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Correction to: On Regular Fréchet-Lie Groups I

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The paper with the above title contains the misprints and an omission. The omisson occurs in Lemma 3.5.

The correction in the statement of Lemma 3.5 is: Page 380 in Lemma 3.5: Φ_2 in the statement and the proof should be understood as a mapping involving X_1-variable, i.e., $\Phi_2(x;\xi)$ should be replaced by $\Phi_2(x; X_1, \xi)$.

By the above reason, the proof of Proposition 4.1 is not correct, for $b_2(x;\tilde{\xi})$ in (53) contains X_1-variable. This gap is repaired as follows: Denote $\phi(x;\tilde{\xi}, X_1)=\langle\tilde{\xi}|\tilde{S}(x; X_1, \bar{X}_0(x;\xi(x;\tilde{\xi})))\rangle$, and set

$$\begin{aligned}\psi(x;\tilde{\xi}, Y, \zeta, X_1)&=\langle\zeta|Y\rangle+\phi(x;\tilde{\xi}+\zeta, X_1)-\phi(x;\tilde{\xi}, X_1)\\&=\left\langle\zeta\,\middle|\,Y+\int_0^1\frac{\partial\phi}{\partial\tilde{\xi}}(x;\tilde{\xi}+t\zeta, X_1)dt\right\rangle.\end{aligned}$$

Note that if $\varphi=\mathrm{id.}$, then $\phi=\langle\tilde{\xi}|X_1\rangle$, hence $\psi=\langle\zeta|Y+X_1\rangle$. Therefore, one may assume that $\int_0^1\partial\phi/\partial\tilde{\xi}(x;\tilde{\xi}+t\zeta, X_1)dt$ is sufficiently close to X_1 in the C^2-topology.

By Lemma 3.5, the given operator can be written by

$$\iint a(x;\tilde{\xi}, X_1)e^{-i\phi(x;\tilde{\xi},X_1)}\nu_1(x, z)u(z)đzđ\tilde{\xi},\quad z=\cdot_x X_1\,, \tag{1}$$

where ν_1 is the cut off function defined in (46). (Cf. (34)~(40)). Since the breadth of ν_1 is sufficiently small, one may assume $\phi(x;\tilde{\xi}, X_1)\equiv\langle\tilde{\xi}|X_1\rangle$ for $|X_1|>>0$. For amplitude $a\in\tilde{\Sigma}^{\beta}_{C}$ we consider the following equation:

$$a(x;\tilde{\xi}, X_1)=\iint e^{-i\psi(x;\tilde{\xi},Y,\zeta,X_1)}b(x;\tilde{\xi}, Y)đYđ\zeta\;, \tag{2}$$

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where $(x; \tilde{\xi})$ is understood as a parameter. To do that, we have to fix a function space $\tilde{S}^{\beta}_{C}$ as the totality of $g(x; \xi, X_1) \in C^{\infty}(T^*N \oplus TN)$ such that g is rapidly decreasing in X and g has the following asymptotic expansion:

$$g(x; r\hat{\xi}, X) \sim g_{\beta}(x; \hat{\xi}, X)\mu(r)^{\beta} + g_{\beta-1}(x; \hat{\xi}, X)\mu(r)^{\beta-1} + \cdots ,$$

where $g_{\beta-j}$'s are C^{∞} functions on $S^*N \oplus TN$, rapidly decreasing in X.

Note that by virture of the cut off function ν and the asymptotic expansion (11) (cf. p. 365), one may assume $a(x; \tilde{\xi}, X_1)$ in (1) is an element of $\tilde{S}^{\beta}_{C}$. If (2) can be solved in $\tilde{S}^{\beta}_{C}$ for a given $a \in \tilde{S}^{\beta}_{C}$, then (1) can be replaced as follows:

$$\iint a e^{-i\phi} \nu_1 u \, dz \, d\tilde{\xi} = \int b'(x; \xi_1) \widetilde{\nu u}(\varphi(x; \xi_1)) d\xi_1 + (K \circ u)(x) ,$$

where

$$(3) \qquad b'(x; \xi_1) = \iint b(x; \xi_1 - \zeta, Y) e^{-i\langle \zeta | Y \rangle} dY d\zeta .$$

If $b \in \tilde{S}^{\beta}_{C}$, then we obtain $b' \in \Sigma^{\beta}_{C}$. Thus, we have only to solve (2).

Now, remark that (2) is a Fourier-integral operator of order 0 on $\boldsymbol{R}^n$ for each fixed $(x; \tilde{\xi})$. Apply the adjoint operator to both sides of (2). The left hand side is

$$(4) \qquad b''(x; \tilde{\xi}, Z) = \iint e^{i\psi(x; \tilde{\xi}, Z, \eta, X_1)} a(x; \tilde{\xi}, X_1) dX_1 d\eta .$$

Since $X_2 = \int_0^1 (\partial\phi/\partial\tilde{\xi})(x; \tilde{\xi} + t\eta, X_1) dt$ is sufficiently close to X_1, b'' can be written as

$$(5) \qquad b''(x; \tilde{\xi}, Z) = \iint e^{i\langle \eta | Z + X_2 \rangle} a(x; \tilde{\xi}, X_1(x; \tilde{\xi}, \eta, X_2)) \frac{dX_1}{dX_2}(x; \tilde{\xi}, \eta, X_2) dX_2 d\eta ,$$

and belongs to $\tilde{S}^{\beta}_{C}$. Thus, we have only to solve

$$(6) \qquad b''(x; \tilde{\xi}, Z_1) = \iiiint e^{i\psi(x; \tilde{\xi}, Z, \eta, X_1) - i\psi(x; \tilde{\xi}, Y, \zeta, X_1)} b(x; \tilde{\xi}, Y) dY d\zeta dX_1 d\eta .$$

Note that (6) is a pseudo-differential equation.

For each fixed $(x; \tilde{\xi}, Z, \eta, Y)$, compute out the critical point and value of the above phase function. Then, using that $\int_0^1 (\partial\phi/\partial\tilde{\xi})(x; \tilde{\xi} + \zeta + t(\eta - \zeta), X_1) dt$ is sufficiently close to X_1, one can obtain that

$$(7) \qquad b''(x; \tilde{\xi}, Z)=\iint\left[\iint\frac{d(\zeta, X_1)}{d(\zeta', X')}e^{-i\langle\zeta'|X'\rangle}đX'đ\zeta'\right]e^{-i\langle\eta|Y-Z\rangle}b(x; \tilde{\xi}, Y)đYđ\eta .$$

Note that

$$c(x; \tilde{\xi}, Z, \eta, Y)=\iint\frac{d(\zeta, X_1)}{d(\zeta', X')}e^{-i\langle\zeta'|X'\rangle}đX'đ\zeta' ,$$

is sufficiently close to 1. Thus, we see that (7) is an invertible pseudo-differential operator. Therefore, $b(x; \tilde{\xi}, Y)$ is obtained in the following form

$$(8) \qquad b(x; \tilde{\xi}, Y)=\iint f(x; \tilde{\xi}, Z, \eta, Y)e^{-i\langle\eta|Z-Y\rangle}b''(x; \tilde{\xi}, Z)đZđ\eta .$$

Hence by (5), we obtain $b \in \tilde{S}^{\beta}_{C}$ and $b' \in \Sigma^{\beta}_{C}$. This computation is not so easy one, but the tiresome computation using (4)–(8) and (3) leads us directly to the conclusion.

Other miscellaneous errata are as follows:

p. 353 $l \uparrow 3$: $h(s, t)\equiv 1$ should be read $h(0, t)\equiv 1$.

p. 358 $l \downarrow 9$: $b(x, \hat{\xi})$ should be read $b(x; r\hat{\xi})$.

p. 385 in Proposition 5.3: The domain of integrations should be T^*_x.

p. 387 $l \downarrow 9$: The second line in the computation of I_λ should be read

$$= \lim_{s\to\infty}\int_0^1\int_{S^*_xN}(i\hat{\xi})^{\beta}(1-\kappa)\left(\frac{1}{i}\frac{d}{dt}\right)^{\lambda}(t^{|\beta|+n-1}b(x; t\hat{\xi}))e^{-its\langle\hat{\xi}|Z-(1/s)X'_0(x;\hat{\xi})\rangle}đ\hat{\xi}đt .$$

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