the action of $B\langle w_3w_6\rangle B$, we may assume $a=0$. Furthermore, by the action of $B\langle w_1w_7w_{11}\rangle B$ we may assume $a=b=0$. Thus we put $y(c,d)=y(0,0,c,d)$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{135}\mathfrak{X}_{108}\mathfrak{X}_{140}$, then $uy(c,d)^{-1}=y(c,\ d+u_{29}u_{17}c^2+u_{22}u_2c^3+u_{22}{}^2c^2+u_{53}c+u_{104}^2+u_{104})$. Hence x is conjugate to z_{135} or z_{137}. Let $P=B\langle w_1,\ w_3,\ w_4,\ w_5,\ w_6,\ w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{137})$. Now the proof is easy.

LEMMA 82. *Let* $J_{38}=I(\alpha_{34},\ \alpha_{36},\ \alpha_{37},\ \alpha_{38},\ \alpha_{40},\ \alpha_{106},\ \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{38}$. *Then* $x\underset{c}{\sim}z_{139}=x_{37}(1)x_{38}(1)x_{40}(1)x_{34}(1)x_{36}(1)x_{106}(1)x_{107}(1)$. $Z_G(z_{139})\subset B\langle w_2w_8w_{12}w_{101}\rangle B$, $Z(z_{139})=1$, $L(z_{139})=A_1$, $|Z_{G(k)}(z_{139})|=(q^2-1)q^{46}$.

The proof is easy.

LEMMA 83. *Let* $J_{39}=I(\alpha_{20},\ \alpha_{21},\ \alpha_{49},\ \alpha_{106},\ \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{39}$. *Then* $x\underset{c}{\sim}z_{140}=x_{20}(1)x_{21}(1)x_{49}(1)x_{106}(1)x_{107}(1)$. $Z_G(z_{140})\subset B\langle w_2w_3w_5,\ w_4,\ w_7\rangle B$, $Z(z_{140})=1$, $L(z_{140})=A_1+G_2$, $|Z_{G(k)}(z_{140})|=(q^2-1)(q^6-1)q^{42}$.

The proof is easy.

LEMMA 84. *Let* $J_{40}=I(\alpha_{37},\ \alpha_{43},\ \alpha_{44},\ \alpha_{45},\ \alpha_{103})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{40}$ *and* $t_{42}t_{45}-t_{37}t_{49}\neq 0$. *Then* $x\underset{c}{\sim}z_{141}=x_{37}(1)x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{103}(1)$. $Z_G(z_{141})\subset B\langle w_2,\ w_8w_{19}\rangle B$, $Z(z_{141})=1$, $L(z_{141})=2A_1$, $|Z_{G(k)}(z_{141})|=(q^2-1)^2q^{48}$.

The proof is easy.

LEMMA 85. $G'(J_{34})=G(z_{130})=G(J_{34})-\{G(J_{37})\cup G(J_{38})\}$, $G'(J_{35})=G(z_{131})=G(J_{35})-\{G(J_{38})\cup G(J_{39})\}$, $G'(J_{36})=G(z_{132})=G(J_{36})-\{G(J_{39})\cup G(J_{40})\}$.

The proof is easy.

LEMMA 86. *Let* $J_{41}=I(\alpha_{37},\ \alpha_{38},\ \alpha_{39},\ \alpha_{40},\ \alpha_{41},\ \alpha_{108},\ \alpha_{109})$ *and let* x *be an element of* U *such that* $I(x)=J_{41}$. *Then* $x\underset{c}{\sim}z_{142}=x_{37}(1)x_{38}(1)x_{39}(1)x_{40}(1)x_{41}(1)x_{108}(1)x_{109}(1)$. $Z_G(z_{142})\subset B\langle w_1w_4w_{17}w_{102}\rangle B$, $Z(z_{142})=1$, $L(z_{142})=A_1$, $|Z_{G(k)}(z_{142})|=(q^2-1)q^{50}$.

The proof is easy.

LEMMA 87. *Let* $J_{42}=I(\alpha_{38}, \alpha_{39}, \alpha_{40}, \alpha_{41}, \alpha_{108}, \alpha_{109})$ *and let* x *be an element of* U *such that* $I(x)=J_{42}$. *Then* $x \underset{c}{\sim} z_{143}=x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{108}(1) x_{109}(1)$. $Z_G(z_{143})\subset B\langle w_{37}, w_1w_4w_{17}w_{102}\rangle B$, $Z(z_{143})=1$, $L(z_{143})=2A_1$, $|Z_{G(k)}(z_{143})|=(q^2-1)^2q^{50}$.

The proof is easy.

LEMMA 88. *Let* $J_{43}=I(\alpha_{42}, \alpha_{43}, \alpha_{49}, \alpha_{102})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{43}$ *and* $t_{43}t_{47}-t_{42}t_{48}\neq 0$. *Then* $x \underset{c}{\sim} z_{144}=x_{42}(1) x_{43}(1)x_{48}(1)x_{49}(1)x_{102}(1)$. *A k-rational point in* $G(z_{144})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$z_{144}=x_{42}(1)x_{43}(1)x_{48}(1)x_{102}(1)$,

$z_{145}=x_{42}(1)x_{43}(1)x_{44}(1)x_{48}(1)x_{51}(\eta)x_{49}(1)x_{102}(1)$, *when* $\mathrm{ch}(K)=2$,

$z_{146}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)x_{49}(1)x_{102}(1)$, *when* $\mathrm{ch}(K)\neq 2$.

$Z_G(z_{144})\subset B\langle w_2, w_4, w_3w_5\rangle B$, $Z(z_{144})\cong Z_2$, $L(z_{144})=A_3$, $|Z_{G(k)}(z_{142})|=2(q^2-1)(q^3-1)(q^4-1)q^{49}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^3+1)(q^4-1)q^{49}$ $(i=145, 146)$,

The proof is easy.

LEMMA 89. $G(J_{37})=G(z_{135})\cup G(z_{137})\cup B(J_{40})\cup G(J_{41})$,

$$G'(J_{38})=G(z_{139})=G(J_{38})-G(J_{41}) ,$$
$$G'(J_{39})=G(z_{140})=G(J_{39})-G(J_{42}) ,$$
$$G'(J_{40})=G(z_{141})=G(J_{40})-\{G(J_{42})\cup G(J_{43})\} ,$$
$$G'(J_{41})=G(z_{142})=G(J_{41})-G(J_{42}) .$$

The lemma follows from Lemmas 1–4.

LEMMA 90. *Let* $J_{44}=I(\alpha_{47}, \alpha_{48}, \alpha_{49}, \alpha_{53}, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{44}$. *Then* $x \underset{c}{\sim} z_{147}=x_{47}(1)x_{48}(1)x_{49}(1)x_{53}(1)x_{101}(1)$. *A k-rational point in* $G(z_{147})$ *is conjugate in* $G(k)$ *to* z_{147} *or* $z_{148}=z_{147}x_{143}(\eta)$. $Z_G(z_{147})\subset B\langle w_1w_6, w_3w_5, w_4\rangle B$, $Z(z_{147})\cong Z_{(2,p)}$, $L(z_{147})=C_3$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)(q^4-1)(q^6-1)q^{52}$ $(i=147, 148)$.

PROOF. By the action of B, $x \underset{c}{\sim} y(a)=z_{147}x_{143}(a)$ for some $a\in K$. If $u=\prod x_i(u_i)$ stabilizes the set $z_{147}\mathfrak{X}_{143}$, then $uy(a)u^{-1}=y(a+u_{101}^2-u_{101}+2u_{114})$. Hence $x \underset{c}{\sim} z_{147}$. Thus we put $P=B\langle w_1, w_3, w_4, w_5, w_6\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{31}, \alpha_{53}, \alpha_{101}))$, $V_1=U(I(\alpha_{56}, \alpha_{102}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{147})$. Now the proof is easy.

Lemma 91. *Let* $J_{45}=I(\alpha_{42}, \alpha_{43}, \alpha_{44}, \alpha_{45}, \alpha_{107}, \alpha_{108})$ *and let* x *be an element of* U *such that* $I(x)=J_{45}$. *Then* $x \underset{c}{\sim} z_{149}=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{107}(1)x_{108}(1)$. *A* k*-rational point in* $G(z_{149})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{149}=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{107}(1)x_{108}(1) ,$$
$$z_{150}=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{48}(1)x_{51}(\eta)x_{106}(1)x_{107}(1) , \quad \textit{when } \mathrm{ch}(K)=2 ,$$
$$z_{151}=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{51}(\zeta)x_{106}(1)x_{107}(1) , \quad \textit{when } \mathrm{ch}(K)\neq 2 .$$

$Z_G(z_{149})\subset B\langle w_2w_5w_7w_8, w_{12}w_{13}\rangle B$, $Z(z_{149})\cong Z_2$, $L(z_{149}) = T_1+ A_1$, $|Z_{G(k)}(z_{149})|=2(q-1)(q^2-1)q^{53}$, $|Z_{G(k)}(z_i)|=2(q+1)(q^2-1)q^{53}(i=150, 151)$.

Proof. By the action of B, we get $x \underset{c}{\sim} z_{149}$. By Lemma 8, $W(z_{149})\subset \langle w_2w_5w_7w_8, w_{12}w_{13}\rangle$. Thus we put $P=B\langle w_1, w_2, w_3, w_5, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{35}, \alpha_{105}))$, $V_1=U(I(\alpha_{50}, \alpha_{49}, \alpha_{111}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{149})$. Now the proof is easy.

Lemma 92. *Let* $J_{46}=I(\alpha_{47}, \alpha_{48}, \alpha_{49}, \alpha_{101})$ *and let* x *be an element of* U *such that* $I(x)=J_{46}$. *Then* $x \underset{c}{\sim} z_{152}=x_{47}(1)x_{48}(1)x_{49}(1)x_{101}(1)$. *A* k*-rational point in* $G(z_{152})$ *is conjugate in* $G(k)$ *to* z_{152} *or* $z_{153}=z_{152}x_{143}(\eta)$. $Z_G(z_{152})\subset B\langle w_1w_6, w_3w_5, w_4, w_2\rangle B$, $Z(z_{152})\cong Z_{(2,p)}$, $L(z_{152})=F_4$, $|Z_{G(k)}(z_i)|=(2, p)(q^2-1)(q^6-1)(q^8-1)(q^{12}-1)q^{52}(i=152, 153)$.

The proof is similar to that of Lemma 90.

Lemma 93. *Let* $J_{47}=I(\alpha_{42}, \alpha_{48}, \alpha_{49}, \alpha_{106}, \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{47}$. *Then* $x \underset{c}{\sim} z_{154}=x_{42}(1)x_{48}(1)x_{49}(1)x_{106}(1)x_{107}(1)$. *A* k*-rational point in* $G(z_{154})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{154}=x_{42}(1)x_{48}(1)x_{49}(1)x_{106}(1)x_{107}(1) ,$$
$$z_{155}=x_{42}(1)x_{43}(1)x_{44}(1)x_{48}(1)x_{51}(\eta)x_{49}(1)x_{106}(1)x_{107}(1) , \quad \textit{when } \mathrm{ch}(K)=2 ,$$
$$z_{156}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)x_{106}(1)x_{107}(1) , \quad \textit{when } \mathrm{ch}(K)\neq 2 .$$

$Z_G(z_{154})\subset B\langle w_4, w_2w_3w_5w_7\rangle B$, $Z(z_{154})\cong Z_2$, $L(z_{154})=T_1+A_2$, $|Z_{G(k)}(z_{154})|=2(q-1)(q^2-1)(q^3-1)q^{54}$, $|Z_{G(k)}(z_i)|=2(q+1)(q^2-1)(q^3+1)q^{54}(i=155, 156)$.

Proof. By Lemma 8, we get $W(x)\subseteq \langle w_4, w_2w_3w_5w_7\rangle$. By the action of B, we get $x \underset{c}{\sim} z_{154}$. Thus we put $P=B\langle w_2, w_3, w_4, w_5, w_7\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{27}, \alpha_{49}, \alpha_{103}))$, $V_1=U(I(\alpha_{52}, \alpha_{108}, \alpha_{116}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{154})$. Now the proof is easy.

LEMMA 94. *Let* $J_{48}=I(\alpha_{37}, \alpha_{38}, \alpha_{39}, \alpha_{49}, \alpha_{108}, \alpha_{113})$ *and let* x *be an element of* U *such that* $I(x)=J_{48}$. *Then* $x \underset{c}{\sim} z_{157}=x_{37}(1)x_{38}(1)x_{39}(1)x_{49}(1)x_{108}(1)x_{113}(1)$. $Z_G(z_{157})\subset B\langle w_4w_7, w_2w_3w_6w_{101}\rangle B$, $Z(z_{157})=1$, $L(z_{157})=B_2$, $|Z_{G(k)}(z_{157})| = (q^2-1)(q^4-1)q^{54}$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\langle w_4w_7, w_2w_3w_6w_{101}\rangle$. By the action of B, $x\underset{c}{\sim} z_{157}$. Thus we put $P=B\langle w_2, w_3, w_4, w_6, w_7, w_{101}\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{21}, \alpha_{40}))$, $V_1=U(I(\alpha_{43}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{157})$. Now the proof is easy.

LEMMA 95. $G'(J_{42})=G(z_{143})=G(J_{42})-G(J_{45})$,

$$G'(J_{43})=G(z_{144})=G(J_{43})-\{G(J_{44})\cup G(J_{47})\}\ ,$$
$$G'(J_{45})=G(z_{148})=G(J_{45})-\{G(J_{47})\cup G(J_{48})\}\ .$$

If $\mathrm{ch}(K)=2$, $\overline{G(z_{135})}-G(z_{135})=G(J_{44})\cup G(J_{48})$. *Here* $\overline{G(z_{135})}$ *is the Zariski closure of* $G(z_{135})$.

PROOF. By calculations, we get

$$G(z_{135})=\left\{y=\prod x_i(u_i)\ \middle|\ \begin{array}{l} I(y)=J_{37},\ f_1(y)u_{104}=u_{39}u_{40}u_{105}\ , \\ \qquad f_2(y)u_{104}=u_{39}u_{40}u_{106}\ , \\ \qquad f_3(y)u_{104}=u_{39}u_{40}u_{108} \end{array}\right\}$$

where

$$f_1(y)=u_{40}u_{44}+u_{41}u_{43}+u_{38}u_{45}\ ,$$
$$f_2(y)=u_{40}u_{47}+u_{41}u_{46}+u_{42}u_{45}+u_{37}u_{49}\ ,$$
$$f_3(y)=u_{47}u_{52}+u_{38}u_{51}+u_{39}u_{50}+u_{42}u_{48}+u_{43}u_{47}+u_{44}u_{46}\ .$$

Hence the closure $\overline{G(z_{135})}$ is contained in the variety

$$V=\left\{y=\prod x_i(u_i)\ \middle|\ \begin{array}{ll} I(y)\subseteqq J_{37}\ , & f_1(y)u_{104}=u_{39}u_{40}u_{105}\ , \\ & f_2(y)u_{104}=u_{39}u_{40}u_{106}\ , \\ & f_3(y)u_{104}=u_{39}u_{40}u_{108}\ , \\ & f_1(y)u_{106}=f_2(y)u_{105}\ , \\ & f_1(y)u_{108}=f_3(y)u_{105}\ , \\ & f_2(y)u_{108}=f_3(y)u_{106}\ , \end{array}\right\}.$$

Let y be an element of $V-G(z_{135})$. If $u_{39}u_{40}\neq 0$, y is in $G(J_{48})$. Thus we may assume $u_{39}u_{40}=0$. If $f_i(y)\neq 0$, for some i, y is in $G(J_{48})$. Thus we may assume $f_1(y)=f_2(y)=f_3(y)=u_{39}u_{40}=0$. Then y is in $G(J_{45})$. On the other hand, $x_{37}(1)x_{38}(1)x_{39}(1)x_{40}(1)x_{104}(1)x_{111}(1)\underset{c}{\sim} z_{157}$ and $x_{37}(1)x_{38}(1)x_{40}(1)x_{41}(1)$

$x_{104}(1) \underset{c}{\sim} z_{147}$. This shows the last assertion. The rest of proof is easy.

LEMMA 96. *Let* $J_{49}=I(\alpha_{42}, \alpha_{48}, \alpha_{106}, \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{49}$. *Then* $x \underset{c}{\sim} z_{158}=x_{42}(1)x_{48}(1)x_{106}(1)x_{107}(1)$. *A k-rational point in* $G(z_{158})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{158}=x_{42}(1)x_{48}(1)x_{106}(1)x_{107}(1)\ ,$$
$$z_{159}=x_{42}(1)x_{43}(1)x_{44}(1)x_{48}(1)x_{51}(\eta)x_{106}(1)x_{107}(1)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{160}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)x_{106}(1)x_{107}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{158})\subset B\langle w_2w_3w_5w_7, w_4, w_{12}\rangle B$, $Z(z_{158})\cong Z_2$, $L(z_{158})=A_4$, $|Z_{G(k)}(z_{158})|=2(q^2-1)(q^3-1)(q^4-1)(q^5-1)q^{54}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^3+1)(q^4-1)(q^5+1)q^{54}(i=159, 160)$.

PROOF. By Lemma 8, $W(x)\subseteqq\langle w_2w_3w_5w_7, w_4, w_{12}\rangle$. By the action of B, we get $x \underset{c}{\sim} z_{158}$. Thus we put $P=B\langle w_2, w_3, w_4, w_5, w_6, w_7\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{158})$. Now the proof is easy.

LEMMA 97. *Let* $J_{50}=I(\alpha_{42}, \alpha_{44}, \alpha_{49}, \alpha_{110}, \alpha_{113})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{50}$ *and* $t_{44}t_{46}-t_{42}t_{48}\neq 0$. *Then* $x \underset{c}{\sim} z_{161}=x_{42}(1)x_{44}(1)x_{48}(1)x_{49}(1)x_{110}(1)x_{113}(1)$. *A k-rational point in* $G(z_{161})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{161}=x_{42}(1)x_{44}(1)x_{48}(1)x_{49}(1)x_{110}(1)x_{113}(1)\ ,$$
$$z_{162}=x_{42}(1)x_{43}(1)x_{44}(1)x_{48}(1)x_{51}(\eta)x_{49}(1)x_{110}(1)x_{113}(1)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{163}=x_{42}(1)x_{43}(1)x_{44}(1)x_{51}(\zeta)x_{49}(1)x_{110}(1)x_{113}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{161})\subset B\langle w_1, w_3, w_4, w_5, w_6, w_7, w_{101}\rangle B$, $Z(z_{161})\cong Z_2$, $L(z_{161})=A_2$, $|Z_{G(k)}(z_{161})|=2(q^2-1)(q^3-1)q^{59}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^3+1)q^{59}(i=162, 163)$.

PROOF. By Lemma 8, we get $W(x)\subseteqq\{1, c=w_3w_7, a=w_5w_{27}w_{105}, b=w_5w_1w_{18}w_{101}, ac, ca, cac, cb, bc, cbc, acb, cacb\}$. By the action of B, $x \underset{c}{\sim} z_{161}$. Thus we put $P=B\langle w_1, w_3, w_4, w_5, w_6, w_7, w_{101}\rangle B$, $R=V=\mathrm{Ru}(P)$, $V_1=D(\mathrm{Ru}(P))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{161})$. Now the proof is easy.

LEMMA 98. *Let* $J_{51}=I(\alpha_{46}, \alpha_{47}, \alpha_{48}, \alpha_{49}, \alpha_{112}, \alpha_{113})$ *and let* x *be an element of* U *such that* $I(x)=J_{51}$. *Then* $x \underset{c}{\sim} z_{164}=x_{46}(1)x_{47}(1)x_{48}(1)x_{49}(1)x_{112}(1)x_{113}(1)$. $Z_G(z_{164})\subset B\langle w_2, w_8w_{13}w_{104}\rangle B$, $Z(z_{164})=1$, $L(z_{164})=2A_1$, $|Z_{G(k)}(z_{164})|=(q^2-1)^2q^{62}$.

PROOF. By Lemma 8, $W(x)\subseteqq\langle w_2, w_8w_{13}w_{104}\rangle$. By the action of B,

$x \underset{c}{\sim} z_{164}$. Thus we put $P = B\langle w_1, w_2, w_3, w_5, w_6, w_7, w_{101}\rangle B$, $R = \mathrm{Ru}(P)$, $V = U(I(\alpha_{28}))$, $V_1 = U(I(\alpha_{50}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{164})$. Now the proof is easy.

LEMMA 99. *Let* $J_{52} = I(\alpha_{46}, \alpha_{47}, \alpha_{48}, \alpha_{112}, \alpha_{116})$ *and let* x *be an element of* U *such that* $I(x) = J_{52}$.

1) *If* $\mathrm{ch}(K) \neq 2$, $x \underset{c}{\sim} z_{165} = x_{46}(1)x_{47}(1)x_{48}(1)x_{112}(1)x_{116}(1)$. *A k-rational point in* $G(z_{165})$ *is conjugate in* $G(k)$ *to* z_{165} *or* $z_{166} = x_{46}(1)x_{47}(1)x_{48}(1)x_{112}(1)x_{49}(1)x_{119}(-\zeta)$. $Z_G(z_{165}) \subset B\langle w_2, w_3w_5w_{102}, w_{10}w_{11}\rangle B$, $Z(z_{165}) \cong Z_2$, $L(z_{165}) = T_1 + B_2$, $|Z_{G(k)}(z_{165})| = 2(q-1)(q^2-1)(q^4-1)q^{63}$, $|Z_{G(k)}(z_{166})| = 2(q+1)(q^2-1)(q^4-1)q^{63}$.

2) *If* $\mathrm{ch}(K) = 2$, x *is conjugate to* z_{165} *or* $z_{167} = z_{165}x_{119}(1)$. $Z_G(z_{165}) \subset B\langle w_2, w_3w_5w_{102}, w_{10}w_{11}\rangle B$, $Z(z_{165}) = 1$, $Z_G(z_{167}) \subset B\langle w_2, w_{10}w_{11}\rangle B$, $Z(z_{167}) = 1$, $L(z_{165}) = A_1 + B_2$, $L(z_{167}) = B_2$, $|Z_{G(k)}(z_{165})| = (q^2-1)^2(q^4-1)q^{64}$, $|Z_{G(k)}(z_{167})| = (q^2-1)(q^4-1)q^{64}$.

PROOF. By Lemma 8, we get $W(x) \subseteq \langle w_2, w_3w_5w_{102}, w_{10}w_{11}\rangle$. Thus we put $P = B\langle w_2, w_3, w_4, w_5, w_7, w_{101}\rangle B$, $R = \mathrm{Ru}(P)$, $V = U(I(\alpha_{27}, \alpha_{49}))$, $V_1 = U(I(\alpha_{52}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{167})$. The rest of proof is similar to that of Lemma 29.

LEMMA 100. $G'(J_{44}) = G(z_{147}) = G(J_{44}) - \{G(J_{46}) \cup G(J_{51})\}$, $G'(J_{48}) = G(z_{157}) = G(J_{48}) - G(J_{50})$, $G'(J_{49}) = G(z_{158}) = G(J_{49}) - G(J_{52})$, $G'(J_{50}) = G(z_{161}) = G(J_{50}) - G(J_{51})$, $G'(J_{51}) = G(z_{164}) = G(J_{51}) - G(J_{52})$, $G'(J_{47}) = G(z_{154}) = G(J_{47}) - \{G(J_{49}) \cup G(J_{50})\}$.

LEMMA 101. *Let* $J_{53} = I(\alpha_{47}, \alpha_{48}, \alpha_{53}, \alpha_{116})$ *and let* $x = \prod x_i(t_i)$ *be an element of* U *such that* $I(x) = J_{53}$ *and* $t_{48}t_{114} - t_{47}t_{115} \neq 0$. *Then* $x \underset{c}{\sim} z_{168} = x_{53}(1)x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1)$. *A k-rational point in* $G(z_{168})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{168} = x_{53}(1)x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1)\ ,$$
$$z_{169} = x_{47}(1)x_{48}(1)x_{49}(1)x_{53}(1)x_{112}(1)x_{113}(1)x_{118}(1)x_{121}(\tau)\ ,$$
$$z_{170} = x_{47}(1)x_{48}(1)x_{53}(1)x_{112}(1)x_{115}(1)x_{117}(\eta)x_{116}(1)\ , \qquad \textit{when } \mathrm{ch}(K) = 2\ ,$$
$$z_{171} = x_{47}(1)x_{48}(1)x_{53}(1)x_{112}(1)x_{117}(\zeta)x_{116}(1)\ , \qquad \textit{when } \mathrm{ch}(K) \neq 2\ .$$

$Z_G(z_{168}) \subset B\langle w_4, w_1w_6, w_3w_5, w_{101}\rangle B$, $Z(z_{168}) \cong S_3$, $L(z_{168}) = 3A_1$, $|Z_{G(k)}(z_{168})| = 6(q^2-1)^3q^{66}$, $|Z_{G(k)}(z_{169})| = 3(q^6-1)q^{66}$, $|Z_{G(k)}(z_i)| = 2(q^2-1)(q^4-1)q^{66}$ $(i = 170, 171)$.

PROOF. By Lemma 8, $W(x) \subset \langle w_4, w_1w_6, w_3w_5, w_{101}\rangle$. By the action of B, we get $x \underset{c}{\sim} z_{168}$. Thus we put $P = B\langle w_1, w_3, w_4, w_5, w_6, w_{101}\rangle B$, $R = \mathrm{Ru}(P)$, $V = U(I(\alpha_{31}, \alpha_{53}))$, $V_1 = U(I(\alpha_{56}, \alpha_{119}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{168})$. Now the proof is easy.

LEMMA 102. *Let* $J_{54}=I(\alpha_{53}, \alpha_{54}, \alpha_{55}, \alpha_{112}, \alpha_{113})$ *and let* x *be an element of* U *such that* $I(x)=J_{54}$. *Then* $x \underset{c}{\sim} z_{172}=x_{53}(1)x_{54}(1)x_{55}(1)x_{112}(1)x_{113}(1)$. $Z_G(z_{172})\subset B\langle w_{18}, w_2w_7, w_1w_5\rangle B$, $Z(z_{172})=1$, $L(z_{172})=A_1+B_2$, $|Z_{G(k)}(z_{172})|=(q^2-1)^2(q^4-1)q^{68}$.

The proof is easy.

LEMMA 103. *Let* $J_{55}=I(\alpha_{47}, \alpha_{48}, \alpha_{116})$ *and let* $x=\prod x_i(t_i)$ *be an element of* U *such that* $I(x)=J_{55}$ *and* $t_{47}t_{115}-t_{48}t_{114}\neq 0$. *Then* $x \underset{c}{\sim} z_{173}=x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1)$. *A k-rational point in* $G(z_{173})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{173}=x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1) ,$$

$$z_{174}=x_{47}(1)x_{48}(1)x_{49}(1)x_{112}(1)x_{113}(1)x_{118}(1)x_{121}(\tau) ,$$

$$z_{175}=x_{47}(1)x_{48}(1)x_{112}(1)x_{115}(1)x_{117}(\eta)x_{116}(1) , \quad \textit{when } \mathrm{ch}(K)=2 ,$$

$$z_{176}=x_{47}(1)x_{48}(1)x_{112}(1)x_{117}(\zeta)x_{116}(1) , \quad \textit{when } \mathrm{ch}(K)\neq 2 .$$

$Z_G(z_{173})\subset B\langle w_2, w_4, w_3w_5, w_1w_6, w_{101}\rangle B$, $Z(z_{173})\cong S_3$, $L(z_{173})=D_4$, $|Z_{G(k)}(z_{173})|=6(q^2-1)(q^4-1)^2(q^6-1)q^{66}$, $|Z_{G(k)}(z_{174})|=3(q^2-1)(q^6-1)(q^8+q^4+1)q^{66}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^6-1)(q^8-1)q^{66}(i=175, 176)$.

The proof is similar to that of Lemma 101.

LEMMA 104. *Let* $J_{56}=I(\alpha_{53}, \alpha_{54}, \alpha_{55}, \alpha_{117}, \alpha_{118}, \alpha_{119})$ *and let* x *be an element of* U *such that* $I(x)=J_{56}$. *Then* $x \underset{c}{\sim} z_{177}=x_{53}(1)x_{54}(1)x_{55}(1)x_{117}(1)x_{118}(1)x_{119}(1)$. $Z_G(z_{177})\subset B\langle w_1w_2w_7, w_{10}w_6w_{101}\rangle B$, $Z(z_{177})=1$, $L(z_{177})=B_2$, $|Z_{G(k)}(z_{177})|=(q^2-1)(q^4-1)q^{74}$.

The proof is easy.

LEMMA 105. *Let* $J_{57}=I(\alpha_{55}, \alpha_{56}, \alpha_{112}, \alpha_{113})$ *and let* x *be an element of* U *such that* $I(x)=J_{57}$. *Then* $x \underset{c}{\sim} z_{178}=x_{55}(1)x_{56}(1)x_{112}(1)x_{113}(1)$. $Z_G(z_{178})\subset B\langle w_1w_4, w_3, w_7, w_{17}\rangle B$, $Z(z_{178})=1$, $L(z_{178})=A_1+B_3$, $|Z_{G(k)}(z_{178})|=(q^2-1)^2(q^4-1)(q^6-1)q^{70}$.

The proof is easy.

LEMMA 106. $G'(J_{46})=G(z_{151})=G(J_{46})-G(J_{55})$,

$$G'(J_{53})=G(z_{168})=G(J_{53})-\{G(J_{54})\cup G(J_{55})\} ,$$

$$G(J_{52})=G(z_{165})\cup G(z_{167})\cup G(J_{53}) ,$$

$$G'(J_{54})=G(z_{172})=G(J_{54})-\{G(J_{56})\cup G(J_{57})\} ,$$

$$G'(J_{55})=G(z_{173})=G(J_{55})-G(J_{57}) .$$

The lemma follows from Lemmas 1–6.

LEMMA 107. *Let* $J_{58}=I(\alpha_{56}, \alpha_{57}, \alpha_{117}, \alpha_{118}, \alpha_{119})$ *and let* x *be an element of* U *such that* $I(x)=J_{58}$. *Then* $x \underset{c}{\sim} z_{179}=x_{56}(1)x_{57}(1)x_{117}(1)x_{118}(1)x_{119}(1)$. $Z_G(z_{179}) \subset B\langle w_2w_3w_6, w_{11}, w_9w_{10}w_{101}\rangle B$, $Z(z_{179})=1$, $L(z_{179})=A_1+G_2$, $|Z_{G(k)}(z_{179})|=(q^2-1)^2(q^6-1)q^{76}$.

The proof is easy.

LEMMA 108. *Let* $J_{59}=I(\alpha_{63}, \alpha_{106}, \alpha_{107})$ *and let* x *be an element of* U *such that* $I(x)=J_{59}$. *Then* $x \underset{c}{\sim} z_{108}=x_{63}(1)x_{106}(1)x_{107}(1)$. $Z_G(z_{180}) \subset B\langle w_2w_3, w_4, w_5, w_6w_7\rangle B$, $Z(z_{180})=1$, $L(z_{180})=B_5$, $|Z_{G(k)}(z_{180})|=(q^2-1)(q^4-1)(q^6-1)(q^8-1)(q^{10}-1)q^{70}$.

The proof is easy.

LEMMA 109. *Let* $J_{60}=I(\alpha_{56}, \alpha_{57}, \alpha_{117}, \alpha_{118})$ *and let* x *be an element of* U *such that* $I(x)=J_{60}$. *Then* $x \underset{c}{\sim} z_{181}=x_{56}(1)x_{57}(1)x_{117}(1)x_{118}(1)$. *A k-rational point in* $G(z_{181})$ *is conjugate in* $G(k)$ *to one of the following elements;*

$$z_{181}=x_{56}(1)x_{57}(1)x_{117}(1)x_{118}(1)\ ,$$
$$z_{182}=x_{53}(1)x_{54}(1)x_{55}(1)x_{117}(1)x_{57}(1)x_{124}(\eta)x_{122}(1)\ , \qquad \textit{when } \mathrm{ch}(K)=2\ ,$$
$$z_{183}=x_{53}(1)x_{54}(1)x_{55}(1)x_{117}(1)x_{124}(\zeta)x_{122}(1)\ , \qquad \textit{when } \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{181}) \subset B\langle w_2w_3w_6, w_{11}, w_{19}, w_9w_{10}w_{101}, w_4w_{13}w_{102}\rangle B$, $Z(z_{181})\cong Z_2$, $L(z_{181})=2G_2$, $|Z_{G(k)}(z_{181})|=2(q^2-1)^2(q^6-1)^2q^{76}$, $|Z_{G(k)}(z_i)|=2(q^4-1)(q^{12}-1)q^{76}(i=182, 183)$.

PROOF. By Lemma 8, we get $W(x)\subseteq\langle w_2w_3w_6, w_{11}, w_4w_{13}w_{102}\rangle$. By the action of B, we get $x \underset{c}{\sim} z_{181}$. We put $P=B\langle w_2, w_3, w_4, w_5, w_6, w_7, w_{101}\rangle B$, $R=\mathrm{Ru}(P)$, $V=U(I(\alpha_{32}))$, $V_1=U(I(\alpha_{63}))$. Then (P, R, V, V_1) gives a structure of $Z_G(z_{181})$. Now the proof is easy.

LEMMA 110. *Let* $J_{61}=I(\alpha_{58}, \alpha_{59}, \alpha_{123}, \alpha_{124}, \alpha_{125})$ *and let* x *be an element of* U *such that* $I(x)=J_{61}$. *Then* $x \underset{c}{\sim} z_{184}=x_{58}(1)x_{59}(1)x_{123}(1)x_{124}(1)x_{125}(1)$. $Z_G(z_{184}) \subset B\langle w_1, w_4w_7, w_2w_{12}w_{101}\rangle B$, $Z(z_{184})=1$, $L(z_{184})=A_1+G_2$, $|Z_{G(k)}(z_{184})|=(q^2-1)^2(q^6-1)q^{84}$.

The proof is easy.

LEMMA 111. *Let* $J_{62}=I(\alpha_{60}, \alpha_{126}, \alpha_{127}, \alpha_{128})$ *and let* x *be an element of* U *such that* $I(x)=J_{62}$. *Then* $x \underset{c}{\sim} z_{185}=x_{60}(1)x_{126}(1)x_{127}(1)x_{128}(1)$. $Z_G(z_{185}) \subset B\langle w_1, w_3, w_9w_{11}, w_2w_5w_{102}\rangle B$, $L(z_{185})=A_1+B_3$, $Z(z_{185})=1$, $|Z_{G(k)}(z_{185})|=(q^2-1)^2(q^4-1)(q^6-1)q^{88}$.

The proof is easy.

LEMMA 112. $G'(J_{56})=G(z_{177})=G(J_{56})-G(J_{58})$,

$$G'(J_{57})=G(z_{178})=G(J_{57})-\{G(J_{58})\cup G(J_{59})\}\ ,$$
$$G'(J_{58})=G(z_{179})=G(J_{58})-G(J_{60}),\ G'(J_{59})=G(z_{180})=G(J_{59})-G(J_{62})\ ,$$
$$G'(J_{60})=G(z_{181})=G(J_{60})-G(J_{61}),\ G'(J_{61})=G(z_{184})=G(J_{61})-G(J_{62})\ .$$

If $\mathrm{ch}(K)=2$, *the Zariski closure* $\overline{G(z_{165})}=G(z_{165})\cup G(J_{54})$.

PROOF. By the action of B, we get $\overline{G(z_{165})}=\{y=\prod x_i(u_i)|f(y)=0,\ I(x)\supseteqq J_{52}\}$, where $f(y)=u_{116}(u_{112}u_{51}+u_{47}u_{115}+u_{48}u_{114})+u_{46}u_{112}u_{119}$. Let y be an element of $\overline{G(z_{165})}-G(z_{165})$. If $u_{116}=u_{119}=0$, y is in $G(J_{60})$. If $u_{116}=u_{46}u_{112}=0$, y is in $G(J_{56})$. Thus we may assume $u_{116}\neq 0$. If $u_{46}u_{112}=0$, y is in $G(J_{54})$. Thus we may assume $u_{46}u_{112}u_{116}\neq 0$. Then y is in $G(J_{56})$. This shows the last assertion. The rest of proof is easy.

LEMMA 113. *Let* $J_{63}=I(\alpha_{63},\ \alpha_{127},\ \alpha_{130})$ *and let* x *be an element of* U *such that* $I(x)=J_{63}$. *Then* $x\underset{c}{\sim}z_{186}=x_{63}(1)x_{127}(1)x_{130}(1)$. *A k-rational point in* $G(z_{186})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{186}=x_{63}(1)x_{127}(1)x_{130}(1)\ ,$$
$$z_{187}=x_{63}(1)x_{126}(1)x_{127}(1)x_{128}(1)x_{131}(1)x_{133}(\eta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)=2\ ,$$
$$z_{188}=x_{63}(1)x_{126}(1)x_{127}(1)x_{128}(1)x_{133}(\zeta)\ ,\qquad \textit{when}\ \mathrm{ch}(K)\neq 2\ .$$

$Z_G(z_{186})\subset B\langle w_3,\ w_4,\ w_6,\ w_2w_5w_7\rangle B$, $Z(z_{186})\cong Z_2$, $L(z_{186})=A_5$, $|Z_{G(k)}(z_{186})|=2(q^2-1)(q^3-1)(q^4-1)(q^5-1)(q^6-1)q^{92}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^3+1)(q^4-1)(q^5+1)(q^6-1)q^{92}$ $(i=187,\ 188)$.

The proof is easy.

LEMMA 114. *Let* $J_{64}=I(\alpha_{63},\ \alpha_{135},\ \alpha_{136},\ \alpha_{137})$ *and let* x *be an element of* U *such that* $I(x)=J_{64}$. *Then* $x\underset{c}{\sim}z_{189}=x_{63}(1)x_{135}(1)x_{136}(1)x_{137}(1)$. $Z_G(z_{189})\subset B\langle w_1w_{101},\ w_3w_7,\ w_4w_6,\ w_5\rangle B$, $Z(z_{189})=1$, $L(z_{189})=C_4$, $|Z_{G(k)}(z_{189})|=(q^2-1)(q^4-1)(q^6-1)(q^8-1)q^{100}$.

The proof is easy.

LEMMA 115. *Let* $J_{65}=I(\alpha_{127},\ \alpha_{130})$ *and let* x *be an element of* U *such that* $I(x)=J_{65}$. *Then* $x\underset{c}{\sim}z_{190}=x_{127}(1)x_{130}(1)$. *A k-rational point in* $G(z_{190})$ *is conjugate in* $G(k)$ *to one of the following elements*;

$$z_{190}=x_{127}(1)x_{130}(1)\ ,$$

$z_{191}=x_{126}(1)x_{127}(1)x_{128}(1)x_{131}(1)x_{133}(\eta)$, *when* $\mathrm{ch}(K)=2$,

$z_{192}=x_{126}(1)x_{127}(1)x_{128}(1)x_{133}(\zeta)$, *when* $\mathrm{ch}(K)\neq 2$.

$Z_G(z_{190})\subset B\langle w_1, w_3, w_4, w_6, w_2w_5w_7\rangle B$, $Z(z_{190})\cong Z_2$, $L(z_{190})=E_6$, $|Z_{G(k)}(z_{190})|=2(q^2-1)(q^5-1)(q^6-1)(q^8-1)(q^9-1)(q^{12}-1)q^{92}$, $|Z_{G(k)}(z_i)|=2(q^2-1)(q^5+1)(q^6-1)(q^8-1)(q^9+1)(q^{12}-1)q^{92}(i=191, 192)$.

The proof is easy.

LEMMA 116. *Let* $J_{66}=I(\alpha_{141}, \alpha_{142}, \alpha_{143})$ *and let* x *be an element of* U *such that* $I(x)=J_{66}$. *Then* $x \underset{c}{\sim} z_{193}=x_{141}(1)x_{142}(1)x_{143}(1)$. $Z_G(z_{193})\subset B\langle w_1w_6, w_3w_5, w_4, w_2, w_{101}\rangle B$, $Z(z_{193})=1$, $L(z_{193})=A_1+F_4$, $|Z_{G(k)}(z_{193})|=(q^2-1)^2(q^6-1)(q^8-1)(q^{12}-1)q^{106}$.

The proof is easy.

LEMMA 117. *Let* $J_{67}=I(\alpha_{150}, \alpha_{151})$ *and let* x *be an element of* U *such that* $I(x)=J_{67}$. *Then* $x \underset{c}{\sim} z_{194}=x_{150}(1)x_{151}(1)$. $Z_G(z_{194})\subset B\langle w_2w_3, w_4, w_5, w_6, w_7, w_{101}\rangle B$, $Z(z_{194})=1$, $L(z_{194})=B_6$, $|Z_{G(k)}(z_{194})|(q^2-1)(q^4-1)(q^6-1)(q^8-1)(q^{10}-1)(q^{12}-1)q^{114}$.

The proof is easy.

LEMMA 118. *Let* $J_{68}=I(\alpha_{157})$ *and let* x *be an element of* $\mathfrak{X}_{157}{}^*$. *Then* $x \underset{c}{\sim} z_{195}=x_{157}(1)$. $Z_G(z_{195})\subset B\langle w_1, w_2, w_3, w_4, w_5, w_6, w_7\rangle B$, $Z(z_{195})=1$, $L(z_{195})=E_7$, $|Z_{G(k)}(z_{195})|(q^2-1)(q^6-1)(q^8-1)(q^{10}-1)(q^{12}-1)(q^{14}-1)(q^{18}-1)q^{120}$.

The proof is easy.

LEMMA 119. $G'(J_{62})=G(z_{185})=G(J_{62})-G(J_{63})$,

$$G'(J_{63})=G(z_{186})=G(J_{63})-\{G(J_{64})\cup G(J_{65})\},$$
$$G'(J_{64})=G(z_{189})=G(J_{64})-G(J_{66}),\ G'(J_{65})=G(z_{190})=G(J_{65})=G(J_{66})\ ,$$
$$G'(J_{66})=G(z_{193})=G(J_{66})-G(J_{67}),\ G'(J_{67})=G(z_{194})=G(J_{67})-G(J_{68})\ ,$$
$$G'(J_{68})=G(z_{195})\cup\{1\}\ .$$

The proof is easy. By the series of lemmas, we proved that

THEOREM 3. *Let* G *be a semisimple algebraic group of type* E_8 *which splits over finite field* k. *Then the conjugate classes of unipotent elements in* G *are giver in Table* 3. *Furthermore the unipotent classes of* $G(k)$ *are given in Table* 6.

THEOREM 4. *Let* G *be as above. Then the structures of the central-*

izers of unipotent elements are given in Table 10.

THEOREM 5. *Let G be as above. Then the inclusion relations of the Zariski closures of the conjugate classes of unipotent elements in G are given in Table* 8.

TABLE 1

The representatives of unipotent classes in the group E_6.

$E_6=x_1(1)x_2(1)x_3(1)x_4(1)x_5(1)x_6(1)$,
$E_6(a_1)=x_2(1)x_4(1)x_5(1)x_6(1)x_8(1)x_{16}(1)$,
$D_5=x_1(1)x_2(1)x_{10}(1)x_{11}(1)x_{12}(1)$,
$A_5+A_1=x_8(1)x_9(1)x_{12}(1)x_{10}(1)x_{11}(1)x_{16}(1)$,
$D_5(a_1)=x_8(1)x_9(1)x_{16}(1)x_{12}(1)x_{18}(1)$,
$A_5=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)$,
$A_4+A_1=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)$,
$D_4=x_2(1)x_{14}(1)x_{16}(1)x_{18}(1)$,
$A_4=x_{14}(1)x_{15}(1)x_{17}(1)x_{12}(1)$,
$D_4(a_1)=x_{14}(1)x_{16}(1)x_{22}(1)x_{24}(1)$,
$A_3+A_1=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)$,
$2A_2+A_1=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)$,
$A_3=x_{14}(1)x_{22}(1)x_{24}(1)$,
$A_2+2A_1=x_{26}(1)x_{27}(1)x_{28}(1)x_{29}(1)$,
$2A_2=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)$,
$A_2+A_1=x_{26}(1)x_{27}(1)x_{35}(1)$,
$A_2=x_{26}(1)x_{35}(1)$,
$3A_1=x_{37}(1)x_{38}(1)x_{40}(1)$,
$2A_1=x_{42}(1)x_{43}(1)$,
$A_1=x_{53}(1)$,
$\phi=1$

TABLE 2

The representatives of unipotent classes in the group E_7.

$E_7=x_1(1)x_2(1)x_3(1)x_4(1)x_5(1)x_6(1)x_7(1)$,

$E_7(a_1)=x_1(1)x_3(1)x_{10}(1)x_9(1)x_5(1)x_6(1)x_7(1)$,

$E_7(a_2)=x_1(1)x_2(1)x_8(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)$,

$D_6+A_1=x_1(1)x_9(1)x_{10}(1)x_{17}(1)x_{22}(1)x_{12}(1)x_7(1)$,

$E_6=x_1(1)x_8(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)$,

$E_6(a_1)=x_8(1)x_9(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)$,

$D_6=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)$,

$D_6(a_1)+A_1=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{23}(1)x_{13}(1)$,

$A_6=x_{14}(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{12}(1)x_{13}(1)$,

$D_6(a_1)=x_1(1)x_{15}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)$,

$D_5+A_1=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)$,

$D_6(a_2)+A_1=x_{14}(1)x_{15}(1)x_{27}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{23}(1)$,

$D_5=x_1(1)x_{15}(1)x_{16}(1)x_{18}(1)x_{19}(1)$,

$(A_5+A_1)'=x_8(1)x_{15}(1)x_{16}(1)x_{20}(1)x_{24}(1)x_{25}(1)$,

$D_6(a_2)=x_{15}(1)x_{20}(1)x_{21}(1)x_{23}(1)x_{18}(x)x_{17}(1)$,

$(A_5+A_1)''=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_{28}(1)x_7(1)$,

$A_5'=x_{28}(1)x_{12}(1)x_{13}(1)x_{20}(1)x_{21}(1)$,

$D_5(a_1)+A_1=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{19}(1)$,

$A_5''=x_{20}(1)x_{21}(1)x_{23}(1)x_{24}(1)x_7(1)$,

$A_4+A_2=x_{20}(1)x_{21}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{25}(1)$,

$D_5(a_1)=x_8(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)$,

$A_4=x_{20}(1)x_{21}(1)x_{24}(1)x_{30}(1)$,

$A_4+A_1=x_{20}(1)x_{21}(1)x_{24}(1)x_{28}(1)x_{30}(1)$,

$D_4+A_1=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)$,

$A_3+A_2+A_1=x_{26}(1)x_{27}(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)$,

$D_4=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)$,

$A_3+A_2=x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{32}(1)x_{37}(1)$,

$(A_3+A_2)_2=x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{32}(1)$, *when* $\mathrm{ch}\,(K)=2$,

$D_4(a_1)+A_1=x_{28}(1)x_{30}(1)x_{31}(1)x_{33}(1)x_{34}(1)$,

$D_4(a_1)=x_{28}(1)x_{30}(1)x_{33}(1)x_{34}(1)$,

$A_3+2A_1=x_{30}(1)x_{31}(1)x_{32}(1)x_{38}(1)x_{40}(1)$,

$(A_3+A_1)'=x_{26}(1)x_{27}(1)x_{40}(1)x_{41}(1)$,

$(A_3+A_1)''=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(1)$,

$A_3=x_{20}(1)x_{21}(1)x_{49}(1)$,

$2A_2+A_1=x_{34}(1)x_{36}(1)x_{37}(1)x_{38}(1)x_{40}(1)$,

$2A_2=x_{34}(1)x_{36}(1)x_{38}(1)x_{40}(1)$,

$A_2+3A_1=x_{37}(1)x_{38}(1)x_{39}(1)x_{40}(1)x_{41}(1)$,

$A_2+2A_1=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)$,

$A_2+A_1=x_{44}(1)x_{46}(1)x_{49}(1)$,

$A_2=x_{44}(1)x_{46}(1)$,

$4A_1=x_{47}(1)x_{48}(1)x_{49}(1)x_{53}(1)$,

$(3A_1)'=x_{53}(1)x_{54}(1)x_{55}(1)$,

$(3A_1)''=x_{47}(1)x_{48}(1)x_{49}(1)$,

$2A_1=x_{58}(1)x_{59}(1)$,

$A_1=x_{63}(1)$,

$\phi=1$

TABLE 3

The representatives of unipotent classes in the group E_8.

$E_8=x_1(1)x_2(1)x_3(1)x_4(1)x_5(1)x_6(1)x_7(1)x_{101}(1),$

$E_8(a_1)=x_1(1)x_2(1)x_9(1)x_{10}(1)x_5(1)x_6(1)x_7(1)x_{101}(1),$

$E_8(a_2)=x_1(1)x_3(1)x_2(1)x_9(1)x_{11}(1)x_{12}(1)x_{13}(1)x_{101}(1),$

$E_7+A_1=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_7(1)x_{101}(1),$

$E_7=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_6(1)x_7(1)x_{101}(1),$

$D_8=x_8(1)x_9(1)x_{10}(1)x_{11}(1)x_{16}(1)x_{12}(1)x_{13}(1)x_{102}(1),$

$E_7(a_1)+A_1=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_7(1)x_{101}(1),$

$E_7(a_1)=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{24}(1)x_{13}(1)x_{101}(1),$

$D_8(a_1)=x_8(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{24}(1)x_{13}(1)x_{102}(1),$

$D_7=x_1(1)x_{15}(1)x_{16}(1)x_{17}(1)x_{18}(1)x_{19}(1)x_{108}(1),$

$E_7(a_2)+A_1=x_{14}(1)x_{15}(1)x_{21}(1)x_{17}(1)x_{18}(1)x_{29}(1)x_{19}(1)x_{101}(1),$

$E_7(a_2)=x_{20}(1)x_{21}(1)x_{17}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1),$

$A_8=x_{14}(1)x_{20}(1)x_{21}(1)x_{22}(1)x_{18}(1)x_{33}(1)x_{13}(1)x_{102}(1),$

$E_6+A_1=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_7(1)x_{101}(1),$

$D_7(a_1)=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{25}(1)x_{35}(1)x_{102}(1),$

$(D_7(a_1))_2=x_{14}(1)x_{22}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{25}(1)x_{102}(1),$ *when* $\mathrm{ch}(K)=2,$

$E_6=x_{20}(1)x_{21}(1)x_{28}(1)x_{24}(1)x_7(1)x_{101}(1),$

$D_8(a_3)=x_{20}(1)x_{21}(1)x_{22}(1)x_{28}(1)x_{24}(1)x_{25}(1)x_{102}(1)x_{103}(1),$

$D_6+A_1=x_8(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{25}(1)x_{30}(1)x_{102}(1),$

$A_7=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{10}(1)x_{40}(1)x_{108}(1),$

$(A_7)_3=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{24}(1)x_{19}(1)x_{108}(1),$ *when* $\mathrm{ch}(K)=3,$

$E_6(a_1)+A_1=x_{20}(1)x_{21}(1)x_{28}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1),$

$D_6=x_1(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{101}(1),$

$D_7(a_2)=x_{20}(1)x_{21}(1)x_{28}(1)x_{29}(1)x_{25}(1)x_{31}(1)x_{103}(1),$

$E_6(a_1)=x_{20}(1)x_{21}(1)x_{23}(1)x_{31}(1)x_{101}(1)x_{102}(1),$

$D_5+A_2=x_{26}(1)x_{28}(1)x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{102}(1)x_{103}(1),$

$(D_5+A_2)_2=x_{26}(1)x_{28}(1)x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{102}(1),$ *when* $\mathrm{ch}(K)=2,$

$A_6+A_1=x_{20}(1)x_{21}(1)x_{28}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{105}(1),$

$D_6(a_1)+A_1=x_{28}(1)x_{27}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{39}(1)x_{102}(1),$

$D_6(a_1)=x_{28}(1)x_{29}(1)x_{33}(1)x_{31}(1)x_{34}(1)x_{101}(1),$

$A_6=x_{29}(1)x_{21}(1)x_{29}(1)x_{30}(1)x_{31}(1)x_{105}(1),$

$D_5+A_1=x_{32}(1)x_{33}(1)x_{30}(1)x_{32}(1)x_{40}(1)x_{101}(1),$

$2A_4=x_{26}(1)x_{29}(1)x_{35}(1)x_{38}(1)x_{30}(1)x_{44}(1)x_{47}(1)x_{105}(1),$

$A_5+A_2=x_{32}(1)x_{37}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{105}(1)x_{107}(1),$

$A_5+2A_1=x_{32}(1)x_{37}(1)x_{27}(1)x_{33}(1)x_{40}(1)x_{36}(1)x_{105}(1),$

$D_6(a_2)=x_{32}(1)x_{29}(1)x_{31}(1)x_{34}(1)x_{106}(1)x_{108}(1),$

$D_5=x_{30}(1)x_{31}(1)x_{32}(1)x_{33}(1)x_{101}(1),$

$D_5(a_1)+A_2=x_{37}(1)x_{33}(1)x_{40}(1)x_{34}(1)x_{36}(1)x_{41}(1)x_{105}(1),$

$(A_5+A_1)'=x_{26}(1)x_{27}(1)x_{40}(1)x_{41}(1)x_{106}(1)x_{107}(1),$

$(A_5+A_1)''=x_{32}(1)x_{27}(1)x_{37}(1)x_{33}(1)x_{36}(1)x_{105}(1),$

$D_4+A_2=x_{37}(1)x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{104}(1)x_{108}(1),$

$(D_4+A_2)_2=x_{37}(1)x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{104}(1),$

$A_4+A_3=x_{37}(1)x_{38}(1)x_{40}(1)x_{34}(1)x_{36}(1)x_{106}(1)x_{107}(1),$

$A_5=x_{20}(1)x_{21}(1)x_{49}(1)x_{106}(1)x_{107}(1),$

$D_5(a_1)+A_1=x_{37}(1)x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{103}(1),$

$A_4+A_2+A_1=x_{37}(1)x_{38}(1)x_{39}(1)x_{40}(1)x_{41}(1)x_{108}(1)x_{109}(1),$

TABLE 3. (Continued)

$A_4+A_2=x_{38}(1)x_{40}(1)x_{39}(1)x_{41}(1)x_{108}(1)x_{109}(1),$
$D_5(a_1)=x_{42}(1)x_{43}(1)x_{48}(1)x_{49}(1)x_{102}(1),$
$D_4+A_1=x_{47}(1)x_{48}(1)x_{49}(1)x_{53}(1)x_{101}(1),$
$A_4+2A_1=x_{42}(1)x_{43}(1)x_{44}(1)x_{45}(1)x_{107}(1)x_{108}(1),$
$D_4=x_{47}(1)x_{48}(1)x_{49}(1)x_{101}(1),$
$A_4+A_1=x_{42}(1)x_{48}(1)x_{49}(1)x_{106}(1)x_{107}(1),$
$2A_3=x_{37}(1)x_{38}(1)x_{39}(1)x_{49}(1)x_{108}(1)x_{113}(1),$
$A_4=x_{42}(1)x_{48}(1)x_{106}(x)x_{107}(1),$
$D_4(a_1)+A_2=x_{42}(1)x_{44}(1)x_{48}(1)x_{49}(1)x_{110}(1)x_{113}(1),$
$A_3+A_2+A_1=x_{46}(1)x_{47}(1)x_{48}(1)x_{49}(1)x_{112}(1)x_{113}(1),$
$A_3+A_2=x_{46}(1)x_{47}(1)x_{48}(1)x_{112}(1)x_{116}(1)x_{119}(1),$
$(A_3+A_2)_2=x_{46}(1)x_{47}(1)x_{48}(1)x_{112}(1)x_{116}(1),$ *when* ch $(K)=2,$
$D_4(a_1)+A_1=x_{58}(1)x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1),$
$A_3+2A_1=x_{53}(1)x_{54}(1)x_{55}(1)x_{112}(1)x_{113}(1),$
$D_4(a_1)=x_{47}(1)x_{48}(1)x_{115}(1)x_{116}(1),$
$2A_2+2A_1=x_{53}(1)x_{54}(1)x_{55}(1)x_{117}(1)x_{118}(1)x_{119}(1),$
$A_3+A_1=x_{55}(1)x_{56}(1)x_{112}(1)x_{113}(1),$
$2A_2+A_1=x_{56}(1)x_{57}(1)x_{117}(1)x_{118}(1)x_{119}(1),$
$A_3=x_{63}(1)x_{106}(1)x_{107}(1),$
$2A_2=x_{56}(1)x_{57}(1)x_{117}(1)x_{118}(1),$
$A_2+2A_1=x_{60}(1)x_{126}(1)x_{127}(1)x_{128}(1),$
$A_2+3A_1=x_{58}(1)x_{59}(1)x_{123}(1)x_{124}(1)x_{125}(1),$
$A_2+A_1=x_{63}(1)x_{128}(1)x_{129}(1),$
$A_2=x_{127}(1)x_{130}(1),$
$4A_1=x_{63}(1)x_{135}(1)x_{136}(1)x_{137}(1),$
$3A_1=x_{141}(1)x_{142}(1)x_{143}(1),$
$2A_1=x_{150}(1)x_{151}(1),$
$A_1=x_{157}(1),$
$\phi=1$

TABLE 4
The representatives of the unipotent classes in the simply connected Chevalley groups $E_7(q)$.

A	B	A	B	A	B	A	B	A	B
y_1		y_2	$2\nmid q$	y_3	$3\mid q$	y_4	$3\mid q$	y_5	$3\mid q$
y_6	$3\mid q$	y_7	$2\mid q$	y_8	$2\mid q$	y_9	$2\mid q$	y_{10}	
y_{11}	$2\nmid q$	y_{12}	$2\mid q$	y_{13}		y_{14}	$2\nmid q$	y_{15}	$2\mid q$
y_{16}		y_{17}	$2\nmid q$	y_{18}	$2\nmid q$	y_{19}	$2\nmid q$	y_{20}	$2\mid q$
y_{21}		y_{22}	$2\mid q$	y_{23}	$3\mid q$	y_{24}	$3\mid q$	y_{25}	
y_{26}	$2\nmid q$	y_{27}	$2\mid q$	y_{28}		y_{29}	$2\nmid q$	y_{30}	$2\mid q$
y_{31}	$2\nmid q$	y_{32}	$2\nmid q$	y_{33}	$2\nmid q$	y_{34}	$2\nmid q$	y_{35}	$2\mid q$
y_{36}		y_{37}	$2\mid q$	y_{38}		y_{39}	$2\nmid q$	y_{40}	$2\mid q$
y_{41}		y_{42}	$2\nmid q$	y_{43}	$2\mid q$	y_{44}	$2\mid q$	y_{45}	$2\mid q$
y_{46}		y_{47}	$2\nmid q$	y_{48}	$2\nmid q$	y_{49}	$2\nmid q$	y_{50}	$2\nmid q$
y_{51}	$2\nmid q$	y_{52}		y_{53}	$2\mid q$	y_{54}	$2\nmid q$	y_{55}	$2\nmid q$
y_{56}	$2\mid q$	y_{57}	$2\mid q$	y_{58}		y_{59}	$2\nmid q$	y_{60}	
y_{61}		y_{62}	$2\nmid q$	y_{63}		y_{64}	$2\nmid q$	y_{65}	
y_{66}	$2\nmid q$	y_{67}		y_{68}		y_{69}	$2\nmid q$	y_{70}	$2\mid q$
y_{71}		y_{72}	$2\nmid q$	y_{73}	$2\mid q$	y_{74}		y_{75}	$2\nmid q$
y_{76}	$2\mid q$	y_{77}		y_{78}	$2\nmid q$	y_{79}	$2\mid q$	y_{80}	
y_{81}	$2\nmid q$	y_{82}	$2\mid q$	y_{83}		y_{84}	$2\nmid q$	y_{85}	
y_{86}	$2\mid q$	y_{87}		y_{88}	$2\nmid q$	y_{89}	$2\nmid q$	y_{90}	$2\nmid q$
y_{91}	$2\mid q$	y_{92}		y_{93}		y_{94}	$2\nmid q$	y_{95}	$2\mid q$
y_{96}		y_{97}	$2\nmid q$	y_{98}		y_{99}		y_{100}	$2\nmid q$
y_{101}		y_{102}		y_{103}		y_{104}		y_{105}	$2\nmid q$
y_{106}		y_{107}		y_{108}	$2\nmid q$	y_{109}	$2\mid q$	y_{110}	
y_{111}	$2\nmid q$	y_{112}	$2\mid q$	y_{113}		y_{114}	$2\nmid q$	y_{115}	
y_{116}	$2\nmid q$	y_{117}		y_{118}		y_{119}		1	

A = representatives.
B = the conditions to take A as a representative.

TABLE 5

The representatives of the unipotent classes in the Chevalley groups $E_8(q)$.

A	B	A	B	A	B	A	B	A	B
z_1		z_2	$3\|q$	z_3	$3\|q$	z_4	$2\|q$	z_5	$2\|q$
z_6	$2\|q$	z_7	$5\|q$	z_8	$5\|q$	z_9	$5\|q$	z_{10}	$5\|q$
z_{11}		z_{12}	$2\|q$	z_{13}	$2\|q$	z_{14}	$2\|q$	z_{15}	$3\|q$
z_{16}	$3\|q$	z_{17}		z_{18}	$2\|q$	z_{19}	$2\|q$	z_{20}	$2\|q$
z_{21}		z_{22}	$2\nmid q$	z_{23}	$2\|q$	z_{24}	$2\|q$	z_{25}	$2\|q$
z_{26}	$3\|q$	z_{27}	$3\|q$	z_{28}	$3\|q$	z_{29}	$3\|q$	z_{30}	
z_{31}	$2\|q$	z_{32}	$2\|q$	z_{33}	$2\|q$	z_{34}	$3\|q$	z_{35}	$3\|q$
z_{36}		z_{37}	$2\nmid q$	z_{38}	$2\|q$	z_{39}		z_{40}	$2\|q$
z_{41}	$2\nmid q$	z_{42}		z_{43}	$2\|q$	z_{44}		z_{45}	$2\nmid q$
z_{46}	$2\|q$	z_{47}	$2\|q$	z_{48}	$2\|q$	z_{49}	$2\|q$	z_{50}	
z_{51}	$2\|q$	z_{52}		z_{53}	$2\|q$	z_{54}	$2\nmid q$	z_{55}	$2\|q$
z_{56}	$2\|q$	z_{57}		z_{58}	$2\|q$	z_{59}		z_{60}	$2\|q$
z_{61}		z_{62}		z_{63}	$2\nmid q$	z_{64}	$2\|q$	z_{65}	
z_{66}	$2\|q$	z_{67}	$3\|q$	z_{68}	$3\|q$	z_{69}		z_{70}	$2\nmid q$
z_{71}	$2\|q$	z_{72}	$2\|q$	z_{73}		z_{74}	$2\|q$	z_{75}	$3\|q$
z_{76}	$3\|q$	z_{77}	$2\|q$	z_{78}	$2\|q$	z_{79}	$2\|q$	z_{80}	$3\|q$
z_{81}	$3\|q$	z_{82}	$2, 3\nmid q$	z_{83}	$2, 3\nmid q$	z_{84}	$2, 3\nmid q$	z_{85}	
z_{86}	$2\|q$	z_{87}	$2\nmid q$	z_{88}		z_{89}	$3\|q$	z_{90}	
z_{91}	$2\|q$	z_{92}	$2\nmid q$	z_{93}		z_{94}	$2\|q$	z_{95}	
z_{96}	$2\nmid q$	z_{97}	$2\|q$	z_{98}		z_{99}	$2\nmid q$	z_{100}	$2\|q$
z_{101}		z_{102}	$2\nmid q$	z_{103}	$2\|q$	z_{104}	$2\|q$	z_{105}	
z_{106}		z_{107}	$2\nmid q$	z_{108}		z_{109}	$2\nmid q$	z_{110}	$2\|q$
z_{111}	$2\|q$	z_{112}	$2\|q$	z_{113}		z_{114}	$2\|q$	z_{115}	
z_{116}	$2\|q$	z_{117}		z_{118}		z_{119}	$2\|q$	z_{120}	$2\nmid q$
z_{121}		z_{122}		z_{123}	$2\|q$	z_{124}	$2\nmid q$	z_{125}	
z_{126}	$2\nmid q$	z_{127}	$2\|q$	z_{128}		z_{129}	$2\|q$	z_{130}	
z_{131}		z_{132}		z_{133}	$2\|q$	z_{134}	$2\nmid q$	z_{135}	
z_{136}	$2\nmid q$	z_{137}	$2\|q$	z_{138}	$2\|q$	z_{139}		z_{140}	
z_{141}		z_{142}		z_{143}		z_{144}		z_{145}	$2\|q$
z_{146}	$2\nmid q$	z_{147}		z_{148}	$2\|q$	z_{149}		z_{150}	$2\|q$
z_{151}	$2\nmid q$	z_{152}		z_{153}	$2\|q$	z_{154}		z_{155}	$2\|q$
z_{156}	$2\nmid q$	z_{157}		z_{158}		z_{159}	$2\|q$	z_{160}	$2\nmid q$
z_{161}		z_{162}	$2\|q$	z_{163}	$2\nmid q$	z_{164}		z_{165}	
z_{166}	$2\nmid q$	z_{167}	$2\|q$	z_{168}		z_{169}		z_{170}	$2\|q$
z_{171}	$2\nmid q$	z_{172}		z_{173}		z_{174}		z_{175}	$2\|q$
z_{176}	$2\nmid q$	z_{177}		z_{178}		z_{179}		z_{180}	
z_{181}		z_{182}	$2\|q$	z_{183}	$2\nmid q$	z_{184}		z_{185}	
z_{186}		z_{187}	$2\|q$	z_{188}	$2\nmid q$	z_{189}		z_{190}	
z_{191}	$2\|q$	z_{192}	$2\nmid q$	z_{193}		z_{194}		z_{195}	
z_{196}		z_{197}		z_{198}		z_{199}		z_{200}	
1		z_{201}							

A=representatives.

B=the conditions to take A as a representative.

Since $Z(z_{117})\simeq S_5$, $G(z_{117})(k)$ splits into 7-conjugate classes in $G(k)$. Thus we denote the representatives z_{117} and z_i $(196\leqq i\leqq 201)$ of the above conjugate classes.

TABLE 6
The inclusion relations among unipotent classes in the group E_6.

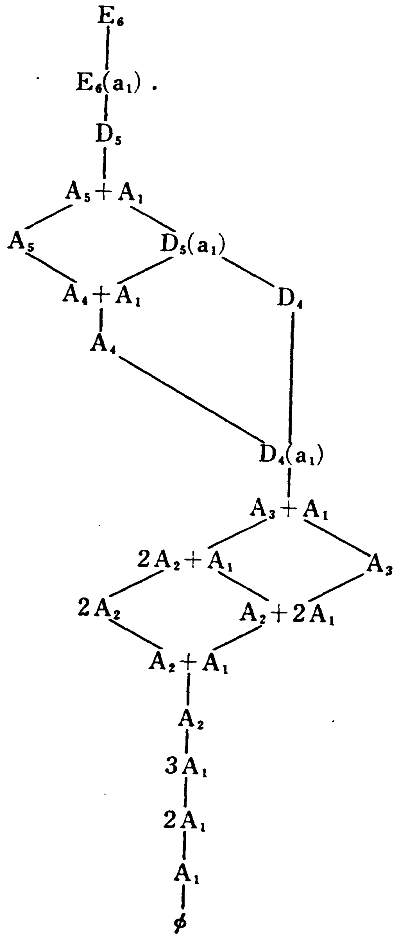


TABLE 7
The inclusion relations among unipotent classes in the group E_7.

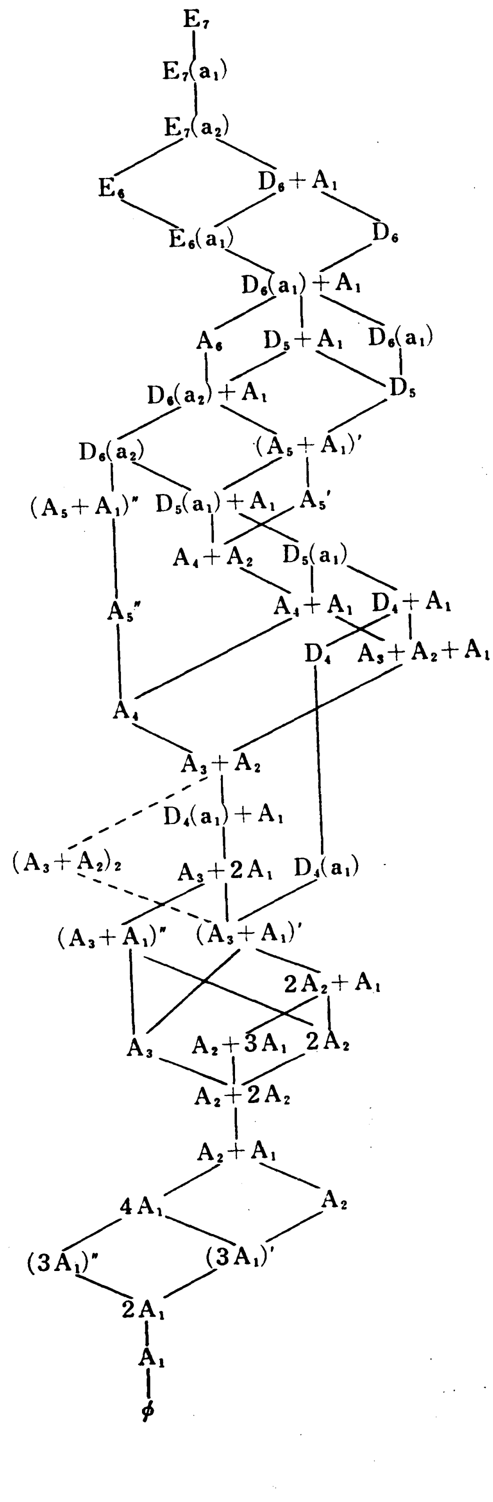


TABLE 8

The inclusion relations among unipotent classes in the group E_8.

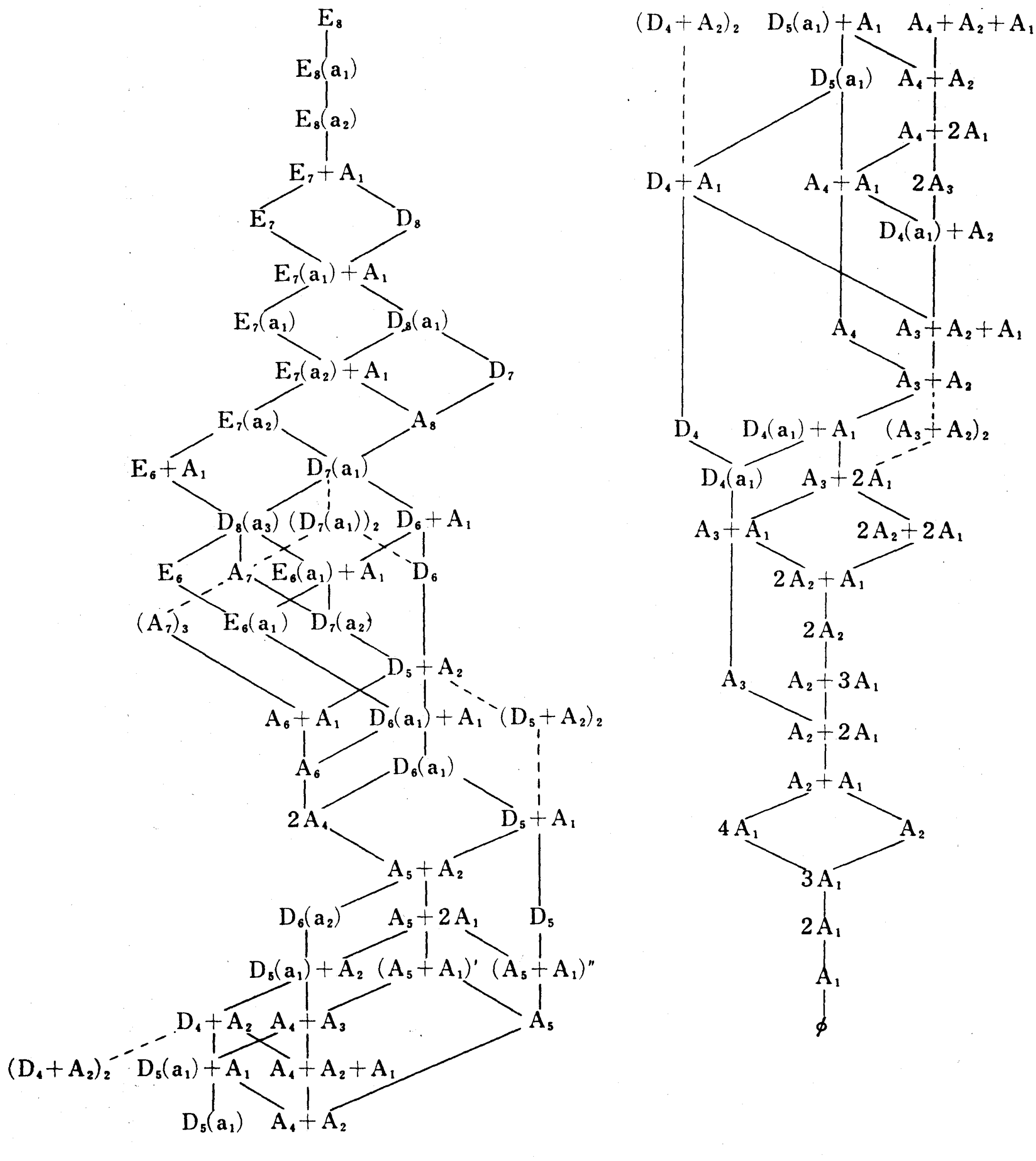


TABLE 9
The structures of the centralizers of unipotent elements in the group E_7.

Representative	$Z(x)$	$L(x)$	dim Ru $(Z_G(x))$
E_7	$Z_{2(6,p)}$	0	7
$E_7(a_1)$	Z_2	0	9
$E_7(a_2)$	Z_2	0	11
D_6+A_1	$Z_2\times Z_{(2,p-1)}$	0	13
E_6	$Z_{(6,p)}$	A_1	10
$E_6(a_1)$	Z_2	T_1	14
D_6	Z_2	A_1	12
$D_6(a_1)+A_1$	$Z^2_{(2,p-1)}$	0	17
A_6	$Z_{(2,p)}$	$\begin{cases} T_1 \ (p=2) \\ A_1 \ (p\neq 2) \end{cases}$	18 $(p=2)$ 16 $(p\neq 2)$
$D_6(a_1)$	Z_2	A_1	16
D_5+A_1	Z_2	A_1	16
$D_6(a_2)+A_1$	$S_3\times Z_{(2,p-1)}$	0	21
D_5	$Z_{(2,p)}$	$2A_1$	15
$(A_5+A_1)'$	Z_2	A_1	20
$(A_5+A_1)''$	$Z_{(2,p-1)}$	A_1	22
$D_6(a_2)$	$Z_{(2,p-1)}$	A_1	20
A_5'	1	$2A_1$	19
$D_5(a_1)+A_1$	$Z_{(2,p-1)}$	A_1	22
A_5''	$Z_{(2,p-1)}$	G_2	17
A_4+A_2	1	A_1	24
$D_5(a_1)$	Z_2	T_1+A_1	23
A_4	Z_2	T_1+A_2	24
A_4+A_1	Z_2	T_2	27
D_4+A_1	Z_2	B_2	21
$A_3+A_2+A_1$	$Z_{(2,p-1)}$	A_1	30
A_3+A_2	$Z_{(2,p-1)}$	$\begin{cases} T_1+A_1 \ (p\neq 2) \\ A_1 \ (p=2) \end{cases}$	31 $(p\neq 2)$ 32 $(p=2)$
$(A_3+A_2)_2$	1 $(p=2)$	$2A_1$ $(p=2)$	31 $(p=2)$
D_4	$Z_{(2,p)}$	C_3	16
$D_4(a_1)+A_1$	$Z_2\times Z_{(2,p-1)}$	$2A_1$	31
$D_4(a_1)$	S_3	$3A_1$	30
A_3+2A_1	$Z_{(2,p-1)}$	$2A_1$	33
$(A_3+A_1)'$	1	$3A_1$	32
$(A_3+A_1)''$	$Z_{(2,p-1)}$	B_3	26
A_3	1	A_1+B_3	25
$2A_2+A_1$	1	$2A_1$	37
$2A_2$	1	A_1+G_2	32
A_2+3A_1	$Z_{(2,p-1)}$	G_2	35
A_2+2A_1	1	$3A_1$	42
A_2+A_1	Z_2	T_1+A_3	41
A_2	Z_2	A_5	32
$4A_1$	$Z_{(2,p-1)}$	C_3	42
$(3A_1)'$	1	A_1+C_3	45
$(3A_1)''$	$Z_{(2,p-1)}$	F_4	27
$2A_1$	1	A_1+B_4	42
A_1	1	D_6	33
ϕ	1	E_7	0

TABLE 10
The structures of the centralizers of unipotent elements in the group E_8.

Representative	$Z(x)$	$L(x)$	dim Ru $(Z_G(x))$
E_8	$Z_{(60,p^2)}$	0	8
$E_8(a_1)$	$Z_{(12,p^2)}$	0	10
$E_8(a_2)$	$Z_{(4,p^2)}$	0	12
E_7+A_1	$Z_2\times Z_{(6,p)}$	0	14
E_7	$Z_{(12,p^2)}$	A_1	13
D_8	Z_2	0	16
$E_7(a_1)+A_1$	Z_2	0	18
$E_7(a_1)$	$Z_{(2,p)}$	A_1	17
$D_8(a_1)$	$\{Z_2\ (p\neq 2)$	0 $(p\neq 2)$	20 $(p\neq 2)$
	$\{D_8\ (p=2)$	0 $(p=2)$	20 $(p=2)$
$E_7(a_2)+A_1$	$S_3\times Z_{(2,p)}$	0	22
D_7	$Z_{(2,p)}$	A_1	19
$E_7(a_2)$	$Z_{(2,p)}$	A_1	21
A_8	S_3	0	24
E_6+A_1	$Z_{(6,p)}$	A_1	23
$D_7(a_1)$	$\{Z_2\ (p\neq 2)$	T_1 $(p\neq 2)$	25 $(p\neq 2)$
	$\{1\ (p=2)$	0 $(p=2)$	26 $(p=2)$
$(D_7(a_1))_2$	$Z_2\ (p=2)$	A_1 $(p=2)$	25 $(p=2)$
E_6	$Z_{(6,p)}$	G_2	18
$D_8(a_3)$	$\{S_3\ (p\neq 3)$	0 $(p\neq 3)$	28 $(p\neq 3)$
	$\{Z_2\ (p=3)$	0 $(p=3)$	28 $(p=3)$
D_6+A_1	Z_2	A_1	25
A_7	1	$\{A_1\ (p\neq 3)$	27 $(p\neq 3)$
		$\{0\ (p=3)$	30 $(p=3)$
$(A_7)_3$	1 $(p=3)$	A_1 $(p=3)$	29 $(p=3)$
$E_6(a_1)+A_1$	Z_2	T_1	29
D_6	$Z_{(2,p)}$	B_2	22
$D_7(a_2)$	Z_2	T_1	31
$E_6(a_1)$	Z_2	A_2	26
D_5+A_2	$\{Z_2\ (p\neq 2)$	T_1 $(p\neq 2)$	33 $(p\neq 2)$
	$\{1\ (p=2)$	0 $(p=2)$	34 $(p=2)$
$(D_5+A_2)_2$	$Z_2\ (p=2)$	A_1 $(p=2)$	33 $(p=2)$
A_6+A_1	1	A_1	33
$D_6(a_1)+A_1$	$Z_{(2,p-1)}$	A_1	33
$D_6(a_1)$	$Z_2\times Z_{(2,p)}$	$2A_1$	32
A_6	$Z_{(2,p)}$	$\{2A_1\ (p\neq 2)$	32 $(p\neq 2)$
		$\{T_1+A_1\ (p=2)$	34 $(p=2)$
D_5+A_1	$Z_{(2,p)}$	$2A_1$	34
$2A_4$	S_5	0	40
A_5+A_2	S_3	A_1	39
A_5+2A_1	Z_2	A_1	41
$D_6(a_2)$	Z_2	$2A_1$	38
D_5	$Z_{(2,p)}$	B_3	27
$D_5(a_1)+A_2$	1	A_1	43
$(A_5+A_1)'$	1	$2A_1$	40

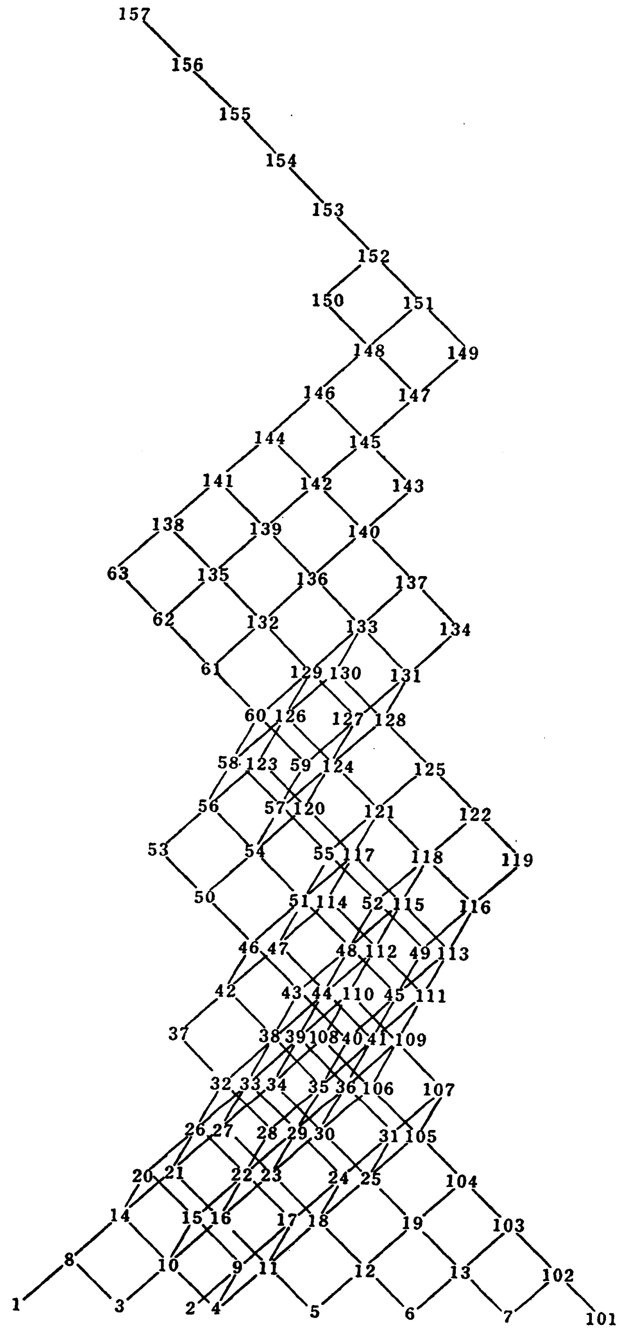
TABLE 10. (Continued)

Representative	$Z(x)$	$L(x)$	dim Ru $(Z_G(x))$
$(A_5+A_1)''$	Z_2	G_2	36
D_4+A_2	Z_2	A_2 $(p\neq 2)$ A_1 $(p=2)$	42 $(p\neq 2)$ 47 $(p=2)$
$(D_4+A_2)_2$	Z_2 $(p=2)$	G_2 $(p=2)$	42 $(p=2)$
A_4+A_3	1	A_1	45
A_5	1	A_1+G_2	35
$D_5(a_1)+A_1$	1	$2A_1$	46
$A_4+A_2+A_1$	1	A_1	49
A_4+A_2	1	$2A_1$	48
$D_5(a_1)$	Z_2	A_3	43
D_4+A_1	$Z_{(2,p)}$	C_3	43
A_4+2A_1	Z_2	T_1+A_1	52
D_4	$Z_{(2,p)}$	F_4	28
A_4+A_1	Z_2	T_1+A_2	51
$2A_3$	1	B_2	50
A_4	Z_2	A_4	44
$D_4(a_1)+A_2$	Z_2	A_2	56
$A_3+A_2+A_1$	1	$2A_1$	60
A_3+A_2	$Z_{(2,p-1)}$	T_1+B_2 $(p\neq 2)$ B_2 $(p=2)$	59 $(p\neq 2)$ 60 $(p=2)$
$(A_3+A_2)_2$	1 $(p=2)$	A_1+B_2 $(p=2)$	59 $(p=2)$
$D_4(a_1)+A_1$	S_3	$3A_1$	63
A_3+2A_1	1	A_1+B_2	63
$D_4(a_1)$	S_3	D_4	54
$2A_2+2A_1$	1	B_2	70
A_3+A_1	1	A_1+B_3	60
$2A_2+A_1$	1	A_1+G_2	69
A_3	1	B_5	45
$2A_2$	Z_2	$2G_2$	64
A_2+3A_1	1	A_1+G_2	77
A_2+2A_1	1	A_1+B_3	78
A_2+A_1	Z_2	A_5	77
$4A_1$	1	C_4	84
A_2	Z_2	E_6	56
$3A_1$	1	A_1+F_4	81
$2A_1$	1	B_6	78
A_1	1	E_7	57
ϕ	1	E_8	0

TABLE 11
The notations of positive roots of the root system E_8.

$\alpha_8=\alpha_1+\alpha_3$,	$\alpha_9=\alpha_2+\alpha_4$,	$\alpha_{10}=\alpha_3+\alpha_4$,	$\alpha_{11}=\alpha_4+\alpha_5$,
$\alpha_{12}=\alpha_5+\alpha_6$,	$\alpha_{13}=\alpha_6+\alpha_7$,	$\alpha_{14}=\alpha_8+\alpha_4$,	$\alpha_{15}=\alpha_9+\alpha_3$,
$\alpha_{16}=\alpha_{10}+\alpha_5$,	$\alpha_{17}=\alpha_9+\alpha_5$,	$\alpha_{18}=\alpha_{11}+\alpha_6$,	$\alpha_{19}=\alpha_{12}+\alpha_7$,
$\alpha_{20}=\alpha_{14}+\alpha_2$,	$\alpha_{21}=\alpha_{14}+\alpha_5$,	$\alpha_{22}=\alpha_{15}+\alpha_5$,	$\alpha_{23}=\alpha_{16}+\alpha_6$,
$\alpha_{24}=\alpha_{17}+\alpha_6$,	$\alpha_{25}=\alpha_{18}+\alpha_7$,	$\alpha_{26}=\alpha_{20}+\alpha_5$,	$\alpha_{27}=\alpha_{21}+\alpha_6$,
$\alpha_{28}=\alpha_{22}+\alpha_4$,	$\alpha_{29}=\alpha_{22}+\alpha_6$,	$\alpha_{30}=\alpha_{23}+\alpha_7$,	$\alpha_{31}=\alpha_{24}+\alpha_7$,
$\alpha_{32}=\alpha_{26}+\alpha_4$,	$\alpha_{33}=\alpha_{26}+\alpha_6$,	$\alpha_{34}=\alpha_{27}+\alpha_7$,	$\alpha_{35}=\alpha_{28}+\alpha_6$,
$\alpha_{36}=\alpha_{29}+\alpha_7$,	$\alpha_{37}=\alpha_{32}+\alpha_2$,	$\alpha_{38}=\alpha_{32}+\alpha_6$,	$\alpha_{39}=\alpha_{33}+\alpha_7$,
$\alpha_{40}=\alpha_{35}+\alpha_5$,	$\alpha_{41}=\alpha_{35}+\alpha_7$,	$\alpha_{42}=\alpha_{37}+\alpha_6$,	$\alpha_{43}=\alpha_{38}+\alpha_5$,
$\alpha_{44}=\alpha_{38}+\alpha_7$,	$\alpha_{45}=\alpha_{40}+\alpha_7$,	$\alpha_{46}=\alpha_{42}+\alpha_5$,	$\alpha_{47}=\alpha_{42}+\alpha_7$,
$\alpha_{48}=\alpha_{43}+\alpha_7$,	$\alpha_{49}=\alpha_{45}+\alpha_6$,	$\alpha_{50}=\alpha_{46}+\alpha_4$,	$\alpha_{51}=\alpha_{46}+\alpha_7$,
$\alpha_{52}=\alpha_{48}+\alpha_6$,	$\alpha_{53}=\alpha_{50}+\alpha_2$,	$\alpha_{54}=\alpha_{50}+\alpha_7$,	$\alpha_{55}=\alpha_{51}+\alpha_6$,
$\alpha_{56}=\alpha_{53}+\alpha_7$,	$\alpha_{57}=\alpha_{54}+\alpha_6$,	$\alpha_{58}=\alpha_{56}+\alpha_6$,	$\alpha_{59}=\alpha_{57}+\alpha_5$,
$\alpha_{60}=\alpha_{58}+\alpha_5$,	$\alpha_{61}=\alpha_{60}+\alpha_4$,	$\alpha_{62}=\alpha_{61}+\alpha_3$,	$\alpha_{63}=\alpha_{62}+\alpha_1$,
$\alpha_{102}=\alpha_{101}+\alpha_7$,	$\alpha_{103}=\alpha_{102}+\alpha_6$,	$\alpha_{104}=\alpha_{103}+\alpha_5$,	$\alpha_{105}=\alpha_{104}+\alpha_4$,
$\alpha_{106}=\alpha_{105}+\alpha_3$,	$\alpha_{107}=\alpha_{105}+\alpha_2$,	$\alpha_{108}=\alpha_{106}+\alpha_1$,	$\alpha_{109}=\alpha_{106}+\alpha_2$,
$\alpha_{110}=\alpha_{108}+\alpha_2$,	$\alpha_{111}=\alpha_{109}+\alpha_4$,	$\alpha_{112}=\alpha_{110}+\alpha_4$,	$\alpha_{113}=\alpha_{111}+\alpha_5$,
$\alpha_{114}=\alpha_{112}+\alpha_3$,	$\alpha_{115}=\alpha_{112}+\alpha_5$,	$\alpha_{116}=\alpha_{113}+\alpha_6$,	$\alpha_{117}=\alpha_{114}+\alpha_5$,
$\alpha_{118}=\alpha_{115}+\alpha_6$,	$\alpha_{119}=\alpha_{116}+\alpha_7$,	$\alpha_{120}=\alpha_{117}+\alpha_4$,	$\alpha_{121}=\alpha_{117}+\alpha_6$,
$\alpha_{122}=\alpha_{118}+\alpha_7$,	$\alpha_{123}=\alpha_{120}+\alpha_2$,	$\alpha_{124}=\alpha_{120}+\alpha_6$,	$\alpha_{125}=\alpha_{121}+\alpha_7$,
$\alpha_{126}=\alpha_{124}+\alpha_2$,	$\alpha_{127}=\alpha_{124}+\alpha_5$,	$\alpha_{128}=\alpha_{124}+\alpha_7$,	$\alpha_{129}=\alpha_{126}+\alpha_5$,
$\alpha_{130}=\alpha_{126}+\alpha_7$,	$\alpha_{131}=\alpha_{127}+\alpha_7$,	$\alpha_{132}=\alpha_{129}+\alpha_4$,	$\alpha_{133}=\alpha_{129}+\alpha_7$,
$\alpha_{134}=\alpha_{131}+\alpha_6$,	$\alpha_{135}=\alpha_{132}+\alpha_3$,	$\alpha_{136}=\alpha_{132}+\alpha_7$,	$\alpha_{137}=\alpha_{133}+\alpha_6$,
$\alpha_{138}=\alpha_{135}+\alpha_1$,	$\alpha_{139}=\alpha_{135}+\alpha_7$,	$\alpha_{140}=\alpha_{136}+\alpha_6$,	$\alpha_{141}=\alpha_{139}+\alpha_1$,
$\alpha_{142}=\alpha_{140}+\alpha_3$,	$\alpha_{143}=\alpha_{140}+\alpha_6$,	$\alpha_{144}=\alpha_{142}+\alpha_1$,	$\alpha_{145}=\alpha_{142}+\alpha_5$,
$\alpha_{146}=\alpha_{145}+\alpha_1$,	$\alpha_{147}=\alpha_{145}+\alpha_4$,	$\alpha_{148}=\alpha_{147}+\alpha_1$,	$\alpha_{149}=\alpha_{147}+\alpha_2$,
$\alpha_{150}=\alpha_{148}+\alpha_3$,	$\alpha_{151}=\alpha_{148}+\alpha_2$,	$\alpha_{152}=\alpha_{151}+\alpha_3$,	$\alpha_{153}=\alpha_{152}+\alpha_4$,
$\alpha_{154}=\alpha_{153}+\alpha_5$,	$\alpha_{155}=\alpha_{154}+\alpha_6$,	$\alpha_{156}=\alpha_{155}+\alpha_7$,	$\alpha_{157}=\alpha_{156}+\alpha_{101}$.

TABLE 13
The root adjacency graph of type E_8.



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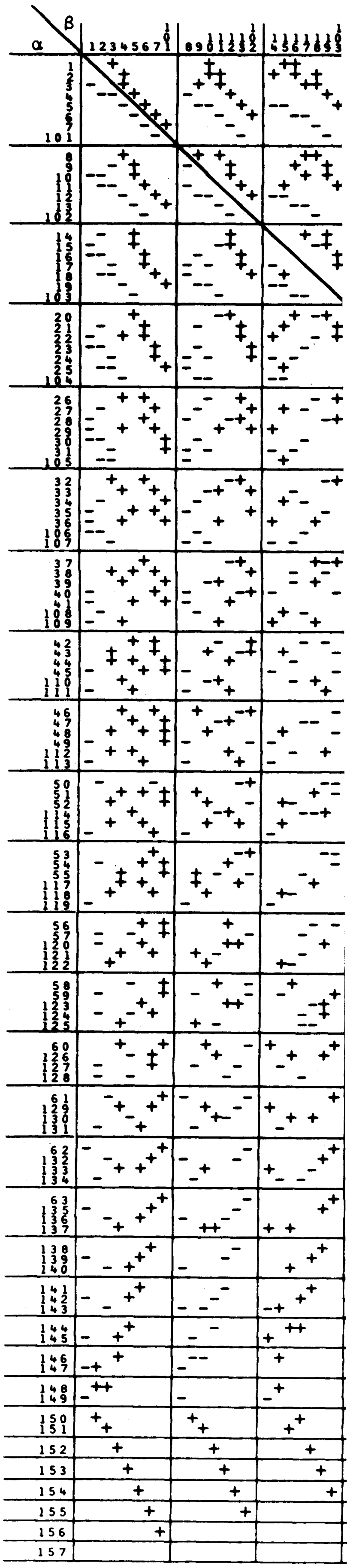
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