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# Remark on "On Normal Integral Bases"

Fuminori KAWAMOTO

*Gakushuin University*
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We can easily extend Theorem 2 of [1] to the following theorem. All results in [1] which follow from Theorem 2 are extended consequently. We will use the same notations as in [1].

THEOREM. *Suppose that $l$ is an odd prime and $a$ $(\neq \pm 1)$ is a rational integer without $l$-th power factor such that $a^{l-1} \equiv 1 \bmod l^2$. Then $\boldsymbol{Q}(\zeta_l, \sqrt[l]{a})/\boldsymbol{Q}(\zeta_l)$ has always a normal integral basis.*

PROOF. Let $g$ be a primitive root $\bmod l$ $(2 \leqq g \leqq l-1)$. Then, for any $j$ $(0 \leqq j \leqq l-1)$, there is some integer $e_j$ such that $g^{e_j} \equiv a^{l-1-j} b_j \bmod l$. Here we may put $e_0 = 0$, because of our hypothesis. Let $\mathfrak{p}$ be a unique prime ideal lying above $l$ in $\boldsymbol{Q}(\zeta)$ and $\varepsilon = (\zeta^g - 1)/(\zeta - 1)$, which is a unit of $\boldsymbol{Q}(\zeta)$. We put $u_j = (-1)^{l-1-j} \varepsilon^{-l e_j}$ $(0 \leqq j \leqq l-1)$. Since $\varepsilon \equiv g \bmod \mathfrak{p}$, we have $\varepsilon^l \equiv g^l \equiv g \bmod l$. Consequently, for any $i$ $(0 \leqq i \leqq l-1)$,

$$\sum_{j=0}^{l-1} \binom{l-1}{j} \zeta^{ij} u_j a^{l-1-j} b_j \equiv \sum_{j=0}^{l-1} \binom{l-1}{j} \zeta^{ij} (-1)^{l-1-j}$$
$$= (\zeta^i - 1)^{l-1} \equiv 0 \bmod l \, .$$

Hence, by Theorem 2 in [1], $(1/l) \sum_{j=0}^{l-1} \varepsilon^{l e_j} (\sqrt[l]{a}^{\,j}/b_j)$ is a generator of normal integral basis. This proves our theorem.

## Reference

[1] F. KAWAMOTO, On normal integral bases, Tokyo J. Math., **7** (1984), 221–231.

*Present Address:*
DEPARTMENT OF MATHEMATICS
GAKUSHUIN UNIVERSITY
MEJIRO, TOSHIMA-KU, TOKYO 171

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