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A Residue Formula for Chern Classes Associated with Logarithmic Connections

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Introduction

There exist various residue theorems in complex analysis whose archetype is the residue theorem for meromorphic 1-forms on a compact Riemann surface. In this case a 1-form can be considered as a connection form of the trivial line bundle or of any holomorphic line bundle with flat representatives. Thus the fact that the sum of the residues of a meromorphic 1-form is zero means that the sum of the residues of a meromorphic connection of a holomorphic line bundle is equal to the Chern class (or number) of the bundle on a compact Riemann surface.

Now let us generalize the situation to higher dimensional cases. Let E be a holomorphic vector bundle over a compact complex manifold M, and let D be a meromorphic connection of E. Then we hope that there may exist some relations between the residues of D and the Chern classes of E. In this paper it is shown that if D is logarithmic and if the pole Z of D satisfies certain conditions, then there are relations among the residues of D, the pole Z and the Chern classes of E. Our theorem states:

If the pole Z of a logarithmic connection D is normally crossing, and if each irreducible component of Z is smooth and the intersection of any finite number of irreducible components of Z is connected, then the following relation holds in the cohomology group $H^k(M, \Omega^k)$

$$c_k(E)=(-1)^k \sum_{j_1,\cdots,j_k} c_k(\mathrm{Res}_{Z_{j_1}}D, \mathrm{Res}_{Z_{j_2}}D, \cdots, \mathrm{Res}_{Z_{j_k}}D) \prod_{t=1}^{k} c_1([Z_{j_t}]) ,$$

where $c_k(E)$ is the k-th Chern class of E, $\mathrm{Res}_{Z_j} D$ is the residue of D along the irreducible component Z_j of Z, $c_k(A_1, \cdots, A_k)$ is the completely polarized form of the k-th Chern polynomial $c_k(A)$, $c_1([W])$ is the Chern

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class of the line bundle associated with a divisor W, and the summation is taken over all k-uple ordered sets of irreducible components of Z.

In §2 we will state precisely the theorem; its proof will be given in §3.

§1. Logarithmic connections and its residues.

Let M be a compact complex manifold and $\pi: E \to M$ be a holomorphic vector bundle over M of rank q. For an analytic subset Z of pure codimension 1, we denote by $\Omega_M^p(\log Z)$ the sheaf of germs of meromorphic p-forms on M with a simple logarithmic pole along Z.

DEFINITION. A meromorphic connection D of E with a simple logarithmic pole along Z is a C-linear map

$$D: \mathcal{O}(E) \longrightarrow \Omega_M^1(\log Z) \underset{\mathcal{O}_M}{\otimes} \mathcal{O}(E)$$

which satisfies

$$D(f.s) = df \otimes s + f.D(s)$$

for any local sections f of $\mathcal{O}_M$ and s of $\mathcal{O}(E)$.

Such a connection D is called a logarithmic connection of E, and Z is called the pole of D.

In this paper we assume that Z satisfies the following condition:

(H.1) Z is normally crossing.

(H.2) Let $Z = \bigcup_{j \in N} Z_j$ be the decomposition of Z into irreducible components (N is a set of indices). Then each component Z_j is smooth.

Choose a sufficiently fine coordinate covering $\{U_\lambda\}$ of M such that:

(i) the restriction of E onto U_λ is trivial, and

(ii) for each Z_j, a defining function $f_{\lambda j}$ is chosen in U_λ and it is a coordinate function of the local coordinate system in U_λ. (This is possible by the above assumptions. If $Z_j \cap U_\lambda$ is void, we set $f_{\lambda j} = 1$.)

Let $e_\lambda = {}^t(e_{\lambda 1}, e_{\lambda 2}, \cdots, e_{\lambda q})$ be a holomorphic frame of E on U_λ. The connection matrix D_λ of D with respect to the frame e_λ is defined by

$$D(e_\lambda) = D_\lambda \otimes e_\lambda .$$

Then D_λ is a matrix whose elements are sections of $\Omega_M^1(\log Z)$.

Let $e_{\lambda\mu}$ be the transition functions of the frames e_λ. On $U_\lambda \cap U_\mu$, $e_{\lambda\mu}$ and D_λ satisfy the following:

$$e_\lambda = e_{\lambda\mu} e_\mu ,$$

(1) $$D_\lambda e_{\lambda\mu} = de_{\lambda\mu} + e_{\lambda\mu} D_\mu \,.$$

Under the assumptions (H.1) and (H.2), the matrix D_λ is written, for each Z_j, by

(2) $$D_\lambda = A_{\lambda j} \frac{df_{\lambda j}}{f_{\lambda j}} + B_{\lambda j} \,,$$

where $A_{\lambda j}$ is a $q \times q$ matrix of holomorphic functions on U_λ and $B_{\lambda j}$ is a $q \times q$ matrix of logarithmic 1-forms on U_λ with a simple pole along $\bigcup_{l \neq j} Z_l$.

Then the residue of D_λ along Z_j is defined by

(3) $$\mathrm{Res}_{Z_j} D_\lambda = A_{\lambda j}|_{Z_j} \,.$$

This is a matrix-valued holomorphic function on $Z_j \cap U_\lambda$ and does not depend on the choice of the local equation $f_{\lambda j}$ and the above representation of D_λ. The following lemma is an easy consequence of (1) and (2).

LEMMA 1. *On $U_\lambda \cap U_\mu \cap Z_j$, we have*

$$\mathrm{Res}_{Z_j} D_\lambda e_{\lambda\mu}|_{Z_j} = e_{\lambda\mu}|_{Z_j} \mathrm{Res}_{Z_j} D_\mu \,.$$

$\{\mathrm{Res}_{Z_j} D_\lambda\}$ defines an element of of $H^0(Z_j, \mathcal{O}(\mathrm{End}\, E))$, where $\mathrm{End}\, E$ is the endomorphism bundle of E. We denote it by $\mathrm{Res}_{Z_j} D$, or simply by $\mathrm{Res}_j D$, and call it the residue of the connection D along Z_j.

§2. Statement of the theorem.

Let $J = (j_1, j_2, \cdots, j_k)$ be an element of $N^k = N \times N \times \cdots \times N$ (k times). If among $j_1, j_2, \cdots, j_k$ there exist p different indices, say $j_1^*, j_2^*, \cdots, j_p^*$, put $J^* = \{j_1^*, j_2^*, \cdots, j_p^*\}$ and let a_m be the number of j_m^* appearing in J $(1 \leq m \leq p)$:

$$\sum_{m=1}^{p} a_m = k \,.$$

For each $J \in N^k$, define Z_{J^*} as $Z_{J^*} = \bigcap_{m=1}^{p} Z_{j_m^*}$. This is a submanifold of M of codimension p or empty because of the assumptions (H.1) and (H.2). Let $Z_{J^*} = \bigcup_i Z_{J^*}^{(i)}$ be the decomposition of Z_{J^*} into connected components (Z_{J^*} being not necessarily connected).

Recall the definition of Chern polynomials $c_k(A)$. This is, by definition, the coefficient of t^k of the following polynomial:

$$\det (I + At) = \sum_{k=0}^{q} c_k(A) t^k \,,$$

where I is the unit matrix of size q, A is a matrix of size q, and t is an indeterminate. $c_k(A)$ is a homogeneous polynomial of the components of A of degree k. Moreover it is invariant:

$$c_k(BAB^{-1})=c_k(A)$$

for any invertible matrix B.

Let $c_k(A_1, A_2, \cdots, A_k)$ be the completely polarized form of $c_k(A)$. This is uniquely determined by the following properties:

(i) $c_k(A_1, \cdots, A_k)$ is multilinear,

(ii) symmetric in A_j, and

(iii) normalized, i.e., $c_k(A, \cdots, A)=c_k(A)$.

Then $c_k(A_1, \cdots, A_k)$ is also invariant:

$$c_k(BA_1B^{-1}, BA_2B^{-1}, \cdots, BA_kB^{-1})=c_k(A_1, A_2, \cdots, A_k)$$

for any invertible B (see Chern [3]).

Now the following proposition is an easy consequence of Lemma 1 and the compactness of M:

PROPOSITION 2. *For any element* $J=(j_1, \cdots, j_k)\in N^k$,

$$c_k(\mathrm{Res}_{j_1} D, \mathrm{Res}_{j_2} D, \cdots, \mathrm{Res}_{j_k} D)$$

is constant on each component $Z_{J^*}^{(i)}$.

We denote this value by $c_k(\mathrm{Res}_J D)^{(i)}$.

Let W be a submanifold of M of codimension p. For a C^∞-differential form φ of type $(n-p, n-p)$ on M ($n=\dim M$), the functional

$$W[\varphi]=\int_W \varphi$$

is a $\bar\partial$-closed current on M of type (p, p). So it determines a cohomology class in $H^p(M, \Omega^p)$ by Dolbeault's theorem. We denote this class by $c_p(W)$.

Now our main theorem is:

THEOREM 3. *Let E be a holomorphic vector bundle over a compact complex manifold M and let D be a logarithmic connection of E with the pole Z. Moreover Z satisfies the two conditions* (H.1) *and* (H.2). *Then the following relation holds in the cohomology group* $H^k(M, \Omega^k)$

$$c_k(E)=(-1)^k \sum_{J\in N^k} \sum_i \{c_k(\mathrm{Res}_J D)^{(i)} c_p(Z_{J^*}^{(i)})\} \prod_{m=1}^{p} c_1([Z_{j^*_m}])^{a_m-1} ,$$

where $c_k(E)$ *is the k-th Chern class of* E.

REMARKS. (1) Let K be the curvature of a C^∞-Hermitian metric of E. $c_k(E)$ is, by definition, the class in $H^k(M, \Omega^k)$ represented by the (k, k)-form $c_k((-1/2\pi i)K)$. (We adopt the definition from Bott-Chern [2]).

(2) In general, $c_k(\mathrm{Res}_J\, D)$ takes different values on various components $Z_{J^*}^{(i)}$. If Z_{J^*} is connected for each $J \in N^k$, then we have $c_p(Z_{J^*}) = c_1(Z_{j_1^*}) \cdots c_1(Z_{j_p^*})$ and we have the formula stated in the introduction.

§3. Proof of the Theorem 3.

3.1. An Hermitian metric of the bundle E.

Let $H: E \oplus E \to C$ be a C^∞-Hermitian metric of E where H is C-linear on the first factor and anti-linear on the second factor. On a coordinate neighborhood U_λ, H is represented by

$$H_\lambda = (H_{\lambda i \bar{j}}) , \quad H_{\lambda i \bar{j}} = H(e_{\lambda i}, e_{\lambda j}) .$$

Let $\Gamma_\lambda = \partial H_\lambda H_\lambda^{-1}$ be the connection matrix of the metric H and $K_\lambda = d\Gamma_\lambda - \Gamma_\lambda \wedge \Gamma_\lambda = \bar{\partial}\Gamma_\lambda$ be its curvature matrix with respect to the frame e_λ.

Now $K = \{K_\lambda\}$ is the C^∞-curvature of E and $c_k((-1/2\pi i)K)$ is a $\bar{\partial}$-closed (k, k)-form on M.

3.2. Metric of the bundle $[Z_j]$.

Let $[Z_j]$ be the line bundle associated with the divisor Z_j. Put $f_{\lambda\mu j} = f_{\mu j}/f_{\lambda j}$ on $U_\lambda \cap U_\mu$. Then a metric of the bundle $[Z_j]$ is a set of positive C^∞-functions $h_{\lambda j}$ on U_λ such that

$$h_{\lambda j} = |f_{\lambda\mu j}|^2 h_{\mu j} \quad \text{on} \quad U_\lambda \cap U_\mu .$$

Put $\omega_{\lambda j} = \partial h_{\lambda j} h_{\lambda j}^{-1} = \partial \log (h_{\lambda j})$ and $\theta_{\lambda j} = \bar{\partial}\omega_{\lambda j} = \bar{\partial}\partial \log (h_{\lambda j})$ on U_λ. Then on $U_\lambda \cap U_\mu$, $\theta_{\lambda j} = \theta_{\mu j}$ so that $\{\theta_{\lambda j}\}$ determines a global $\bar{\partial}$-closed (1, 1)-form θ_j, which is the curvature form of $[Z_j]$.

LEMMA 4. *The form* $(-1/2\pi i)\theta_j$ *is* $\bar{\partial}$*-cohomologous to the current* Z_j *defined in* §2.

This lemma is well-known and the proof is omitted.

3.3. Now we prove the theorem 3. The left hand side of the formula in the theorem is represented by the differential form $c_k((-1/2\pi i)K) = (-1/2\pi i)^k c_k(K, K, \cdots, K)$, and the right hand side is represented by the current

$$(-1)^k \sum_{J\in N^k} \left\{\sum_t c_k(\mathrm{Res}_J D)^{(t)}\left(\prod_{m=1}^{p}\left(\frac{-1}{2\pi i}\theta_{j_m^*}\right)^{a_m-1}\right)Z_{J^*}^{(t)}\right\} .$$

This follows from Lemma 4. So it is sufficient to prove the above two currents are mutually $\bar{\partial}$-cohomologous.

We fix a C^∞-metric of M and for a subset X of M, we denote the ε-neighborbood of X by X^ε.

LEMMA 5. *Put* $L=D-\Gamma$. *It is an* End E-*valued* 1-*form on* M, *having a pole along* Z. L *satisfies*:

(i) $\bar{\partial}L=-K$ *on* $M-Z$,

(ii) $\mathrm{Res}_{Z_j} L=\mathrm{Res}_{Z_j} D$ *for any component* Z_j.

(We defined residues only for logarithmic forms but the component of the matrix L is a sum of a logarithmic form and is a C^∞-form. So the residue of such a form is also well-defined.)

The proof of the above lemma is easy and omitted.

Now let φ of a C^∞-form on M of type $(n-k, n-k)$. Then

$$\begin{aligned}
&\left(\frac{-1}{2\pi i}\right)^k c_k(K)[\varphi] \\
&=\left(\frac{-1}{2\pi i}\right)^k \int_M c_k(K, K, \cdots, K)\wedge\varphi \\
&=\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} c_k(K, \cdots, K)\wedge\varphi \\
&=\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} c_k(-\bar{\partial}L, K, \cdots, K)\wedge\varphi \\
&=-\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} \bar{\partial}(c_k(L, K, \cdots, K))\wedge\varphi \\
&=-\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} \bar{\partial}(c_k(L, K, \cdots, K)\wedge\varphi) \\
&\quad-\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} c_k(L, K, \cdots, K)\wedge\bar{\partial}\varphi \\
&=-\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} d(c_k(L, K, \cdots, K)\wedge\varphi)-T[\bar{\partial}\varphi] ,
\end{aligned}$$

where $T[\psi]=(-1/2\pi i)^k \lim_{\varepsilon\to 0}\int_{M-Z^\varepsilon} c_k(L, K, \cdots, K)\wedge\psi$ is a well-defined current of type $(k, k-1)$ on M called the principal value of Cauchy. Then, by Stokes' theorem,

$$\left(\frac{-1}{2\pi i}\right)^k c_k(K)[\varphi]$$
$$=\left(\frac{-1}{2\pi i}\right)^k \lim_{\varepsilon\to 0}\int_{\partial Z^\varepsilon} c_k(L, K, \cdots, K)\wedge\varphi - T[\bar\partial\varphi]$$
$$=\left(\frac{-1}{2\pi i}\right)^k (2\pi i)\sum_{j\in N}\int_{Z_j}\mathrm{Res}_{Z_j}(c_k(L, K, \cdots, K)\wedge\varphi) - T[\bar\partial\varphi]$$
$$=\frac{(-1)^k}{(2\pi i)^{k-1}}\sum_{j\in N}\int_{Z_j} c_k(\mathrm{Res}_j D, K, \cdots, K)\wedge\varphi - \bar\partial T[\varphi] .$$

Therefore the current $(-1/2\pi i)^k c_k(K)$ is $\bar\partial$-cohomologous to

$$\frac{(-1)^k}{(2\pi i)^{k-1}}\sum_{j\in N} Z_j\wedge c_k(\mathrm{Res}_j D, K, \cdots, K) .$$

Next, in order to modify the last integral, we prepare two lemmas. Fix an element $J=(j_1, j_2, \cdots, j_k)\in N^k$. On U_λ, the connection matrix D_λ is written by

$$D_\lambda=\sum_{t\in J^*} A_{\lambda t}\frac{df_{\lambda t}}{f_{\lambda t}}+B_\lambda ,$$

where B_λ is a matrix of logarithmic 1-forms on U_λ having a pole along Z_s, $s\notin J^*$. Let $B_{\lambda J}=B_\lambda|_{Z_{J^*}}$ and let

$$\tilde B_{\lambda J}=B_{\lambda J}-\sum_{j\in J^*}\mathrm{Res}_j D_\lambda\omega_{\lambda j}|_{Z_{J^*}} ,$$

where ω_j is the previously chosen metric connection of $[Z_j]$.

LEMMA 6. *$\tilde B_J=\{\tilde B_{\lambda J}\}$ is a connection of $E|_{Z_{J^*}}$, having a pole along $Z_s\cap Z_{J^*}$, $s\notin J^*$.*

The proof of this lemma is done by an easy calculation, so it is omitted.

LEMMA 7. *Put $L_J=\tilde B_J-\Gamma|_{Z_{J^*}}$, where Γ is a metric connection of E. Then L_J is an* End *E-valued 1-form on Z_{J^*}, satisfying*

(i) $\bar\partial L_J=-\sum_{j\in J^*}\mathrm{Res}_j D\theta_j-K$ *on* $Z_{J^*}-\bigcup_{s\in J^*}Z_s$,

(ii) $\mathrm{Res}_{Z_s} L_J=\mathrm{Res}_{Z_s} D$ *for* $s\notin J^*$.

The proof is analogous to the proof of Lemma 5 and is omitted.

In the following, we write R_j for $\mathrm{Res}_j D$. For $J=(j_1, j_2, \cdots, j_l)\in N^l$, $J^*=\{j_1^*, \cdots, j_p^*\}$, $(l\leqq k)$, we consider the integral

$$I(\varphi)=\frac{(-1)^{l-p}}{(2\pi i)^{k-p}}\int_{Z_{J^*}} c_k(R_{j_1}, \cdots, R_{j_l}, K, \cdots, K)(\theta_{j_1^*})^{a_1-1}\cdots(\theta_{j_p^*})^{a_p-1}\varphi$$

$$=\frac{(-1)^{l-p}}{(2\pi i)^{k-p}}\lim_{\varepsilon\to 0}\int_{Z_{J^*}-X^\varepsilon} c_k(R_{j_1}, \cdots, R_{j_l}, K, \cdots, K)P_J(\theta)\varphi ,$$

where $X=(\bigcup_{s\notin J^*} Z)\cap Z_{J^*}$, and we denote the product $(\theta_{j_1^*})^{a_1-1}\cdots(\theta_{j_p^*})^{a_p-1}$ by $P_J(\theta)$. We get, by Lemma 7,

$$\bar\partial L_J=-\sum_{j\in J^*} R_j\theta_j-K \quad\text{on}\quad Z_{J^*}-\bigcup_{s\notin J^*} Z_s .$$

Substituting this, we have

$$\begin{aligned}
I(\varphi)=&\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\lim_{\varepsilon\to 0}\int_{Z_{J^*}-X^\varepsilon} c_k(R_{j_1}, \cdots, R_{j_l}, \bar\partial L_J, K, \cdots, K)P_J(\theta)\varphi\\
&+\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\lim_{\varepsilon\to 0}\sum_{j\in J^*}\int_{Z_{J^*}-X^\varepsilon} c_k(R_{j_1}, \cdots, R_{j_\lambda}, R_j\theta_j, K, \cdots, K)P_J(\theta)\varphi\\
=&\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\lim_{\varepsilon\to 0}\int_{Z_{J^*}-X^\varepsilon} \bar\partial(c_k(R_{j_1}, \cdots, R_{j_l}, L_J, K, \cdots, K)P_J(\theta)\varphi)\\
&+\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\lim_{\varepsilon\to 0}\int_{Z_{J^*}-X^\varepsilon} c_k(R_{j_1}, \cdots, R_{j_l}, L_J, K, \cdots, K)P_J(\theta)\bar\partial\varphi\\
&+\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\sum_{j\in J^*}\int_{Z_{J^*}} c_k(R_{j_1}, \cdots, R_{j_l}, R_j, K, \cdots, K)\theta_j P_J(\theta)\varphi\\
=&\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}(-2\pi i)\sum_{s\notin J^*}\int_{Z_s\cap Z_{J^*}} c_k(R_{j_1}, \cdots, R_{j_l}, \mathrm{Res}_s D, K, \cdots, K)P_J(\theta)\varphi\\
&+T[\bar\partial\varphi]\\
&+\frac{(-1)^{l-p+1}}{(2\pi i)^{k-p}}\sum_{j\in J^*}\int_{Z_{J^*}} c_k(R_{j_1}, \cdots, R_{j_l}, R_j, K, \cdots, K)\theta_j P_J(\theta)\varphi ,
\end{aligned}$$

where $T[\psi]=(-1)^{l-p+1}/(2\pi i)^{k-p}\lim_{\varepsilon\to 0}\int_{Z_{J^*}-X^\varepsilon} c_k(R_{j_1},\cdots,R_{j_l},L_J, K,\cdots,K)P_J(\theta)\psi$ is a principal value current on M. So

$$\begin{aligned}
I(\varphi)=&\frac{(-1)^{(l+1)-(p+1)}}{(2\pi i)^{k-(p+1)}}\sum_{J'\in N^{l+1}}\int_{Z_{J'^*}} c_k(R_{j_1}, \cdots, R_{j_l}, R_s, K, \cdots, K)P_{J'}(\theta)\varphi\\
&+\frac{(-1)^{(l+1)-p}}{(2\pi i)^{k-p}}\sum_{J''\in N^{l+1}}\int_{Z_{J''^*}} c_k(R_{j_1}, \cdots, R_{j_l}, R_j, K, \cdots, K)P_{J''}(\theta)\varphi+\bar\partial T[\varphi] ,
\end{aligned}$$

where $J'=(j_1, \cdots, j_l, s)=(J, s)$, $s\notin J^*$ and $J''=(j_1, \cdots, j_l, j)=(J, j)$, $j\in J^*$. This shows that we have at last

$$\begin{aligned}
&\frac{1}{(2\pi i)^k}\int_M c_k(K, \cdots, K)\varphi\\
&\quad=\sum_{J\in N^k}\frac{(-1)^{k-p}}{(2\pi i)^{k-p}}\int_{Z_{J^*}} c_k(R_{j_1}, \cdots, R_{j_k})P_J(\theta)\varphi+\bar\partial T[\varphi]
\end{aligned}$$

for some $(k, k-1)$-current T on M.

Now

$$\left(\frac{-1}{2\pi i}\right)^{k-p} P_J(\theta) = \left(\frac{-1}{2\pi i}\right)^{k-p} (\theta_{j_1^*})^{a_1-1} \cdots (\theta_{j_p^*})^{a_p-1}$$
$$= \left(\frac{-1}{2\pi i}\theta_{j_1^*}\right)^{a_1-1} \cdots \left(\frac{-1}{2\pi i}\theta_{j_p^*}\right)^{a_p-1}$$

and $c_k(R_{j_1}, \cdots, R_{j_k}) = c_k(\mathrm{Res}_J D)$ is constant on $Z_{J^*}^{(i)}$. This proves the theorem completely.

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