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The Classification of Fano 3-Folds with Torus Embeddings

Keiichi WATANABE and Masayuki WATANABE

Nagoya Institute of Technology and Waseda University

Introduction

Let X be a smooth projective 3-fold over an algebraically closed field k. X is called a Fano 3-fold if the anti-canonical divisor $-K_X$ of X is ample. Recently, Ishkovsky has developped the theory of Fano 3-folds in his papers [1], [2] and has determined the structure of Fano 3-folds with Picard number 1. In this paper, we will consider Fano 3-folds with torus embedding (which we will call toric Fano 3-folds) and determine all the toric Fano 3-folds up to isomorphism. As the result, we see that there are 18 toric 3-folds and their Picard numbers are at most 5.

§1. The statement of the result.

THEOREM. (1) *If V is a toric Fano 3-fold, then the Picard number $\rho=\rho(V)$ of V is not greater than 5 and V is isomorphic to one of the following manifolds.*

(I) ($\rho=1$) (a) $\boldsymbol{P}^3$.

(II) ($\rho=2$) (b) $\boldsymbol{P}^2\times\boldsymbol{P}^1$, (c) $\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$, (d) $\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(2))$, (e) $\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$.

In the following, we denote by DS_k ($6\leqq k\leqq 8$) the toric Del Pezzo surfaces obtained from $\boldsymbol{P}^2$ by blowing up 9-k points of $\boldsymbol{P}^2$ (cf. (2.7)).

(III) ($\rho=3$) (f) $\boldsymbol{P}^1\times\boldsymbol{P}^1\times\boldsymbol{P}^1$, (g) $DS_8\times\boldsymbol{P}^1$, (h) $\boldsymbol{P}(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1, 1))$,

(i) $\boldsymbol{P}(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1, -1))$,

(j) $\boldsymbol{P}(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$, *where ℓ is the total transform of a line in $\boldsymbol{P}^2$,*

(k) F_1^3; *obtained by blowing up a line of $\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(2))$,*

(l) F_2^3; *obtained by blowing up a point of $\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$.*

(*The precise definition of F_i^ρ's will be given in* § 3.)

(IV) ($\rho=4$) (m) $DS_7\times\boldsymbol{P}^1$,

(n) F_1^4; *obtained by blowing up an exceptional line of $\boldsymbol{P}(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1, -1))$,*

(o) F_2^4; *obtained by blowing up an exceptional line of $DS_8\times\boldsymbol{P}^1$,*

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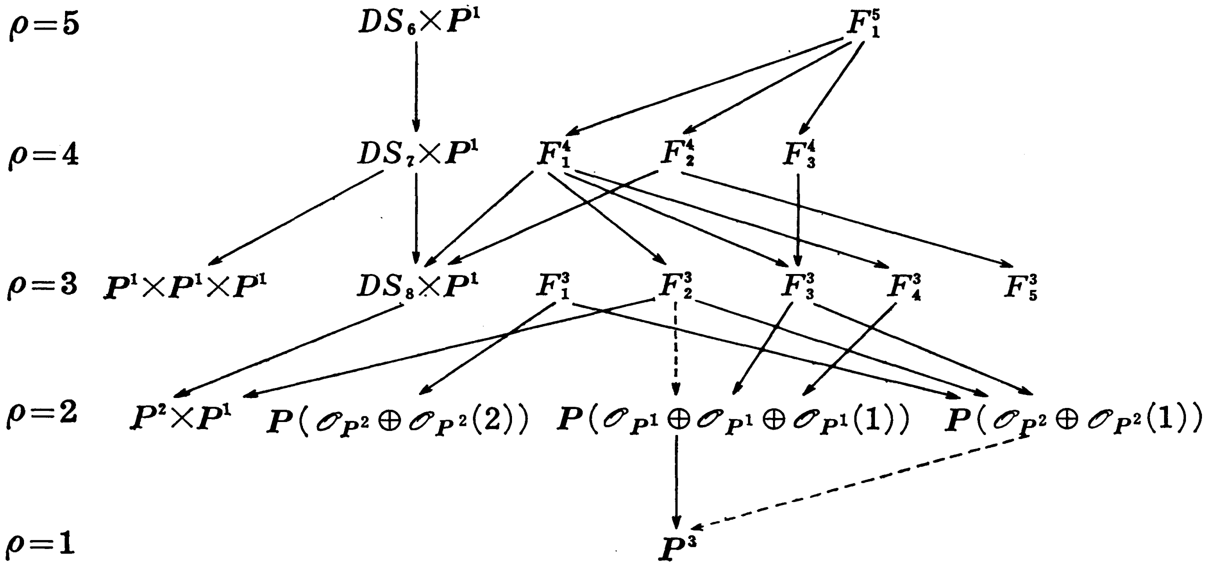
(p) F_3^4; *obtained by blowing up an exceptional line of* $P(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$.

(V) $(\rho=5)$ (q) $DS_6\times P^1$,

(r) F_1^5; *obtained by blowing up an exceptional line of* F_1^4.

In the above statements, a line means a 1-dimensional T-stable subvariety.

(2) *The blowing-up and blowing-down relations between these Fano 3-folds are indicated by the following diagram. (The arrow → indicates a blowing-up with center a line and the dotted arrow ⇢ indicates a blowing-up with center a point.)*



where $F_3^3=P(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$, $F_4^3=P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,-1))$, *and* $F_5^3=P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,1))$.

§ 2. Some preliminaries on toric 3-folds.

To carry out the classification, we need some knowledge of the theory of toric 3-folds, for which we refer to [3]. First we fix our notations.

(2.1) k is an algebraically closed field.

T is a three dimensional algebraic torus over k.

(Z^3, Δ) is a finite partial polyhedral decomposition.

$V=\mathrm{Tem}(\Delta)$ is a non-singular complete toric 3-fold determined by Δ.

$Sk^1=\{\sigma_1, \cdots, \sigma_d\}$ is the set of 1-skeltons of Δ.

$n_i\in Z^3$ is the fundamental generator of σ_i $(i=1, \cdots, d)$.

$\{D_1, \cdots, D_d\}$ is the set of T-stable prime divisors of V, where D_i is the closure of $\mathrm{orb}(\sigma_i)$ $(i=1, \cdots, d)$.

$\rho=\rho(V)$ is the Picard number of V. Note that $\rho=d-3$ in this case ([3], 6.1).

K_V is the canonical divisor of V. We can put $K_V = -\sum_{i=1}^{d} D_i$ (cf. [3], 6.6).

PROPOSITION (2.2) ([3], Proposition 9.1). *There are canonical bijections between the followings.*

(a) *The set of isomorphic classes of non-singular complete toric 3-folds.*

(b) *The set of isomorphic classes of admissible Z^3-weighted triangulations of S, where S is a sphere in R^3 centered at the origin.*

(c) *The set of combinatoric isomorphic classes of admissible doubly Z-weighted triangulations of S.*

By abuse of language, we will denote the Z^3-weighted or the doubly Z-weighted triangulation of S corresponding to Δ by the same letter Δ.

(2.3) For a Z^3-weighted (or doubly Z-weighted) triangulation Δ of S, let v_i be the vertex of Δ corresponding to σ_i and s_i be the number of edges concentrating to v_i ($i=1, \cdots, d$). Note that the number of edges of Δ is $(\sum_{i=1}^{d} s_i)/2$ and the number of triangles of Δ is $(\sum_{i=1}^{d} s_i)/3$. By Euler's theorem, we have $(\sum_{i=1}^{d} s_i)/3 - (\sum_{i=1}^{d} s_i)/2 + d = 2$ or

$$\sum_{i=1}^{d} s_i = 6d - 12 . \tag{2.3.1}$$

If $L = v_i \cdot v_j$ is the edge of Δ connecting the vertexes v_i and v_j, we will denote the line $D_i \cap D_j$ of V by the same letter L.

LEMMA (2.4). *If $-K_V$ is ample, then $d \leqq 10$.*

PROOF. For an edge $L = v_i \cdot v_j$ of Δ (or a line $D_i \cap D_j$ of V), we put $\delta_L = (D_i^2 \cdot D_j) + (D_i \cdot D_j^2) + 1$. Since $-K_V$ is ample, $(-K_V \cdot L) > 0$. On the other hand, if v and v' are the vertexes such that the triangles $\{v, v_i, v_j\}$ and $\{v', v_i, v_j\}$ belong to Δ and if D and D' are divisors of V corresponding to v and v' respectively, we have $(-K_V \cdot L) = (\sum_{m=1}^{d} D_m \cdot D_i \cdot D_j) = (D_i^2 \cdot D_j) + (D_i \cdot D_j^2) + (D_i \cdot D_j \cdot D) + (D_i \cdot D_j \cdot D') = \delta_L + 1$. Hence we get $\delta_L \geqq 0$ for every edge L.

Moreover, we have

$$\sum_L \delta_L = \sum_{1 \leq i < j \leq d} \{(D_i^2 \cdot D_j) + (D_i \cdot D_j^2) + 1\} = \sum_{i=1}^{d} \{\sum_{j \neq i} (D_i \cdot D_j^2)\} + \frac{1}{2} \sum_{i=1}^{d} s_i . \tag{2.4.1}$$

Since each D_i is a non-singular complete toric surface and the set $\{D_j \cap D_i \mid j \neq i,\ D_i \cap D_j \neq \varnothing\}$ is the set of T-stable prime divisors on D_i, we have

(2.4.2) $$\sum_{j\neq i}(D_i\cdot D_j^2)=12-3s_i \quad (\text{cf. [3], §8}).$$

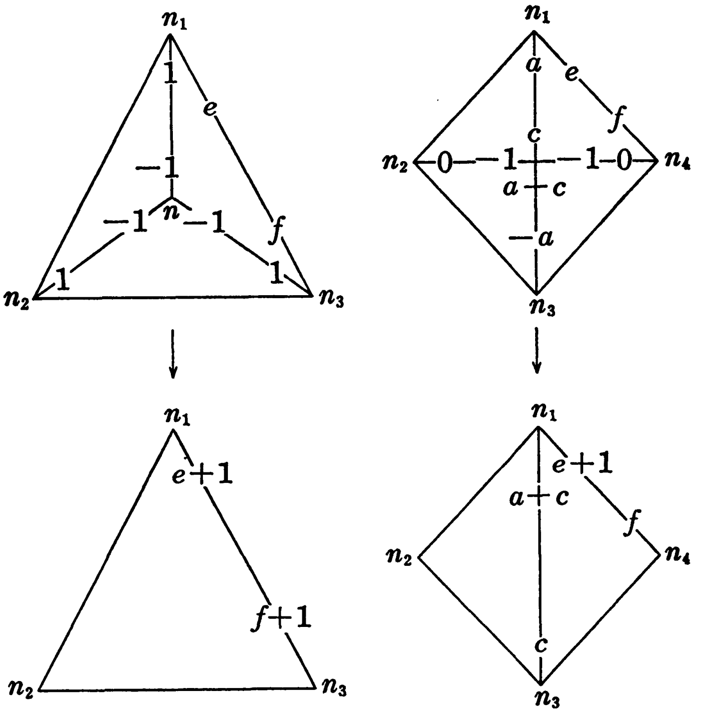
Putting (2.4.1), (2.4.2) and (2.3.1) together, we get

(2.4.3) $$\sum_L \delta_L=\sum_{i=1}^{d}(12-3s_i)+\frac{1}{2}\sum_{i=1}^{d}s_i=30-3d\geqq 0 .$$

PROPOSITION (2.5). *$-K_V$ is ample if and only if for every triangle τ of Δ, $(m(\tau), n_j)>-1$ for $n_j\notin\tau$, where $m(\tau)$ is an element of $(\boldsymbol{Z}^3)^\vee$ (dual of $\boldsymbol{Z}^3$) detemined by $(m(\tau), n_i)=-1$ for $n_i\in\tau$.*

PROOF. This is a special case of ([3], 6.5).

PROPOSITION (2.6) ([3], 9.2). *Let v be a vertex of Δ and $v_1, \cdots, v_s$ be the vertexes of its link going around v in this order. The divisor D corresponding to v can be contracted smoothly to a point (resp. to a line) if and only if $s=3$ and $n=n_1+n_2+n_3$ (resp. $s=4$ and $n=n_1+n_3$ or $n=n_2+n_4$). The contraction takes place as in the following figures.*



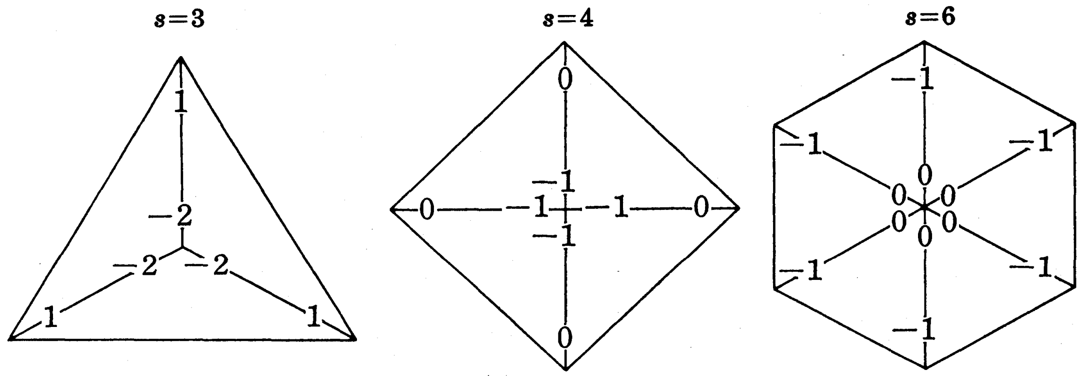
$n=n_1+n_3$

PROPOSITION (2.7). *If X is a 2-dimensional complete smooth torus embedding with ample $-K_X$, then X is isomorphic to one of the following surfaces.* (1) $\boldsymbol{P}^2$, (2) $\boldsymbol{P}^1\times\boldsymbol{P}^1$, (3) DS_k $(6\leqq k\leqq 8)$, *where DS_k is the surface obtained from $\boldsymbol{P}^2$ by blowing up 9-k points of $\boldsymbol{P}^2$. In this case, the* *center of the blowing up is intersection of two coordinate axes of* P^2.

§ 3. The classification.

LEMMA (3.1). *There exists no toric Fano 3-folds for* $d=9$ *or* 10.

PROOF. Let v be a vertex of Δ and s be the number of edges concentrating to v. If $d=10$, by (2.4.3), we must have $\delta_L=0$ for every edge L of Δ. Then, considering the double Z-weighting, we see that the possible value of s is 3, 4 or 6 and that the weighted link around the vertex v must be one of the followings.

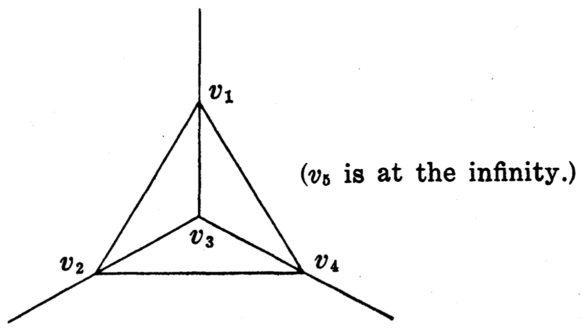


We can easily see that these weighted links cannot make an admissible dobule Z-weighting.

If $d=9$, let δ_v be the sum of δ_L concentrating to v. Then by (2.4.3), $\delta_v \leqq 3$ for every vertex v. There are 33 types of weighted links of a vertex v satisfying the condition $\delta_v \leqq 3$. But we can see that these cannot make an admissible double Z-weighted triangulation.

(3.2) Now, let us begin the classification. As we have seen, $4 \leqq d \leqq 8$. If $d=4$, then $V \cong P^3$ (cf. [3], Theorem 7.1).

(3.3) If $d=5$, the triangulation of the sphere is as follows.



As V is smooth, we may assume $n_1=(1,0,0)$, $n_2=(0,1,0)$, and $n_3=(0,0,1)$. Also, as $\det({}^tn_1, {}^tn_3, {}^tn_4)=\det({}^tn_3, {}^tn_2, {}^tn_4)=\det({}^tn_5, {}^tn_1, {}^tn_4)=\det({}^tn_5, {}^tn_2, {}^tn_1)=\det({}^tn_5, {}^tn_4, {}^tn_2)=1$, we may put $n_4=(-1,-1,c)$ and $n_5=(a,b,-1)$, where a, b, c are integers such that $ac=bc=0$. Moreover, by (2.5), we have inequalities $a+b<2$, $c<3$, $2a-ac-b>-2$, $2b-bc-a>-2$ and $ac+bc-c<0$. From these inequalities, it follows that Δ is isomorphic to one of the following $\boldsymbol{Z}^3$-weightings. (In what follows, we describe the $\boldsymbol{Z}^3$-weighting by the matrix $N=({}^tn_1, \cdots, {}^tn_d)$.)

(1)
$$N=\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}.$$

In this case, the divisor D_4 contracts to a line in $\boldsymbol{P}^3$ and $V\cong\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$. (This is the case (e) of the Theorem.)

(2)
$$N=\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}.$$

In this case, the divisor D_3 contracts to a point in $\boldsymbol{P}^3$ and $V\cong\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$. (This is the case (c) of the Theorem.)

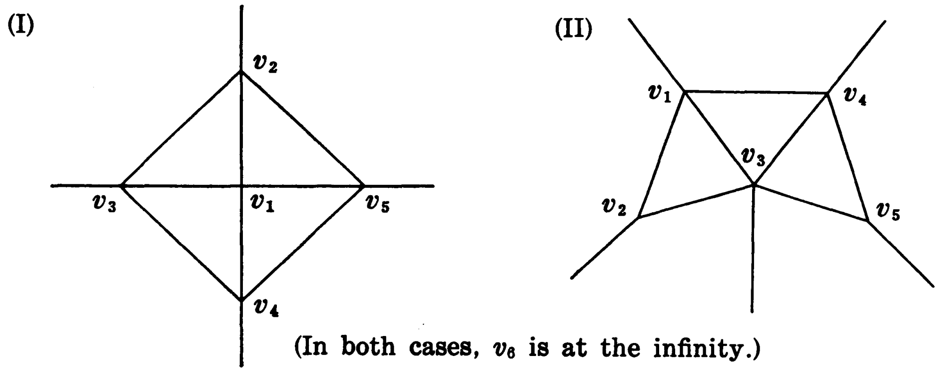
(3)
$$N=\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}.$$

In this case, $V\cong\boldsymbol{P}^2\times\boldsymbol{P}^1$. (This is the case (b) of the Theorem.)

(4)
$$N=\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 \end{bmatrix}.$$

In this case, $V\cong\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(2))$. (This is the case (c) of the Theorem.)

(3.4) $d=6$. There are two non-isomorphic triangulations of S as indicated by following figures.



(In both cases, v_6 is at the infinity.)

By the same argument as in (3.3), we see that Δ is isomorphic to one of the followings. (As in (3.3), the matrix N is $({}^t n_1, \cdots, {}^t n_6)$.)

Case (II). (1)

$$N=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}.$$

The divisor D_1 can be blown down to a line and we get $P(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(2))$. Also, if we contract the divisor D_4 to a line, we get $P(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$. This is F_1^3 (case (k)) of the Theorem.

(2)

$$N=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}.$$

We can contract D_5 (resp. D_1, D_4) to a point (resp. to a line) and get $\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$ (resp. $\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$, $\boldsymbol{P}^2\times\boldsymbol{P}^1$). This is the variety F_2^3 (case (l)) of the Theorem.

Case (I). (3)

$$N=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}.$$

We can contract D_1 (resp. D_3) to a line and get $\boldsymbol{P}(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$ (resp. $\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$). This is the variety $\boldsymbol{P}(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$ (case (j)) of the Theorem.

(4)

$$N=\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}.$$

We can contract D_1 or D_6 to a line and get $\boldsymbol{P}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$ in both cases. This variety is isomorphic to the $\boldsymbol{P}^1$-bundle $\boldsymbol{P}(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,-1))$ over $\boldsymbol{P}^1\times\boldsymbol{P}^1$ (case (i) of the Theorem).

(5)

$$N=\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}.$$

This variety is isomorphic to the P^1-bundle $P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,1))$ over $P^1\times P^1$. (This is the case (h) of the Theorem.) In this case, we can contract D_1 in two different (but isomorphic) ways getting a variety which is not a Fano 3-fold.

(6)

$$N=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0 & -1 & 0\end{bmatrix}.$$

This variety is isomorphic to $P^1\times P^1\times P^1$ (case (f) of the Theorem).

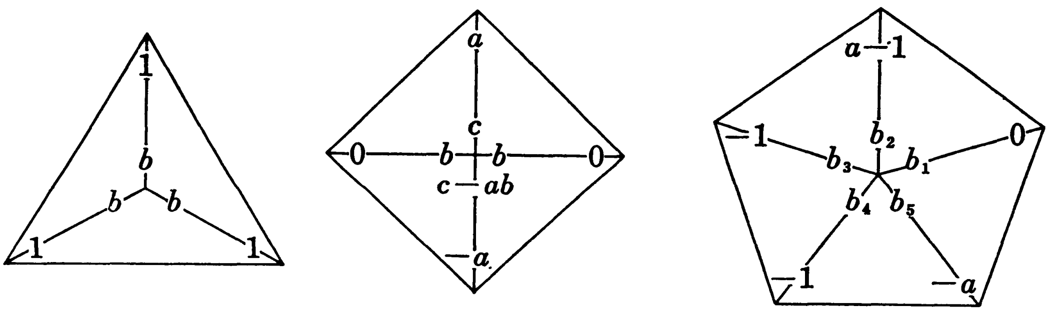
(7)

$$N=\begin{bmatrix}1 & 0 & 0 & 1 & 0 & -1\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0 & -1 & 0\end{bmatrix}.$$

This variety is isomorphic to $DS_8\times P^1$ (case (g) of the Theorem). We can contract D_1 to a line and get $P^2\times P^1$.

(3.5) For the case of $d=7, 8$, we will carry out the classification using the language of the double Z-weightings of S.

If $d=7$ $(\rho=4)$, we have $\sum_L \delta_L=9$. If v is a vertex of Δ, as $\delta_v\leqq 9$, the weighted link of v is one of the followings.

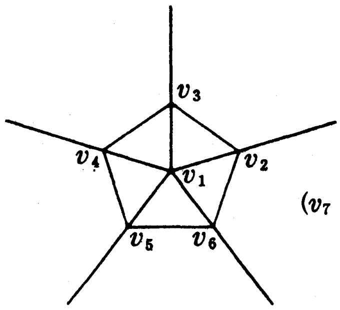


$-2\leqq b\leqq 1$

$$\begin{cases}2b+2c-ab\leqq 5,\\ b\geqq -1,\\ a+c\geqq -1,\\ c-ab-a\geqq -1.\end{cases}$$

$$\begin{cases}b_1=b_3+b_4,\\ -b_1+(a-1)b_3=-ab_4-b_5,\\ b_1\geqq -1,\ b_2+a-1\geqq -1,\\ b_3\geqq 0,\ b_4\geqq 0,\\ b_5-a\geqq -1,\ \sum_{i=1}^5 b_i\leqq 7.\end{cases}$$

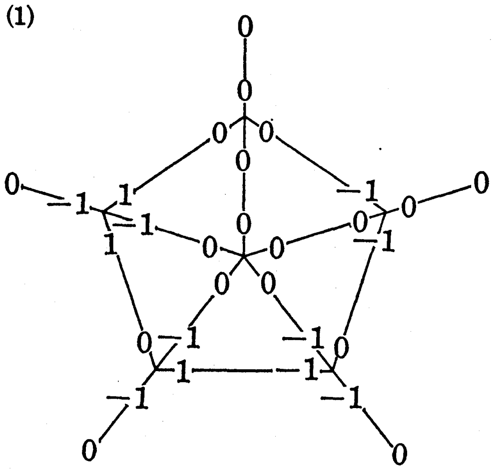
There are 5 non-isomorphic triangulations of the sphere for $d=7$ (cf. [3], p. 77). Among these triangulations, only the following one can have admissible double Z-weightings which satisfy the conditions above.



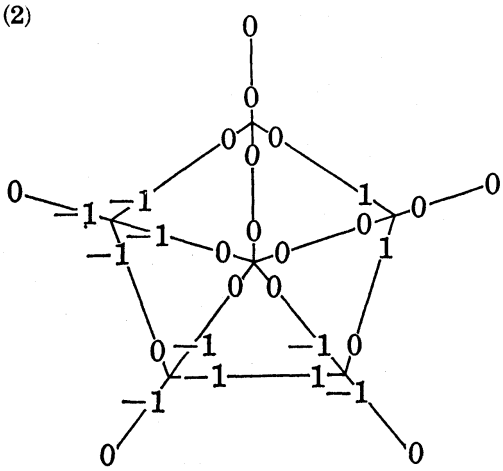
(v_7 is at the infinity.)

There are four non-isomorphic double $\boldsymbol{Z}$-weighting satisfying the above conditions and it can be proved that the corresponding varieties are actually Fano 3-folds using (2.5). The followings are the list of double $\boldsymbol{Z}$-weightings.

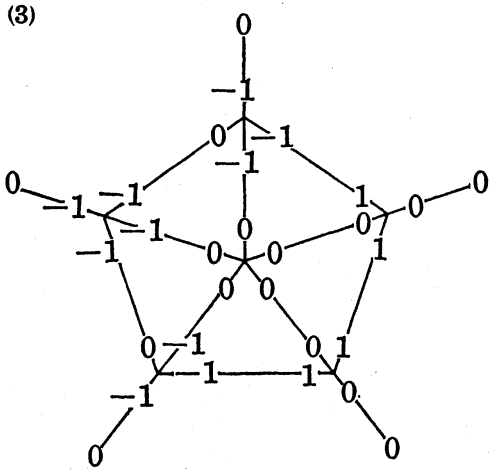
(1)



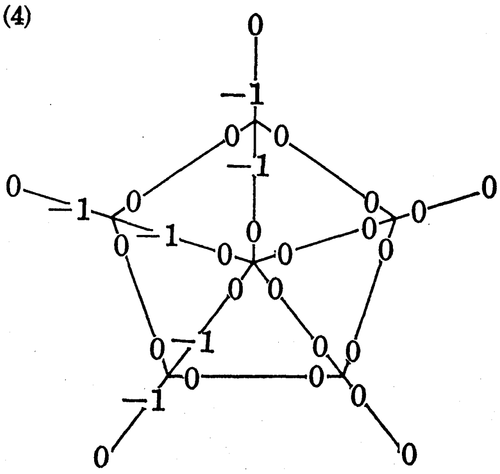
(2)



(3)



(4)



The corresponding Z^3-weightings are as follows;

(1)

$$N=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & -1 & -1 & 0 & 1\\ 0 & 0 & 1 & 0 & -1 & -1 & 0\end{bmatrix}.$$

The divisor D_2, D_4, D_5 and D_6 are contractible to a line and we get F_2^3, $P(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$, $P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1, -1))$ and $DS_8\times P^1$ respectively after contracting. This variety is F_1^4 (case (n)) of the Theorem.

(2)

$$N=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & -1 & -1 & 0 & -1\\ 0 & 0 & 1 & 0 & -1 & -1 & 0\end{bmatrix}.$$

We can contract D_5 (resp. D_6) to a line and get $DS_8\times P^1$ (resp. $P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1, 1)))$. We can contract D_4 in two different ways. But the both resulting varieties are not Fano 3-folds. This variety is F_2^4 (case (o)) of the Theorem.

(3)

$$N=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & -1 & -1 & 0 & -1\\ 0 & 0 & 1 & 1 & 0 & -1 & 1\end{bmatrix}.$$

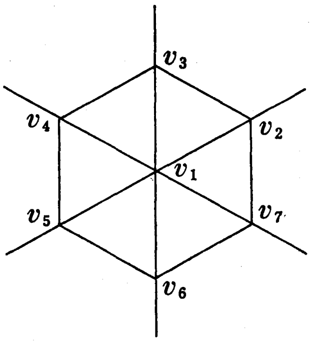
We can contract D_6 to a line and get $P(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$. The divisors D_3 D_4 are also contractible to a line. But the resulting varieties are not Fano 3-folds. This variety is F_3^4 (case (p)) of the Theorem.

(4)

$$N=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & -1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1 & 0 & -1 & 0\end{bmatrix}.$$

This variety is isomorphic to $DS_7\times P^1$ (case (m) of the theorem). We can contract D_3 (resp. D_4) and get $DS_8\times P^1$ (resp. $P^1\times P^1\times P^1$). The divisor D_5 is contractible to a line. But the resulting manifold is not a Fano 3-fold.

(3.6) For $d=8$, classifying by double Z-weightings as in (3.5), we can see that the possibility of Δ is restricted to one of the following two types both of which have the same triangulation as follows.



(v_8 is at infinity.)

(1)

$$N=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & -1 & -1 \end{bmatrix}.$$

We can contract D_2 (resp. D_3, D_4, D_5, D_7) and get F_2^4 (resp. F_3^4, F_2^4, F_1^4, F_1^4). The divisor D_6 is contractible to a line in two ways. But the resulting varieties are not Fano 3-folds. This variety is F_1^5 (case (r)) of the Theorem.

(2)

$$N=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & -1 & 0 \end{bmatrix}.$$

This variety is isomorphic to $DS_6 \times P^1$ (case (q) of the Theorem). We can contract D_2, D_3, D_4, D_5, D_6 or D_7 to a line and get $DS_7 \times P^1$.

This concludes the classification of the toric Fano 3-folds.

COROLLARY (3.7). *Indices and degrees of the above Fano 3-folds are as follows.* (For the definition of indices and degrees of Fano 3-folds, see Iskovskih [1].)

	P^3	$P^2\times P^1$	$P(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(2))$	$P(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(1))$
ρ	1	2	2	2
index	4	1	1	1
degree	1	54	62	54

$P(\mathcal{O}_{P^2}\oplus\mathcal{O}_{P^2}(1))$	$P^1\times P^1\times P^1$	$DS_8\times P^1$	$P(\mathcal{O}_{DS_8}\oplus\mathcal{O}_{DS_8}(\ell))$	$P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,-1))$
2	3	3	3	3
2	2	1	1	1
7	6	48	50	44

$P(\mathcal{O}_{P^1\times P^1}\oplus\mathcal{O}_{P^1\times P^1}(1,1))$	F_1^3	F_2^3	$DS_7\times P^1$	F_1^4	F_2^4	F_3^4	$DS_6\times P^1$	F_1^5
3	3	3	4	4	4	4	5	5
1	1	1	1	1	1	1	1	1
52	50	46	42	40	44	46	36	36

COROLLARY (3.8). *The manifolds appearing in the above classification are not isomorphic as abstract varieties to each other.*

ADDED IN PROOF. After the submission of our paper, V. V. Batirev has obtained the same result as ours which appeared in Izvestia Akademii Nauk, S.S.S.R., Vol. 45, no. 4, 1981.

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Present Address:
DEPARTMENT OF MATHEMATICS
NAGOYA INSTITUTE OF TECHNOLOGY
GOKISO-CHO, SHOWA-KU, NAGOYA 466
and
DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCES AND ENGINEERINGS
WASEDA UNIVERSITY
OKUBO, SHINJUKU-KU, TOKYO 160