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Values of the Unipotent Characters of the Chevalley Group of Type F_4 at Unipotent Elements

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§0. Introduction.

Let $\mathbf{G}$ be a connected reductive algebraic group with connected center and split over a finite field $\mathbf{F}_q$ of characteristic p, and F the corresponding Frobenius morphism. Then the subgroup $G=\mathbf{G}^F$ of $\mathbf{G}$ consisting of elements fixed by F is a finite Chevalley group. The table consisting of the values at unipotent elements of the unipotent characters of G which we shall simply call the unipotent character table of G, is an essential part of the character table of G.

If $\mathbf{G}$ has type F_4 and p is good for $\mathbf{G}$ (that is, $p\geq 5$), the problem of completing the unipotent character table of G is reduced to the determination of the values of one almost character; this was settled independently by N. Kawanaka [Ka2, Ka3] and G. Lusztig ([Lu3], cf. Remark 4.3 of this paper also). The purpose of this paper is to form the table in the cases $p=2$ or 3. And since the unipotent character table when $p\geq 5$ can be obtained at once from that when $p=3$ (cf. Remark 4.5), we could actually form this table as well.

Through the Fourier transform matrix introduced by G. Lusztig [Lu1], the determination of the values of irreducible characters of G is equivalent to the determination of the values of almost characters. And the almost characters are closely related to another set of class functions on G called the characteristic functions associated with character sheaves of G, also due to Lusztig [Lu2]. In fact, he conjectured that if p is almost good for $\mathbf{G}$ the characteristic functions coincide with the almost characters up to multiplication by a scalar. In [Sho4], T. Shoji proved Lusztig's conjecture in the case where $\mathbf{G}$ has connected center; moreover, he showed that for $\mathbf{G}$ of type F_4, Lusztig's conjecture also holds when $p=3$, and a weaker version of it holds when $p=2$ (cf. Theorem 1.7).

Now, $G=F_4(p^n)$ has 37 unipotent characters and using known results, we can compute the values at unipotent elements of 30 of the corresponding almost characters. Our problem therefore is to determine the values of the remaining 7 almost characters,

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all of which belong to a 21-element family. By Shoji's result (Theorem 1.7), 5 (resp., 3) of these 7 can be expressed as a scalar multiple of a characteristic function or a linear combination of 2 characteristic functions, and the rest are 0 at the unipotent elements. Since the values of the characteristic functions at unipotent elements can be determined* using the same algorithm Lusztig devised in [Lu2, V, §24], as proved by Shoji in [Sho4, I], the values of the undetermined almost characters can be expressed in terms of the scalars appearing in Theorem 1.7. And our work is done once we find the values of these scalars.

We can choose a parabolic subgroup P of G so that 5 of the 11 irreducible characters in 1_P^G will lie in the 21-element family of unipotent characters. We construct the Hecke algebra $\mathscr{H}(G, P)$ and form its character table. For a unipotent character χ belonging to 1_P^G and a unipotent element x of G, $\chi(x)$ can then be computed using the character formula Theorem 2.3, provided we know $|\mathscr{C} \cap D_i|$ for all $D_i \in P\backslash G/P$, where $\mathscr{C}$ is the unipotent conjugacy class containing x.

On the other hand, we can also compute $\chi(x)$ by transforming the 21-element family of almost characters to the corresponding unipotent characters using the appropriate Fourier transform matrix. We do this for $\chi \in 1_P^G$ and a unipotent element $x \in G$. Equating this with the value obtained above through Hecke algebras, and repeating this for the appropriate χ's and x's, we obtain equations involving the scalars in Theorem 1.7.

We actually succeed in finding the values of all the scalars we want to determine when $p=3$, and all except two (c and c_7) when $p=2$. In either case, we obtain the values of the yet undetermined almost characters, thus completing the unipotent character table of $G=F_4(p^n)$, $p=2$ or 3. And as remarked in (4.5), this will allow us to form the corresponding table for the case $p \geq 5$.

As a consequence, we improve Shoji's result vis-a-vis Lusztig's conjecture when $G=F_4(2^n)$—namely, that the conjecture holds for one more pair of characteristic function and almost character.

The rest of the paper goes as follows. In §1 we look into the unipotent characters and almost characters and how they are classified into families, particularly for the case $G=F_4(p^n)$. Here we state Shoji's result concerning Lusztig's conjecture relating almost characters with characteristic functions for $G=F_4(p^n)$.

In §2 we form the Hecke algebra $\mathscr{H}(G, P)$ for a certain maximal parabolic subgroup P of $G=F_4(p^n)$ and determine its character table. We state the character formula (Theorem 2.3) relating irreducible characters of G with those of $\mathscr{H}(G, P)$; we note that it involves a factor $|\mathscr{C} \cap D_i|$ where $\mathscr{C}$ is a unipotent class in G and D_i a double coset in $P\backslash G/P$. We compute this number in §3.

In §4, we determine the unknown almost character values of $G=F_4(p^n)$, $p=2$ or 3, leading us into the completion of the unipotent character table of $G=F_4(p^n)$, p

* Actually, when $p=2$, one characteristic function, χ_{A_4}, cannot be computed. But its restriction map χ'_{A_4} (see Theorem 1.7) can be determined and this would turn out to be sufficient for our purpose.

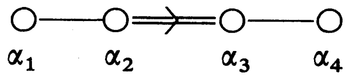
arbitrary. §5 lists our results given in table form. We also include, omitting the details, the unipotent character table of $Sp(8, 2^n)$ which can be formed from the Green functions [Y] and the generalized Green functions.

Throughout the paper, $\mathbf{G}$ will denote a connected reductive algebraic group with connected center and split over a finite field $\mathbf{F}_q$ of characteristic p, unless otherwise specified; F: $\mathbf{G} \to \mathbf{G}$ will be the corresponding Frobenius morphism. G will denote the subgroup $\mathbf{G}^F$ consisting of elements of $\mathbf{G}$ fixed by F. Similarly, for subgroups $\mathbf{T}$, $\mathbf{P}$, $\mathbf{B}$, $\mathbf{U}$, etc., of $\mathbf{G}$, the corresponding subgroups of G will be written T, P, B, U, etc.

Card $S = |S|$ denotes the cardinality of a set S. If H is a group and $S \subset H$, $N_H(S)$ (resp., $Z_H(S)$) or simply $N(S)$ (resp., $Z(S)$) is the normalizer (resp., centralizer) of S in H. If H is finite, Irr H denotes the irreducible characters of H over $\mathbf{C}$, $C(H/\sim)$ the space of class functions on H over $\mathbf{C}$, and $\langle \psi, \phi \rangle_H$ the inner product of $\psi, \phi \in C(H/\sim)$ given by $\langle \psi, \phi \rangle_H = \frac{1}{|H|} \sum_{h \in H} \psi(h)\overline{\phi(h)}$, sometimes simply written $\langle \psi, \phi \rangle$.

If H is an algebraic group, H° will denote the connected component of H containing the identity element and $A_H(u) = Z_H(u)/Z_H(u)^\circ$ for $u \in H_{\text{uni}}$, where H_{uni} stands for the set of unipotent elements in H.

When we take $G = F_4(p^n)$, we follow the notations in [Shi1] and [Sho1] for the unipotent conjugacy classes. The simple roots $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ correspond to the vertices of the Dynkin diagram of the Lie algebra of type F_4 as follows:



Finally, we mention that the main parts of this paper are contained in the Doctor of Science thesis of the first named author [Mar].

§1. Unipotent characters, almost characters, and characteristic functions.

Let $\mathbf{G}$ be as in the introduction and F the corresponding Frobenius map. Fix a prime $l \neq p$ and let $\bar{\mathbf{Q}}_l$ be the algebraic closure of the field of l-adic numbers. Then for an F-stable maximal torus $\mathbf{T}$ and $\theta \in \operatorname{Irr} T$, the generalized character $R_{\mathbf{T}}^{\mathbf{G}}(\theta)$ is defined by Deligne and Lusztig in [DL].

In particular, when θ is the trivial representation of T, we obtain the unipotent characters of G.

(1.1) Definition. An irreducible character χ of G is said to be unipotent if $\langle R_{\mathbf{T}}^{\mathbf{G}}(1), \chi \rangle \neq 0$.

In [Lu1], Lusztig showed that the unipotent characters can be classified into families and each family $\mathscr{F}$ can be parametrized in terms of a certain finite group Γ which must be one of

$$1\,,\quad (\mathbf{Z}/2\mathbf{Z})^n\,,\quad \mathscr{S}_3\,,\quad \mathscr{S}_4\,,\quad \text{and}\quad \mathscr{S}_5\,,$$

(where $\mathscr{S}_n$ denotes the symmetric group of degree n). Specifically,

$$\mathscr{F} \xleftrightarrow{1:1} M(\Gamma)=\{(x,\sigma): x\in\Gamma,\ \sigma\in\operatorname{Irr} Z_\Gamma(x)\}/\sim\ .$$

An operation $\{\,,\}$ is defined on $M(\Gamma)$ so that the $|\mathscr{F}|\times|\mathscr{F}|$-matrix with entries $\{(x,\sigma),(y,\tau)\}$ for $(x,\sigma),(y,\tau)\in M(\Gamma)$ is unitary and hermitian. Lusztig calls such a matrix a Fourier transform matrix. Now, for each $\chi_{(x,\sigma)}\in\mathscr{F}$ an almost character $R_{(x,\sigma)}$ is defined as follows:

$$R_{(x,\sigma)}=\sum_{(y,\tau)\in M(\Gamma)} \Delta(\chi_{(y,\tau)})\{(x,\sigma),(y,\tau)\}\,\chi_{(y,\tau)}$$

where $\Delta(\chi_{(y,\tau)})=\pm 1$ is the modification concerning exceptional characters of Weyl groups of type E_7 and E_8 (cf. [Lu1, 4.21]).

Extending this definition to all irreducible characters, we obtain an orthonormal basis for the space $C(G/\sim)$ of class functions on G. Since the transition matrix from this basis to that consisting of irreducible characters of G is known and is invertible, the problem of determining the values of the irreducible characters is reduced to determining the values of the almost characters.

To describe the situation for the case $G=F_4(p^n)$ more precisely, we recall the following definition:

(1.2) Definition. $\chi\in\operatorname{Irr} G$ is cuspidal if for every proper parabolic subgroup P of G, the truncation map $T_{P/U}(\chi)=0$ where U is the unipotent radical of P and

$$(T_{P/U}(\chi))(p)=\frac{1}{|U|}\sum_{u\in U}\chi(up)\qquad (p\in P)\,.$$

(1.3) Theorem. [Lu1, Theorems 4.23, 8.6] *$G=F_4(p^n)$ has 37 unipotent characters that can be classified as follows: (Here, B is a fixed Borel subgroup of G.)*

(1) *25 characters appearing in 1_B^G and denoted by $[\chi]$ where $\chi\in\operatorname{Irr} W$, W the Weyl group of G;*

(2) *5 characters in δ_P^G where δ is the unique unipotent cuspidal character of M with M Levi subgroup of a parabolic subgroup P of G of type B_2;*

(3) *7 cuspidal characters in G.*

For the 25 characters of $W=W(F_4)$, we follow the usual notation—that is, using Kondo's table in [Ko], the isolated characters of degree 4, 12 and 16 are denoted 4_1, 12_1, and 16_1, respectively, the 4 other characters of degree 4 are denoted 4_2, 4_3, 4_4, and 4_5 in this sequence; the other characters of the same degree d are denoted d_1, d_2, etc., as they appear in the table. On the other hand, the 5 characters in (2) are denoted $B_2[\chi]$ where $\chi\in\operatorname{Irr} N(M)/M$. Now $N(M)/M\cong W(B_2)\cong D_4$, the dihedral group of order

8 which can be naturally embedded in $\mathscr{S}_4$. Irr $W(B_2)=\{1, \varepsilon, \varepsilon', \varepsilon'', r\}$ where 1 and r are the trivial and the degree-2 characters, respectively, and for a transposition h_2 and a 4-cycle h_4 in $W(B_2)$, $\varepsilon'(h_2)=\varepsilon'(h_4)=-1$, $\varepsilon''(h_2)=1$, $\varepsilon''(h_4)=-1$, and $\varepsilon=\varepsilon'\cdot\varepsilon''$. We denote the 7 cuspidal unipotent characters in G by $F_4[\theta]$, $F_4[\theta^2]$, $F_4[i]$, $F_4[-i]$, $F_4^{\mathrm{I}}[1]$, $F_4^{\mathrm{II}}[1]$, and $F_4[-1]$ where θ (resp., i) is a fixed primitive cube (resp., fourth) root of 1 in $\mathbf{C}$.

(1.4) THEOREM. [Lu1, 4.10] *The unipotent characters of $G=F_4(p^n)$ belong to* 11 *families: eight* 1*-element families, two* 4*-element families, and one* 21*-element family.*

We list below the 37 unipotent characters opposite their labeling in the family they belong (for the nontrivial families) [loc. cit., p. 371]. We note in particular that the 7 cuspidal characters lie in the 21-element family. For the notations, see [loc. cit., pp. 79, 80].

1-element families		4-element families	
$[1_1]$	$[1_4]$	$[4_2]$, $[4_5]$	$\leftrightarrow$ $(1, 1)$
$[9_1]$	$[9_4]$	$[2_3]$, $[2_4]$	$\leftrightarrow$ $(1, \varepsilon)$
$[8_1]$	$[8_2]$	$[2_1]$, $[2_2]$	$\leftrightarrow$ $(g_2, 1)$
$[8_3]$	$[8_4]$	$B_2[1]$, $B_2[\varepsilon]$	$\leftrightarrow$ (g_2, ε)

21-element family

$[12_1]$	$\leftrightarrow$	$(1, 1)$	$B_2[\varepsilon'']$	$\leftrightarrow$	(g_2, ε')	$[6_1]$	$\leftrightarrow$	$(g_3, 1)$
$[9_3]$	$\leftrightarrow$	$(1, \lambda^1)$	$[4_4]$	$\leftrightarrow$	(g_2, ε'')	$F_4[\theta]$	$\leftrightarrow$	(g_3, θ)
$[1_3]$	$\leftrightarrow$	$(1, \lambda^2)$	$[9_2]$	$\leftrightarrow$	$(g_2', 1)$	$F_4[\theta^2]$	$\leftrightarrow$	(g_3, θ^2)
$F_4^{\mathrm{II}}[1]$	$\leftrightarrow$	$(1, \lambda^3)$	$F_4^{\mathrm{I}}[1]$	$\leftrightarrow$	(g_2', ε)	$[4_3]$	$\leftrightarrow$	$(g_4, 1)$
$[6_2]$	$\leftrightarrow$	$(1, \sigma)$	$[1_2]$	$\leftrightarrow$	(g_2', ε')	$B_2[\varepsilon']$	$\leftrightarrow$	$(g_4, -1)$
$[16_1]$	$\leftrightarrow$	$(g_2, 1)$	$[4_1]$	$\leftrightarrow$	(g_2', ε'')	$F_4[i]$	$\leftrightarrow$	(g_4, i)
$F_4[-1]$	$\leftrightarrow$	(g_2, ε)	$B_2[r]$	$\leftrightarrow$	(g_2', r)	$F_4[-i]$	$\leftrightarrow$	$(g_4, -i)$

Again, let us consider the general case for $\mathbf{G}$. In [Lu3], Lusztig defines character sheaves as certain complexes of perverse sheaves on $\mathbf{G}$. A character sheaf A is said to be F-stable if A is isomorphic to the inverse image F^*A. For each F-stable character sheaf A and an isomorphism $\varphi: F^*A \xrightarrow{\sim} A$, a function $\chi_{A,\varphi} \in C(G/\sim)$, called the characteristic function associated with A and φ, is defined. Now, Lusztig proves:

(1.5) THEOREM. [Lu2] *Suppose p is almost good for $\mathbf{G}$—that is, p is good for each factor of exceptional type.*

(i) *$\{\chi_{A,\varphi_A}$: A F-stable character sheaf on $\mathbf{G}$, $\varphi_A: F^*A \xrightarrow{\sim} A$ appropriately chosen$\}$ is an orthonormal basis for $C(G/\sim)$.*

(ii) *Each χ_{A,φ_A} is computable.*

In Lusztig's program of determining the values of all irreducible characters of G, it then remains to find the transition matrix from the characteristic functions to the irreducible characters, for the case p is almost good for $\mathbf{G}$. He was able to determine part of this matrix in [Lu4] (for certain groups, including $G=F_4(p^n)^*$ with $p\geq 5$), and this submatrix turned out to be sufficient to form the unipotent character table of G (cf. Remark 4.3).

Now Lusztig made the following conjecture:

(1.6) CONJECTURE. [Lusztig] If p is almost good for **G**, then the characteristic functions coincide with the almost characters of G up to scalar.

In [Sho4], Shoji proves that Lusztig's conjecture holds if **G** has connected center. Moreover, when $G=F_4(p^n)$, he shows that it also holds when $p=3$ and a weaker version of it holds when $p=2$. We state Shoji's results for the cases that we need.

For the rest of the section let **G** be of type F_4. We note that of the 37 almost characters, 25 could be determined from the Green functions (restrictions of $R_{\mathbf{T}}^{\mathbf{G}}(1)$ to the set G_{uni} of unipotent elements of G) which can be computed due to the algorithm in [Lu2, V, §24] as extended by [Sho4, I, Theorems 2.2 and 7.4]. In particular, it follows that the restrictions on p and p^n for the table of Green functions in [Sho2] can be replaced by $p\geq 3$. If $p=2$, G. Malle [Ma] determined the Green functions of G; also, one of the authors computed them [Shi2] using the results in [Lo] and Table 7 in §5 and with several assumptions that are now assured by [Sho4].

If $p=2$, the 5 almost characters in Theorem 1.3(2) could be determined through the generalized Green functions associated with the Levi subgroup of type B_2, which can be computed as (ordinary) Green functions using the table in [Sp2] p. 330. We note that the linear combinations of almost characters $B_2[1]+B_2[\varepsilon'']+B_2[r]$ and $B_2[\varepsilon]+B_2[\varepsilon']+B_2[r]$ can be calculated directly, again using [Lo] and Table 7. The 5 almost characters thus obtained are exhibited in Table 5.A. If $p\geq 3$, these 5 almost characters are zero at unipotent elements.

So we just need to determine the values of the 7 remaining almost characters, corresponding to the cuspidal characters of G. For this, we look into the following character sheaves of **G**: (For notations on unipotent classes of G, we follow [Sp1]; for representatives x_i, [Shi1] and [Sho1]; and for character sheaves A_i, [Sho4].)

A. When $p=2$:

$A_j \leftrightarrow (u, \rho_j)$ $(j=1, 2)$, $u=x_{31}$ a regular unipotent element of G and ρ_1, ρ_2 linear characters of $A_{\mathbf{G}}(u)$ such that $\rho_1(\bar{u})=i$ and $\rho_2(\bar{u})=-i$ where $\langle\bar{u}\rangle=A_{\mathbf{G}}(u)\cong \mathbf{Z}/4\mathbf{Z}$;

$A_3 \leftrightarrow (u, \rho)$, $u=x_{29}$ unipotent element of type $F_4(a_1)$, ρ the nontrivial character of $A_{\mathbf{G}}(u)\cong \mathbf{Z}/2\mathbf{Z}$;

$A_4 \leftrightarrow (u, \rho)$, $u=x_{24}$ unipotent element of type $F_4(a_2)$, ρ the sign character of $A_{\mathbf{G}}(u)\cong D_4$;

$A_5 \leftrightarrow (u, \rho)$, $u=x_{17}$ unipotent element of type $F_4(a_3)$, ρ the sign character of $A_{\mathbf{G}}(u)\cong \mathscr{S}_3$.

* He initially assumes $p^n\equiv 1 \pmod{12}$, then suggests that this assumption can be extended to $p\geq 5$.

B. When $p=3$:

$A_j \leftrightarrow (u, \rho_j)$ $(j=1, 2)$, $u=x_{25}$ a regular unipotent element of G and ρ_1, ρ_2 linear characters of $A_{\mathbf{G}}(u)$ such that $\rho_1(\bar{u})=\theta$ and $\rho_2(\bar{u})=\theta^2$ (θ a primitive cube root of 1) where $\langle\bar{u}\rangle=A_{\mathbf{G}}(u)\cong\mathbf{Z}/3\mathbf{Z}$;

$A_3 \leftrightarrow (u, \rho)$, $u=x_{14}$ unipotent element of type $F_4(a_3)$, ρ the sign character of $A_{\mathbf{G}}(u)\cong\mathscr{S}_4$.

C. When $p\geq 5$:

$A_1 \leftrightarrow (u, \rho)$, $u=x_{14}$ unipotent element of type $F_4(a_3)$, ρ the sign character of $A_{\mathbf{G}}(u)\cong\mathscr{S}_4$.

(1.7) THEOREM. [Sho4, I, Theorems 6.3, 7.5] *Following the notations above, we have:*

A. *If* $p=2$, *then there exist* $c_i\in\bar{\mathbf{Q}}_l$ $(1\leq i\leq 7)$ *with* $|c_i|=1$ *for* $i=1, 2, 3$, *such that*

$$R_{(g_4,i)}=c_1\chi_{A_1}, \quad R_{(g_4,-i)}=c_2\chi_{A_2}, \quad R_{(g_2,\varepsilon)}=c_3\chi_{A_3},$$

$$R_{(g'_2,\varepsilon)}=c_4\chi_{A_4}+c_5\chi_{A_5}, \quad \textit{and} \quad R_{(1,\lambda^3)}=c_6\chi_{A_4}+c_7\chi_{A_5}.$$

Moreover, $\chi_{A_4}=\chi'_{A_4}+c\chi_{A_5}$ *for some* $c\in\bar{\mathbf{Q}}_l$ *where* $\chi'_{A_4}\in C(G/\sim)$ *obtained by restricting* χ_{A_4} *on* $\mathscr{C}^F$ *where* $\mathscr{C}$ *is the unipotent class of* $\mathbf{G}$ *corresponding to* A_4.

B. *If* $p=3$, *then there exist* $c_i\in\bar{\mathbf{Q}}_l$ *with* $|c_i|=1$, $i=1, 2, 3$ *such that*

$$R_{(g_3,\theta)}=c_1\chi_{A_1}, \quad R_{(g_3,\theta^2)}=c_2\chi_{A_2} \quad \textit{and} \quad R_{(1,\lambda^3)}=c_3\chi_{A_3}.$$

C. *If* $p\geq 5$, *then there exists* $c_1\in\bar{\mathbf{Q}}_l$ *with* $|c_1|=1$ *such that*

$$R_{(1,\lambda^3)}=c_1\chi_{A_1}.$$

(1.8) REMARKS. (i) The almost characters listed in the preceding theorem correspond to cuspidal characters. Those almost characters corresponding to cuspidal characters not listed are 0 at all unipotent elements.

(ii) By Lusztig's algorithm for Theorem 1.5 (ii) (when $p\geq 5$) and its extension by Shoji, we can assume the following*:

	χ_{A_1}	χ_{A_2}
x_{31}	q^2	q^2
x_{32}	iq^2	$-iq^2$
x_{33}	$-q^2$	$-q^2$
x_{34}	$-iq^2$	iq^2

	χ_{A_3}
x_{29}	q^3
x_{30}	$-q^3$

	χ'_{A_4}
x_{24}	q^4
x_{25}	$-q^4$
x_{26}	q^4
x_{27}	$-q^4$
x_{28}	q^4

	χ_{A_5}
x_{17}	q^6
x_{18}	$-q^6$
x_{19}	q^6

A: $p=2$**

* The unipotent elements at which a character function is 0 are omitted.

** In place of χ_{A_4}, we can determine only its restriction map χ'_{A_4} (cf. Theorem 1.7).

	χ_{A_1}	χ_{A_2}
x_{25}	q^2	q^2
x_{26}	θq^2	$\theta^2 q^2$
x_{27}	$\theta^2 q^2$	θq^2

	χ_{A_3}
x_{14}	q^6
x_{15}	q^6
x_{16}	$-q^6$
x_{17}	$-q^6$
x_{18}	q^6

B: $p=3$

	χ_{A_1}
x_{14}	q^6
x_{15}	q^6
x_{16}	$-q^6$
x_{17}	$-q^6$
x_{18}	q^6

C: $p\geq 5$

We notice that χ_{A_3} of $p=3$ coincides with χ_{A_1} of $p\geq 5$.

(iii) Yamagishi has determined the Green functions of $Sp(8, 2^n)$ under certain assumptions [Y] which can now be removed due to Theorem 2.2 in [Sho4]. Since no cuspidal characters exist in this case, we can form the unipotent character table as soon as we know the values of the generalized Green functions associated with the Levi subgroup of type B_2. The values of the almost characters and unipotent characters thus obtained are given in Tables 8 and 9.

§2. The Hecke algebra.

From the last section our problem of constructing the unipotent character table of $G=F_4(p^n)$ is reduced to finding the values of certain scalars. For this purpose we form linear equations involving these scalars by computing the values of certain unipotent characters in the 21-element family at unipotent elements in two ways. The first way is by transforming the almost characters into unipotent characters by using the 21×21-Fourier transform matrix* and then applying Theorem 1.7.

The second way is through the Hecke algebras and is discussed in this section.

Let G be the finite group of Lie type corresponding to **G** as in the last section. Suppose P is a subgroup of G. Let $e=|P|^{-1}\sum_{x\in P}x$, a primitive idempotent in the group algebra **C**G. Then we define the Hecke algebra $\mathscr{H}=\mathscr{H}(G, P)$ associated with G and P to be the subalgebra $e\mathbf{C}Ge$ of **C**G.

If we set

$$\operatorname{ind} x=|P: {}^{x}P\cap P| \quad (x\in G), \qquad \{D_j\}_{1\leq j\leq r}=P\backslash G/P,$$

$$D_j=Px_jP \ (1\leq j\leq r), \quad \text{and} \quad a_j=(\operatorname{ind} x_j)ex_je \ (1\leq j\leq r),$$

then $\{a_j\colon 1\leq j\leq r\}$ is a basis of $\mathscr{H}$ called the standard basis of $\mathscr{H}$.

We enumerate three results on Hecke algebras that we shall need later. For their proof, please refer to [CR1].

* This matrix is in [Ca, p. 456] but the entry corresponding to $\{(1, \sigma), (g_3, 1)\}$ should be $-1/3$.

(2.1) PROPOSITION. [CR1, Theorem 11.25] *Let $\mathscr{H}$ be the Hecke algebra associated with a finite group G and subgroup P.*

(i) *Let $\zeta \in \operatorname{Irr} G$ and $\zeta|_{\mathscr{H}}$ the restriction of ζ to $\mathscr{H}$. Then $\zeta|_{\mathscr{H}} \neq 0$ if and only if $\langle \zeta, 1_P^G \rangle \neq 0$.*

(ii) *The map from $\{\zeta \in \operatorname{Irr} G: \langle \zeta, 1_P^G \rangle \neq 0\}$ to the set of irreducible characters of $\mathscr{H}$ given by $\zeta \mapsto \zeta|_{\mathscr{H}}$ is a bijection. Under this map, $\deg(\zeta|_{\mathscr{H}}) = \langle \zeta, 1_P^G \rangle$.*

(2.2) THEOREM. [CR1, Theorem 11.32] (i) *The central primitive idempotents $\{e\varepsilon_i: \langle \zeta^i, 1_P^G \rangle \neq 0\}$ of $\mathscr{H}$ are given by*

$$e\varepsilon_i = \zeta^i(1) \cdot |G: P|^{-1} \sum_{j=1}^{r} (\operatorname{ind} x_j)^{-1} \zeta^i(\widehat{a_j}) a_j ,$$

where $\widehat{a_j} = (\operatorname{ind} x_j) e x_j^{-1} e$ for $1 \leq j \leq r$.

(ii) (*Orthogonality relations*) *For φ, φ' irreducible characters of $\mathscr{H}$,*

$$\sum_{j=1}^{r} (\operatorname{ind} x_j)^{-1} \varphi(\widehat{a_j}) \varphi'(\mathrm{a}_j) = \begin{cases} 0 & \text{if } \varphi \neq \varphi' , \\ \varphi(e) \cdot |G: P| \cdot \zeta(1)^{-1} & \text{if } \varphi = \varphi' = \zeta|_{\mathscr{H}} . \end{cases}$$

(2.3) THEOREM. [CR1, Theorem 11.34] *If $\zeta \in \operatorname{Irr} G$, $\langle \zeta, 1_P^G \rangle \neq 0$, and $\zeta|_{\mathscr{H}} = \varphi$, then*

$$\zeta(t) = \frac{|Z_G(t)| \cdot \{\sum_{j=1}^{r} (\operatorname{ind} x_j)^{-1} \varphi(a_j) |\mathscr{C} \cap D_j|\}}{|P| \cdot \{\sum_{j=1}^{r} (\operatorname{ind} x_j)^{-1} \varphi(\widehat{a_j}) \varphi(a_j)\}}$$

where $\mathscr{C}$ is the conjugacy class of G that contains $t \in G$.

For the rest of the section, let $G = F_4(p^n)$. Then the Weyl group W of G has the generating set $\{s_1, s_2, s_3, s_4\}$ where s_i is the reflection corresponding to the root α_i $(1 \leq i \leq 4)$. Let $J = \{s_1, s_2, s_4\}$ and $P = P_J$ and W_J the parabolic subgroups corresponding to J of G and W, respectively. Since $\mathscr{H}(G, P) \cong \mathscr{H}(W, W_J)$ (cf. [CIK]), and the decomposition of $1_{W_J}^W$ is explicitly determined (cf. [A]), we know that $\mathscr{H}$ is 17-dimensional,

$$\mathscr{H} \cong \underbrace{\mathbf{C} \oplus \mathbf{C} \oplus \cdots \oplus \mathbf{C}}_{9 \text{ copies}} \oplus M_2(\mathbf{C}) \oplus M_2(\mathbf{C}) ,$$

and that (see §1 for the notations):

$$1_P^G = [1_1] + [4_2] + [12_1] + [16_1] + [6_1] + [8_3] + [2_1] + [9_2] + [4_3] + 2[8_1] + 2[9_1] .$$

It then follows from Proposition 2.1 (ii) that $\mathscr{H}$ has 9 characters of degree 1 and 2 characters of degree 2. We also note that 5 of the 11 characters in 1_P^G lie in the 21-element family of unipotent characters of G.

For $P \backslash G / P$ we choose the representatives w_i as given in Table 1 and then form the standard basis elements of $\mathscr{H}$: $a_i = |P|^{-1} \sum_{x \in D_i} x$, $1 \leq i \leq 17$.

For simplicity, suppose that the irreducible characters of G in 1_P^G are $\zeta^1, \zeta^2, \cdots, \zeta^{11}$ corresponding to the irreducible characters $\varphi^1, \varphi^2, \cdots, \varphi^{11}$, respectively, of $\mathscr{H}$ where φ_{10} and φ_{11} have degree 2.

We notice that to apply the character formula (2.3) on ζ in 1_P^G, we need to know two sets of values, namely $\varphi(a_j)$ and $|\mathscr{C}\cap D_j|$, $1\leq j\leq 17$. Theorem 2.4 takes care of the first; the second is discussed in the next section.

(2.4) THEOREM. *The character table of* $\mathscr{H}(G, P)$ *is as given in Table* 3.

The rest of the section will be devoted to the proof of this theorem. First we note that for $1\leq i, j\leq 17$, $a_ia_j=\sum_{k=1}^r p_{ij}^k a_k$, for $p_{ij}^k\in\mathbf{C}$. Let $B_i=(p_{ij}^k)_{(j,k)}$. B_2 and B_3 in particular can be formed without much difficulty (cf. [Go]).

(2.5) REMARK. Suppose φ is an irreducible character of $\mathscr{H}$ of degree 1. Then $\varphi(a_1)=1$ and $\varphi(a_i)\varphi(a_j)=\varphi(a_ia_j)=\sum_{k=1}^r p_{ij}^k\varphi(a_k)$, and so

$$\varphi(a_i)\begin{pmatrix}1\\ \varphi(a_2)\\ \vdots\\ \varphi(a_r)\end{pmatrix}=B_i\begin{pmatrix}1\\ \varphi(a_2)\\ \vdots\\ \varphi(a_r)\end{pmatrix}.$$

Thus ${}^t(\varphi(a_1)\,\varphi(a_2)\cdots\varphi(a_r))$, the transpose of $(\varphi(a_1)\,\varphi(a_2)\cdots\varphi(a_r))$, is an eigenvector of B_i associated with the eigenvalue $\varphi(a_i)$, $1\leq i\leq r$.

We look at $B_2=(p_{2j}^k)_{(j,k)}$. Using *Mathematica* 2.0 for DEC RISC, we obtain:

(2.6) PROPOSITION. B_2 *has the eigenvalues*

(i) $-1-q+q^2$, $-1-q$, $q(1+2q+2q^2+q^3)$, $-1+q+3q^2+2q^3$, *and* $-1-q+3q^2$, *each of multiplicity* 1;

(ii) $\dfrac{q\sqrt{q(4+9q+4q^2)}-2+3q^2+2q^3}{2}$ *and* $\dfrac{-q\sqrt{q(4+9q+4q^2)}-2+3q^2+2q^3}{2}$, *both of multiplicity* 2;

(iii) $-1+2q^2+q^3$ *of multiplicity* 3; *and*

(iv) $-1-q-q^2$ *of multiplicity* 5.

Let us determine the degree 1 characters of $\mathscr{H}$. Suppose ${}^t(1\,\alpha_2\cdots\alpha_{17})$ is an eigenvector of B_2 with eigenvalue α_2 one of those listed in the last proposition. If α_2 has multiplicity 1 then this eigenvector is uniquely determined; for the other cases, it involves $m-1$ parameters where m is the multiplicity of α_2. For this eigenvector to give rise to a character of $\mathscr{H}$, the following must also hold according to Remark 2.5:

$$\alpha_3\begin{pmatrix}1\\ \alpha_2\\ \vdots\\ \alpha_{17}\end{pmatrix}=B_3\begin{pmatrix}1\\ \alpha_2\\ \vdots\\ \alpha_{17}\end{pmatrix}.$$

In fact, even if we know only the 2nd, 3rd, 5th, and 6th rows, for example, of B_3, we will obtain exactly 9 solutions for $(1\,\alpha_2\cdots\alpha_{17})$: one each from the multiplicity 1 and multiplicity 3 eigenvalues and three from that having multiplicity 5. And since $\mathscr{H}$ has exactly 9 characters of degree 1, the 9 eigenvectors we just obtained should give us these 9 characters of $\mathscr{H}$.

To determine the two degree 2 irreducible characters of $\mathscr{H}$, we first form a linear combination of these two. By Theorem 2.2 (i):

$$e=\sum_{i=1}^{11} e\varepsilon_i=|G:P|^{-1}\left(\sum_{i=1}^{11}\zeta^i(1)\sum_{j=1}^{17}(\operatorname{ind}x_j)^{-1}\varphi^i(\widehat{a_j})a_j\right).$$

This implies

$$\begin{aligned}&|G:P|^{-1}\textstyle\sum_{j=1}^{17}(\operatorname{ind}x_j)^{-1}(\zeta^{10}(1)\varphi^{10}+\zeta^{11}(1)\varphi^{11})(\widehat{a_j})a_j\\ &\quad=e-|G:P|^{-1}\textstyle\sum_{j=1}^{17}(\operatorname{ind}x_j)^{-1}(\sum_{i=1}^{9}\zeta^i(1)\varphi^i)(\widehat{a_j})a_j.\end{aligned}$$

We then obtain

$$\text{(2.8)}\qquad (\zeta^{10}(1)\varphi^{10}+\zeta^{11}(1)\varphi^{11})(a_j)=-\sum_{i=1}^{9}(\zeta^i(1)\varphi^i)(a_j)\qquad (2\leq j\leq 17).$$

Now, the values of the degree 1 irreducible characters $\varphi^1,\cdots,\varphi^9$ of $\mathscr{H}$ are already known from the computations above and the degrees of $\zeta^1,\cdots,\zeta^{11}$ are known in [Lu1]. Thus, as soon as we know one of φ^{10} and φ^{11}, the other can be determined from (2.8).

Suppose ρ^i is the irreducible matrix representation of $\mathscr{H}$ affording the character φ^i $(1\leq i\leq 11)$. Then

$$\rho^{10}(a_i)=\begin{pmatrix}\alpha_i & \beta_i\\ \gamma_i & \delta_i\end{pmatrix}$$

with $\alpha_i,\beta_i,\gamma_i,\delta_i\in\mathbf{C}$, for $1\leq i\leq 17$. Then $\varphi^{10}(a_i)=a_i+\delta_i$ $(1\leq i\leq 17)$ and we may assume

$$\rho^{10}(a_1)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\quad\text{and}\quad\rho^{10}(a_2)=\begin{pmatrix}\alpha_2 & \beta_2\\ 0 & \delta_2\end{pmatrix}.$$

Now B_2 is the matrix of the regular representation of α_2 and B_2 is similar to

$$(\oplus_{i=1}^{9}\rho^i(a_2))\oplus\rho^{10}(a_2)\oplus\rho^{10}(a_2)\oplus\rho^{11}(a_2)\oplus\rho^{11}(a_2).$$

Taking into account the multiplicities of the eigenvalues of B_2, we can conclude that $\alpha_2\neq\delta_2$ and both have multiplicity 2. Thus in the matrix representation above we may assume $\beta_2=0$. We then have

$$\rho^{10}(a_2a_j)=\sum_{k=1}^{17}p_{2j}^k\rho^{10}(a_k)=\sum_{k=1}^{17}p_{2j}^k\begin{pmatrix}\alpha_k & \beta_k\\ \gamma_k & \delta_k\end{pmatrix},\quad\text{and}$$

$$\rho^{10}(a_2a_j)=\rho^{10}(a_2)\rho^{10}(a_j)=\begin{pmatrix}\alpha_2 & 0\\ 0 & \delta_2\end{pmatrix}\begin{pmatrix}\alpha_j & \beta_j\\ \gamma_j & \delta_j\end{pmatrix}=\begin{pmatrix}\alpha_2\alpha_j & \alpha_2\beta_j\\ \delta_2\gamma_j & \delta_2\delta_j\end{pmatrix}.$$

Thus

$$\alpha_2\begin{pmatrix}1\\ \alpha_2\\ \vdots\\ \alpha_{17}\end{pmatrix}=B_2\begin{pmatrix}1\\ \alpha_2\\ \vdots\\ \alpha_{17}\end{pmatrix}, \quad \text{and} \quad \delta_2\begin{pmatrix}1\\ \delta_2\\ \vdots\\ \delta_{17}\end{pmatrix}=B_2\begin{pmatrix}1\\ \delta_2\\ \vdots\\ \delta_{17}\end{pmatrix}.$$

Now for $2\leq j\leq 17$, substituting the values of $\zeta^i(1)$ and $\varphi^i(a_j)$ $(1\leq i\leq 9)$ in the right-hand side of (2.8), we obtain a rational integer. We can then conclude that $\{\alpha_2, \delta_2\}$ must be either $\{-1-q-q^2, -1+2q^2+q^3\}$ or {the two eigenvalues of multiplicity 2 in Proposition 2.6}. Computing the eigenvectors ${}^t(1\ \alpha_2 \cdots \alpha_{17})$ and ${}^t(1\ \delta_2 \cdots \delta_{17})$ associated with α_2 and δ_2, respectively, we obtain φ^{10}; φ^{11} can then be determined at once using (2.8).

§3. $|\mathscr{C}\cap D_i|$.

Let G be a finite group of Lie type corresponding to the connected reductive algebraic group $\mathbf{G}$ and Frobenius map F: $\mathbf{G}\to\mathbf{G}$. Let W be the Weyl group of $\mathbf{G}$. Then W is a finite Coxeter group with a presentation $\langle s_1, s_2, \cdots, s_l : s_i^2=1, (s_is_j)^{m_{ij}}=1, i\neq j\rangle$ where m_{ij} is the order of s_is_j. Let $J\subset\{s_1, s_2, \cdots, s_l\}$ and $W_J=\langle J\rangle$, the standard parabolic subgroup of W corresponding to J. We then have

(3.1) PROPOSITION. [CR2, Theorem 64.38] *Let $I, J\subset\{s_1, s_2, \cdots, s_l\}$. Then every double coset $W_IwW_J\in W_I\backslash W/W_J$ contains a unique element x of minimal length satisfying: For every $y\in W_IxW_J$, there exist $u\in W_I$ and $v\in W_J$ such that $y=uxv$ and $l(uxv)=l(u)+l(x)+l(v)$.*

(3.2) REMARK. An element $x\in W_IwW_J$ satisfying the property stated in Proposition 3.1 is called a distinguished coset representative. We denote by D_{IJ} the set of all such coset representatives for $W_I\backslash W/W_J$.

Suppose that U, B, H, N, and $W=N/H$ are the subgroups associated with G as a group with split BN-pair. Moreover, let (Φ, Δ) be the root system of W with Φ^+ (resp., Φ^-) the set of positive (resp., negative) roots with respect to a fixed ordering. For $\alpha\in\Phi$, let $U_\alpha=\{x_\alpha(t): t\in\mathbf{F}_q\}$; for $w\in W$, $U_w^+=\langle U_\alpha: \alpha\in\Phi^+, w(\alpha)\in\Phi^+\rangle$ and $U_w^-=\langle U_\alpha: \alpha\in\Phi^+, w(\alpha)\in\Phi^-\rangle$. Suppose, as above, that W has presentation $\langle s_1, s_2, \cdots, s_l: s_i^2=1, (s_is_j)^{m_{ij}}=1, i\neq j\rangle$. Fix $J\subset\{s_1, s_2, \cdots, s_l\}$. Let $W_J=\langle J\rangle$ and $P_J=BW_JB$, the standard parabolic subgroups associated with J of W and G, respectively. Then we have the following refinement of the Bruhat decomposition [CR2, Theorem 65.4]:

(3.3) PROPOSITION. *Following the notations above,*

$$G=\bigcup_{w\in D_{\phi J}} U_{w^{-1}}^-wP_J\,.$$

Suppose $D_{JJ}=\{w_1, w_2, \cdots, w_r\}$. Let $D_i=P_Jw_iP_J$ for $1\leq i\leq r$. For $x, g\in G$, let $x^g=g^{-1}xg$. For a fixed $x\in G$, let $\mathscr{C}=\{x^g: g\in G\}$, the conjugacy class of G that contains x, and $Z=Z_G(x)$. We wish to determine $|\mathscr{C}\cap D_i|$ for $1\leq i\leq r$.

By Proposition 3.3, we get:

$$\begin{aligned}\mathrm{Card}\{g\in G: x^g\in D_i\}&=|Z|\cdot\mathrm{Card}\{Zg\in Z\backslash G: x^g\in D_i\}=|Z|\cdot|\mathscr{C}\cap D_i|\\&=\mathrm{Card}\{(u, w, p): w\in D_{\phi J}, u\in U^-_{w^{-1}}, p\in P_J, x^{uwp}\in D_i\}\,.\end{aligned}$$

Since $x^{uwp}\in D_i$ if and only if $x^{uw}\in D_i$, we have:

(3.4) COROLLARY. *For $x\in G$ contained in the conjugacy class $\mathscr{C}$,*

$$|\mathscr{C}\cap D_i|=\frac{|P_J|}{|Z_G(x)|}\cdot\mathrm{Card}\{(u, w): w\in D_{\phi J}, u\in U^-_{w^{-1}}, x^{uw}\in D_i\}\,.$$

By the last corollary, to determine $|\mathscr{C}\cap D_i|$ for a fixed $x\in G$, we just have to know which double coset D_i contains x^{uw} for each $w\in D_{\phi J}$ and $u\in U^-_{w^{-1}}$. To carry this out we note the following:

1. For each $w\in D_{\phi J}$, $u\in U^-_{w^{-1}}$, suppose $x^u=x^-_{u,w^{-1}}x^+_{u,w^{-1}}$ where $x^+_{u,w^{-1}}\in U^+_{w^{-1}}$ and $x^-_{u,w^{-1}}\in U^-_{w^{-1}}$. Since $Px^{uw}P=P(x^-_{u,w^{-1}})^w(x^+_{u,w^{-1}})^wP=P(x^-_{u,w^{-1}})^wP$, we may ignore $x^+_{u,w^{-1}}$.
2. If $(x^-_{u,w^{-1}})^w=x^1_{u,w}w'x^2_{u,w}$ with $x^1_{u,w}, x^2_{u,w}\in P$ and $w'\in W$, then $P(x^-_{u,w^{-1}})^wP=Pw'P$. Therefore $Px^{uw}P=Pw'P$ and x^{uw} is in the double coset D_i if and only if $w'\in D_i$.

Let us now consider $G=F_4(p^n)$. As in the previous section we take $J=\{s_1, s_2, s_4\}$ and form the parabolic subgroup W_J of W. Tables 1 and 2 give the sets $D_{\phi J}$ and D_{JJ} of distinguished coset representatives for W/W_J and $W_J\backslash W/W_J$, respectively.

Also, let $P=P_J$ be the corresponding parabolic subgroup of G, and $D_i=Pw_iP$ $(1\leq i\leq 17)$, the double cosets of G over P.

We apply the procedure outlined above. For example, if $p=2$, $x=x_{31}$ (cf. [Shi1]) a regular unipotent element, and $w=w_{33}\in D_{\phi J}$ (cf. Table 2), an element $u\in U^-_{w^{-1}}$ is of the form

$$u=x_4(t_1)x_3(t_2)x_{3+4}(t_3)x_{2+4}(t_4)x_{1-2-3+4}(t_5)x_{1-2+3+4}(t_6)x_{1+2+3+4}(t_7)x_{1+4}(t_8)$$

where $t_j\in\mathbf{F}_q$, $1\leq j\leq 8$ (for the other notations, please refer to [Shi1]); in this case we can show that:

$$x^{uw}\in D_{14}\Leftrightarrow t_1+t_1^2=0 \quad\text{and}\quad x^{uw}\in D_{15}\Leftrightarrow t_1+t_1^2\neq 0\,.$$

Insofar as we want to know $|\mathscr{C}\cap D_i|$ to determine the values of certain unipotent characters on certain unipotent elements of G, we don't need to find this number for all $x\in G$. Moreover, using some information on the characters of G, we can obtain $|\mathscr{C}\cap D_i|$ for all $i=1, 2, \cdots, 17$ if we know this cardinality for certain i's.

For example, we consider the case $x=x_{29}$, again for $p=2$. Since $x_{29}\sim x_{29}^{-1}$ in

$G, D_6^{-1}=D_7$, and $D_{10}^{-1}=D_{11}$, it follows that $|\mathscr{C}\cap D_6|=|\mathscr{C}\cap D_7|$ and $|\mathscr{C}\cap D_{10}|=|\mathscr{C}\cap D_{11}|$. We also know the values of the characters $[1_1]$, $[8_1]$, $[9_1]$, $[8_3]$, $[9_2]$ and $[2_1]$ on x_{29}. Moreover, using the transition matrix for the 21-family of irreducible characters of G, we have $[6_1](x_{29})=[4_3](x_{29})=0$ and $[12_1](x_{29})=[9_2](x_{29})$. Combining all of these information, $|\mathscr{C}\cap D_i|$, $i=1, 2, \cdots, 17$, can be obtained the moment we know its value for 6 different i's (not containing both 6 and 7, nor both 10 and 11).

(3.5) THEOREM. *Let $G=F_4(p^n)$. When $p=2$ (resp., $p=3$), Table 4.A (resp., 4.B) gives the values $|\mathscr{C}\cap D_i|$ $(1\leq i\leq 17)$ for $\mathscr{C}$ the unipotent classes in G containing $x=x_{24}, x_{29}, x_{31}, x_{32}, x_{33}$, and x_{34} (resp., $x=x_{25}, x_{26}$, and x_{27}).*

(3.6) REMARKS. (i) We also computed $|\mathscr{C}\cap D_i|$ for $\mathscr{C}$ containing x_{17} and x_{30} when $p=2$ (cf. [Mar]).

(ii) It seems that computing $|\mathscr{C}\cap D_i|$ is a difficult problem. As far as we know the only general theorem for $|\mathscr{C}\cap D_i|$ is the following result due to Kawanaka [Ka1, Theorem 7.2]: If p is good for **G** and $\mathscr{C}$ is a regular unipotent class, then $|\mathscr{C}\cap Bg|$ is independent of $g\in G$ and of $\mathscr{C}$. As a corollary we have $|\mathscr{C}\cap D_i|=|\mathscr{C}\cap P|\cdot \operatorname{ind} x_i$. Although this result doesn't hold in the case $G=F_4(p^n)$ and $p=2$ or 3, we can show something similar (cf. Table 4):

$$|\mathscr{R}\cap D_i|=|\mathscr{R}\cap P|\cdot \operatorname{ind} x_i,$$

where $\mathscr{R}$ is the set of all regular unipotent elements in G.

§4. Results.

In this section we restrict G to $F_4(p^n)$ and we follow the notations in §2 and §3.

As we saw after (1.6), we only need to determine the 7 almost characters corresponding to cuspidal characters to complete the unipotent character table of G. Two (resp., four, six) of them are zero at unipotent elements when $p=2$ (resp., $p=3$, $p\geq 5$). The nonzero almost characters are precisely those we listed in Theorem 1.7; to determine the values of these almost characters, we only need to determine c_i $(1\leq i\leq 7)$ and c in (1.7.A) if $p=2$, c_i $(1\leq i\leq 3)$ in (1.7.B) if $p=3$ and c_1 in (1.7.C) if $p\leq 5$.

First let us consider the case $p=2$. To determine $R_{(g_4,i)}$ and $R_{(g_4,-i)}$, we apply Theorem 2.3 with $\zeta=[12_1]$ at the regular unipotent element x_{31}:

$$[12_1](x_{31})=\frac{|Z_G(x_{31})|\cdot\{\sum_{j=1}^{17}(\operatorname{ind} x_j)^{-1}[12_1](a_j)|\mathscr{C}\cap D_j|\}}{|P|\cdot\{\sum_{j=1}^{17}(\operatorname{ind} x_j)^{-1}[12_1](a_j)^2\}}$$

where $\mathscr{C}$ is the unipotent class of G that contains x_{31}. Using Table 3 (for the values of $[12_1](a_j)$) and Table 4.A (for the values of $|\mathscr{C}\cap D_i|$), we get $[12_1](x_{31})=\frac{1}{2}q^2$. Now, forming the irreducible character $[12_1]$ from the 21-element family of almost characters using the 21×21-Fourier transform matrix, and applying Theorem 1.7 and the table in Remark 1.8 (ii), we get

$$[12_1](x_{31})=\tfrac{1}{4}R_{(g_4,i)}(x_{31})+\tfrac{1}{4}R_{(g_4,-i)}(x_{31})=\tfrac{1}{4}(c_1+c_2)q^2\,.$$

Thus $c_1+c_2=2$. Since $|c_1|=|c_2|=1$, $c_1=c_2=1$.

Similarly, we have

$$[12_1](x_{29})=\tfrac{1}{4}q^3=\tfrac{1}{4}R_{(g_2,\varepsilon)}(x_{29})=\tfrac{1}{4}c_3q^3\,,$$

and so $c_3=1$. Also,

$$\begin{aligned}[12_1](x_{24})=\tfrac{1}{8}q^3(4+q)&=(\tfrac{1}{8}R_{(g_2',\varepsilon)}+\tfrac{1}{4}R_{(g_2',r)})(x_{24})\\&=\tfrac{1}{8}(c_4\chi'_{A_4}+(cc_4+c_5)\chi_{A_5})(x_{24})+\tfrac{1}{2}q^3\\&=\tfrac{1}{8}c_4q^4+\tfrac{1}{2}q^3\,,\end{aligned}$$

which implies that $c_4=1$. Thus,

$$R_{(g_2',\varepsilon)}=\chi'_{A_4}+(c+c_5)\chi_{A_5}\quad\text{and}\quad R_{(1,\lambda^3)}=c_6\chi'_{A_4}+(cc_6+c_7)\chi_{A_5}\,.$$

Now since $\langle R_{(g_2',\varepsilon)}, R_{(g_2',\varepsilon)}\rangle=\langle\chi'_{A_4},\chi'_{A_4}\rangle=1$ and $\langle\chi'_{A_4},\chi_{A_5}\rangle=0$, we have $c+c_5=0$ and $c_6=0$.

To show that $c_7=1$, we follow Lusztig's method in [Lu3, 8.12]. Specifically, using the 21-element family transition matrix, we have $[12_1](x_{17})=\frac{1}{24}q^4(5+c_7q^2)$. Since this value must be a rational integer, $c_7=\pm1$ and $3\,|\,5+c_7q^2$; thus, c_7 must be equal to 1.

Thus, we obtain the following

(4.1) THEOREM. *For $G=F_4(2^n)$,*

$$R_{(g_4,i)}=\chi_{A_1}\,,\quad R_{(g_4,-i)}=\chi_{A_2}\,,\quad R_{(g_2,\varepsilon)}=\chi_{A_3}\,,$$
$$R_{(g_2',\varepsilon)}=\chi'_{A_4}\,,\quad \textit{and}\quad R_{(1,\lambda^3)}=\chi_{A_5}\,.$$

Now let us consider the case when $p=3$. Using Tables 3 and 4.B, and applying Theorem 2.3 with $\zeta=[12_1]$ and $t=x_{25}$, we get $[12_1](x_{25})=\frac{2}{3}q^2$. But also, using the transition matrix from the almost characters to unipotent characters in the 21-element family and Theorem 1.7 and Remark 1.8 (ii), we have

$$[12_1](x_{25})=\tfrac{1}{3}(R_{(g_3,\theta)}+R_{(g_3,\theta^2)})(x_{25})=\tfrac{1}{3}(c_1\chi_{A_1}+c_2\chi_{A_2})(x_{25})=\tfrac{1}{3}(c_1+c_2)q^2\,.$$

As above, it then follows that $c_1=c_2=1$ and therefore, $R_{(g_3,\theta)}=\chi_{A_1}$ and $R_{(g_3,\theta^2)}=\chi_{A_2}$. Now following the method to prove $c_6=1$ in Theorem 4.1, we have $[12_1](x_{14})=\frac{1}{24}q^4(23+c_3q^2)$, which must be a rational integer. Therefore $c_3=\pm1$, $8\,|\,23+c_3q^2$ and so $c_3=1$ giving us $R_{(1,\lambda^3)}=\chi_{A_3}$. We therefore have

(4.2) THEOREM. *For $G=F_4(3^n)$,*

$$R_{(g_3,\theta)}=\chi_{A_1}\,,\quad R_{(g_3,\theta^2)}=\chi_{A_2}\,,\quad \textit{and}\quad R_{(1,\lambda^3)}=\chi_{A_3}\,.$$

(4.3) REMARK. The method used to prove $R_{(1,\lambda^3)}=\chi_{A_3}$ is also valid for the corresponding relation $R_{(1,\lambda^3)}=\chi_{A_1}$ if $p\geq5$. In fact using the result of Shoji [Sho4, I, Theorem 5.7], we also have $[12_1](x_{14})=\frac{1}{24}q^4(23+c_1q^2)$, which is a rational integer. Thus we get $c_1=\pm 1$ and $24\,|\,23+c_1q^2$. Since $q^2\equiv 1 \pmod{24}$ for $p\geq 5$, we must have $c_1=1$. This is actually the method devised by Lusztig in [Lu3, 8.12] to determine the unipotent character table of $G=F_4(p^n)$ under the initial assumption $p^n\equiv 1 \pmod{12}$, and this restriction can thus be replaced by $p\geq 5$ as he suggested and as we just showed above.

The values at unipotent elements of the almost unipotent characters which are not obtainable from the Green functions are listed in Table 5.A (resp., 5.B) when $p=2$ (resp., $p=3$). We can then complete the unipotent character table of G.

(4.4) THEOREM. *Table* 6.A (*resp.*, 6.B) *gives the unipotent character table of* $F_4(p^n)$ *when* $p=2$ (*resp.*, $p=3$).

(4.5) REMARK. The corresponding tables for the case $p\geq 5$ can be derived at once from the tables for the case $p=3$. We note that if $p\geq 5$, G has 26 unipotent classes, the first 25 of which coincide with the first 25 of the case $p=3$ (cf. [Sho1]). In fact the values of the unipotent characters on these 25 classes are equal for $p\geq 5$ and for $p=3$; for the 26th class, the regular unipotent class, the character values are all equal to 0 except for the character $[1_1]$ which has the value 1.

§5. Tables.

In the tables, $G=F_4(p^n)$, $p=2$ or 3, unless otherwise specified. For the unipotent conjugacy classes of G, we follow the notations in [Shi1] and [Sho1] when $p=2$ and $p=3$, respectively. When a subset J of the generating set of the Weyl group W of G appears in a table, it is understood to be the subset $J=\{s_1, s_2, s_4\}$; the parabolic subgroup P is P_J and $P\backslash G/P=\{D_j\colon 1\leq j\leq 17\}$.

For simplicity, we use i to denote s_i, $1\leq i\leq 4$, in Tables 1 and 2. ϕ_n is the nth cyclotomic polynomial—for example, $\phi_1=q-1$, $\phi_2=1+q$ and $\phi_6=1-q+q^2$; a dot entry stands for a value of 0.

For Table 7, consider the parabolic subgroup P_I of G where $I=\{s_1, s_2, s_3\}$. Suppose $\mathscr{C}$ is a conjugacy class of G and $\mathscr{C}'$ a conjugacy class of P_I/U_I where U_I is the unipotent radical of P_I. If x is an arbitrary element of $\mathscr{C}$ and $\pi\colon P_I\to P_I/U_I$ is the canonical map we define

$$g_{CC'}=\mathrm{Card}\{gP_I\subset G\colon x^g\in P_I \text{ and } \pi(x^g)\in\mathscr{C}'\}\,.$$

Table 7 gives these values for unipotent conjugacy classes. For the unipotent class representatives z_i of P_I/U_I, we follow [Shi1].

Since $Sp(8, 2^n)=C_4(2^n)\cong B_4(2^n)$ we can use the y_i-notation for $B_4(2^n)$ in [Shi1] for the unipotent classes of $Sp(8, 2^n)$. However we change the ordering slightly as follows: $u_i=y_i$ $(0\leq i\leq 24, i\neq 4, 5, 6, 7)$, $u_4=y_7, u_5=y_6, u_6=y_4$, and $u_7=y_5$. For the almost characters and unipotent characters, we use symbols introduced by Lusztig [Lu1].

List of Tables.

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TABLE 1. The elements of D_{JJ}

$w_1 = \{\}$
$w_2 = 3$
$w_3 = 323$
$w_4 = 3423$
$w_5 = 323123$
$w_6 = 3423123$
$w_7 = 3234123$
$w_8 = 3234323$
$w_9 = 32343123$
$w_{10} = 3234323123$
$w_{11} = 3231234323$
$w_{12} = 34231234123$
$w_{13} = 342312343123$
$w_{14} = 3231234323123$
$w_{15} = 34231234323123$
$w_{16} = 3234323123423123$
$w_{17} = 32312343231234323123$

TABLE 2. The elements of $D_{\phi J}$

$w_1 = \{\}$	$w_{33} = 32343123$	$w_{65} = 432312343123$
$w_2 = 3$	$w_{34} = 31234123$	$w_{66} = 342312343123$
$w_3 = 23$	$w_{35} = 43234123$	$w_{67} = 234231234123$
$w_4 = 43$	$w_{36} = 31234323$	$w_{68} = 3231234323123$
$w_5 = 123$	$w_{37} = 234323123$	$w_{69} = 4231234323123$
$w_6 = 323$	$w_{38} = 123423123$	$w_{70} = 4323123423123$
$w_7 = 423$	$w_{39} = 323423123$	$w_{71} = 3423123423123$
$w_8 = 3123$	$w_{40} = 312343123$	$w_{72} = 3432312343123$
$w_9 = 4123$	$w_{41} = 432343123$	$w_{73} = 2342312343123$
$w_{10} = 4323$	$w_{42} = 231234123$	$w_{74} = 1234231234123$
$w_{11} = 3423$	$w_{43} = 431234123$	$w_{75} = 43231234323123$
$w_{12} = 23123$	$w_{44} = 231234323$	$w_{76} = 34231234323123$
$w_{13} = 43123$	$w_{45} = 1234323123$	$w_{77} = 34323123423123$
$w_{14} = 34123$	$w_{46} = 3234323123$	$w_{78} = 23423123423123$
$w_{15} = 34323$	$w_{47} = 3123423123$	$w_{79} = 23432312343123$
$w_{16} = 23423$	$w_{48} = 4323423123$	$w_{80} = 12342312343123$
$w_{17} = 323123$	$w_{49} = 2312343123$	$w_{81} = 343231234323123$
$w_{18} = 423123$	$w_{50} = 4312343123$	$w_{82} = 234231234323123$
$w_{19} = 343123$	$w_{51} = 4231234123$	$w_{83} = 234323123423123$
$w_{20} = 234123$	$w_{52} = 3231234323$	$w_{84} = 123423123423123$
$w_{21} = 234323$	$w_{53} = 31234323123$	$w_{85} = 123432312343123$
$w_{22} = 123423$	$w_{54} = 43234323123$	$w_{86} = 2343231234323123$
$w_{23} = 4323123$	$w_{55} = 23123423123$	$w_{87} = 1234231234323123$
$w_{24} = 3423123$	$w_{56} = 43123423123$	$w_{88} = 1234323123423123$
$w_{25} = 2343123$	$w_{57} = 32312343123$	$w_{89} = 3234323123423123$
$w_{26} = 1234123$	$w_{58} = 42312343123$	$w_{90} = 12343231234323123$
$w_{27} = 3234123$	$w_{59} = 34231234123$	$w_{91} = 32343231234323123$
$w_{28} = 1234323$	$w_{60} = 43231234323$	$w_{92} = 31234323123423123$
$w_{29} = 3234323$	$w_{61} = 231234323123$	$w_{93} = 312343231234323123$
$w_{30} = 34323123$	$w_{62} = 431234323123$	$w_{94} = 231234323123423123$
$w_{31} = 23423123$	$w_{63} = 323123423123$	$w_{95} = 2312343231234323123$
$w_{32} = 12343123$	$w_{64} = 423123423123$	$w_{96} = 32312343231234323123$

TABLE 3. The character table of $\mathscr{H}(G, P)$

	a_1	a_2	a_3	a_4	a_5	a_6, a_7	a_8	a_9
$[1_1]$	1	$q\phi_2\phi_3$	$q^3\phi_2\phi_3$	$q^4\phi_2^2\phi_3$	$q^6\phi_2$	$q^7\phi_2\phi_3$	$q^7\phi_3$	$q^8\phi_2^2\phi_3$
$[4_2]$	1	$-1+q+3q^2+2q^3$	$q\phi_2^2(-1+q+q^2)$	$q^2\phi_2(-2-q+3q^2+3q^3)$	$q^3(-1+q^2+q^3)$	$q^4(-2-2q+q^2+3q^3+q^4)$	$q^4(-1-q+q^2+2q^3)$	$q^5\phi_1\phi_2(3+5q+3q^2)$
$[12_1]$	1	$-1-q+3q^2$	$2q^2(-2+q)$	$q(1+2q-8q^2+3q^3)$	$q^2(1-2q)$	$q^2(-1+5q-3q^2)$	$q^2(-1+3q-3q^2)$	$q^2(1-5q+12q^2-5q^3+q^4)$
$[16_1]$	1	$-1-q+q^2$	$-2q^2$	$-q\phi_1(1+3q+q^2)$	q^2	$q^2(-1+q+q^2)$	$q^2(-1+q+q^2)$	$-q^2\phi_1\phi_2\phi_6$
$[6_1]$	1	$-\phi_2$	$-q^2\phi_2$	$q\phi_2^2$	$q^2\phi_2$	$-q^2\phi_2$	$-q^2$	$q^2\phi_2^2\phi_6$
$[8_3]$	1	$\phi_2(-1+q+q^2)$	$q\phi_2(-1-q+q^2)$	$q\phi_1\phi_2^2(-1+q+q^2)$	$-q^3\phi_2$	$q^3\phi_2(1-q-q^2)$	$q^3\phi_{12}$	$-2q^4\phi_1\phi_2^2$
$[2_1]$	1	$-\phi_3$	$q\phi_3\phi_4$	$-q^3\phi_2\phi_3$	$q^3(-1-q^2+q^3)$	$-q^6\phi_3$	$q^4\phi_3$	$q^5\phi_2^2\phi_3$
$[9_2]$	1	$-\phi_3$	$2q^3$	$q\phi_2(1+q-q^2)$	$q^2(1-2q)$	$q^2(-1+q+q^2)$	$q^2(-1-q+q^2)$	$q^2\phi_2^2(1-3q+q^2)$
$[4_3]$	1	$-\phi_3$	·	$q\phi_2\phi_3$	q^2	$-q^2\phi_3$	$-q^2\phi_3$	$-q^2\phi_1\phi_2\phi_3$
$[8_1]$	2	$-2-q+q^2+q^3$	$q^2(-1+2q+2q^2+q^3)$	$-q\phi_2(-1+q+3q^2)$	$q^2(1-2q-q^2+q^4)$	$q^3(2+q-q^2-q^3)$	$-q^2\phi_2\phi_6$	$q^3\phi_1\phi_2^2\phi_6$
$[9_1]$	2	$-2+3q^2+2q^3$	$q\phi_1(1+4q+3q^2+q^3)$	$q\phi_2(1-2q-3q^2+2q^3+q^4)$	$q^2(1-q+q^4)$	$q^3(1-q-2q^2+q^4)$	$q^3\phi_2(1-q+q^3)$	$q^4\phi_2^2(1-4q+q^2)$

	a_{10}, a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}
$[1_1]$	$q^{10}\phi_2\phi_3$	$q^{11}\phi_3$	$q^{12}\phi_2^2\phi_3$	$q^{13}\phi_2$	$q^{14}\phi_2\phi_3$	$q^{16}\phi_2\phi_3$	q^{20}
$[4_2]$	$q^6(-1-3q-q^2+2q^3+2q^4)$	$q^7(-2-q+q^2+q^3)$	$q^8\phi_2(-3-3q+q^2+2q^3)$	$q^8(-1-q+q^3)$	$q^9\phi_2^2(-1-q+q^2)$	$q^{11}(-2-3q-q^2+q^3)$	$-q^{14}$
$[12_1]$	$q^4(-3+5q-q^2)$	$q^4(-3+3q-q^2)$	$q^4(3-8q+2q^2+q^3)$	$q^5(-2+q)$	$2q^5(1-2q)$	$q^6(3-q-q^2)$	q^8
$[16_1]$	$q^4(-1-q+q^2)$	$q^4(-1-q+q^2)$	$-q^4\phi_1(1+3q+q^2)$	$-q^6$	$2q^6$	$q^6(-1+q+q^2)$	$-q^8$
$[6_1]$	$-q^5\phi_2$	$-q^6$	$q^5\phi_2^2$	$q^5\phi_2$	$-q^5\phi_2$	$-q^7\phi_2$	q^8
$[8_3]$	$q^5\phi_2(1+q-q^2)$	$-q^4\phi_{12}$	$q^5\phi_1\phi_2^2(1+q-q^2)$	$q^7\phi_2$	$q^7\phi_2(-1+q+q^2)$	$q^8\phi_2(-1-q+q^2)$	$-q^{11}$
$[2_1]$	$-q^6\phi_3$	$q^8\phi_3$	$-q^8\phi_2\phi_3$	$q^8(1-q-q^3)$	$q^9\phi_3\phi_4$	$-q^{12}\phi_3$	q^{14}
$[9_2]$	$q^4(1+q-q^2)$	$q^4(1-q-q^2)$	$q^4\phi_2(-1+q+q^2)$	$q^5(-2+q)$	$2q^5$	$-q^6\phi_3$	q^8
$[4_3]$	$q^4\phi_3$	$q^4\phi_3$	$-q^4\phi_2\phi_3$	$-q^6$	·	$q^6\phi_3$	$-q^8$
$[8_1]$	$q^5(1+q-q^2-2q^3)$	$q^6\phi_2\phi_6$	$q^7\phi_2(3+q-q^2)$	$q^5(-1+q^2+2q^3-q^4)$	$q^6(-1-2q-2q^2+q^3)$	$q^8(-1-q+q^2+2q^3)$	$-2q^{11}$
$[9_1]$	$q^5(1-2q^2-q^3+q^4)$	$q^5\phi_2(1-q^2+q^3)$	$q^6\phi_2(1+2q-3q^2-2q^3+q^4)$	$q^6(1-q^3+q^4)$	$-q^7\phi_1(1+3q+4q^2+q^3)$	$q^9(2+3q-2q^3)$	$2q^{12}$

TABLE 4. $|\mathscr{C}_j \cap D_j| \cdot |Z_G(x_j)|/|P|$ where $\mathscr{C}_j$ is the conjugacy class containing x_j

$i \backslash j$	24	29
1	$1+q+3q^2$	$1+q$
2	$q^2(-1+6q+12q^2+8q^3)$	$q^2(2+3q+2q^2)$
3	$q^3(-3-2q+5q^2+9q^3+2q^4)$	$q^3(1+4q+4q^2+3q^3)$
4	$q^4(-6-7q+7q^2+23q^3+10q^4)$	$q^3(-1-q+3q^2+5q^3+5q^4+q^5)$
5	$q^5(-2+q+5q^2+2q^3)$	$-q^4+2q^7$
6	$q^6(-4-7q+5q^2+10q^3+4q^4)$	$q^5(-1+3q^3+2q^4+q^5)$
7	$q^6(-4-7q+5q^2+10q^3+4q^4)$	$q^5(-1+3q^3+2q^4+q^5)$
8	$q^5(1-q-4q^2+3q^3+6q^4)$	$q^7(1+q)^2$
9	$q^6(-2-2q-16q^2+14q^4+8q^5+q^6)$	$q^6(-2+q+q^2+8q^3+6q^4+3q^5+q^6)$
10	$q^7(-2-q-7q^2+8q^4+6q^5+q^6)$	$q^7(-1-2q^2+2q^3+3q^4+2q^5+q^6)$
11	$q^7(-2-q-7q^2+8q^4+6q^5+q^6)$	$q^7(-1-2q^2+2q^3+3q^4+2q^5+q^6)$
12	$q^7(-1+2q-4q^2-2q^3+4q^4+2q^5+q^6)$	$q^9(-1+q+2q^2+q^3+q^4)$
13	$q^7(2-3q-q^2-10q^3-17q^4-q^5+12q^6+8q^7+3q^8+q^9)$	$q^8(-3-3q^2-2q^3+2q^4+4q^5+4q^6+3q^7+q^8)$
14	$q^8(1-4q^2-2q^3+q^4+4q^5+q^6)$	$q^8(-1+q-q^2+q^5+q^6)$
15	$q^9(-1+q-3q^2-10q^3-6q^4+3q^5+3q^6+2q^7+q^8)$	$q^8(1-2q-q^3+q^5+2q^6+2q^7+2q^8+q^9)$
16	$q^9(2+q+2q^2+2q^3-6q^4-8q^5-q^6+4q^7+2q^8+2q^9+q^{10})$	$q^9(1-q-q^2-q^3-2q^4-q^5+q^7+2q^8+2q^9+q^{10})$
17	$q^{14}(2-3q^2+q^6)$	$q^{11}(1+q^9)$

$i \backslash j$	31	32 or 34	33
1	1	1	1
2	$q(1+3q+4q^2+4q^3)$	$q(1+q)$	$q(1+3q+4q^2)$
3	$-q^2+3q^5+q^6$	$q^2(1+2q+2q^2+q^3+q^4)$	$q^2(-1+4q^2+3q^3+q^4)$
4	$q^3(-1+q^2+10q^3+5q^4+q^5)$	$q^3(1+2q+q^2+q^4+q^5)$	$q^3(-1+9q^2+6q^3+5q^4+q^5)$
5	$q^6(2+q)$	q^7	$q^6(2+q)$
6	$q^5(1-3q+2q^2+4q^3+3q^4+q^5)$	$q^5(1+q+q^4+q^5)$	$q^5(-3+q+2q^2+4q^3+3q^4+q^5)$
7	$q^5(1-3q+2q^2+4q^3+3q^4+q^5)$	$q^5(1+q+q^4+q^5)$	$q^5(-3+q+2q^2+4q^3+3q^4+q^5)$
8	$q^5(1-3q+q^2+2q^3+q^4)$	$q^5(1+q+q^2+q^4)$	$q^5(-3+q+q^2+2q^3+q^4)$
9	$q^5(-2+6q-6q^2-q^3+4q^4+5q^5+3q^6+q^7)$	$q^6(2+4q+3q^2+2q^3+3q^4+3q^5+q^6)$	$q^5(2-10q-2q^2-q^3+4q^4+5q^5+3q^6+q^7)$
10	$q^6(-2+2q-q^2+2q^4+3q^5+2q^6+q^7)$	$q^8(1+q^3+2q^4+q^5)$	$q^6(2-2q-q^2+2q^4+3q^5+2q^6+q^7)$
11	$q^6(-2+2q-q^2+2q^4+3q^5+2q^6+q^7)$	$q^8(1+q^3+2q^4+q^5)$	$q^6(2-2q-q^2+2q^4+3q^5+2q^6+q^7)$
12	$q^6(-2+2q-q^2-q^3+q^5+q^6+q^7)$	$q^8(1+q+q^3+q^4+q^5)$	$q^6(2-2q-q^2-q^3+q^5+q^6+q^7)$
13	$q^6(2-4q-q^3-2q^4+3q^6+4q^7+4q^8+3q^9+q^{10})$	$q^9(1+q)^2(1-q^2+q^3+q^4+q^5)$	$q^6(-2+4q-q^3-2q^4+3q^6+4q^7+4q^8+3q^9+q^{10})$
14	$q^9(-1+q^3+q^4+q^5)$	$q^9(1-q^3+q^4+q^5)$	$q^9(1+q^2)(-1+q^2+q^3)$
15	$q^8(-2-q^2-2q^3-2q^4+q^6+2q^7+2q^8+q^9)$	$q^{10}(1+2q+2q^2+q^4+2q^5+2q^6+q^7)$	$q^8(2-q^2-2q^3-2q^4+q^6+2q^7+2q^8+q^9)$
16	$q^8(2-q^4-q^5+q^7+q^8+2q^9+2q^{10}+q^{11})$	$q^{12}(1+q-q^3+q^4+2q^5+2q^6+q^7)$	$q^8(-2-q^4-q^5+q^7+q^8+2q^9+2q^{10}+q^{11})$
17	$q^{15}(-1+q^5)$	$q^{15}(1+q^5)$	$q^{15}(-1+q^5)$

A: $p=2$

$i \backslash j$	25
1	1
2	$q(1+2q+2q^2+3q^3)$
3	$q^3(1+4q^2+q^3)$
4	$q^4(1-3q+6q^2+3q^3+q^4)$
5	$q^5(-1+q)(2+q)$
6	$q^5(4-2q+q^2+2q^3+2q^4+q^5)$
7	$q^5(4-2q+q^2+2q^3+2q^4+q^5)$
8	$q^5(2-2q+q^2+q^3+q^4)$
9	$q^5(-4+8q-4q^2+q^3+3q^4+4q^5+3q^6+q^7)$
10	$q^6(-2+4q+q^4+2q^5+2q^6+q^7)$
11	$q^6(-2+4q+q^4+2q^5+2q^6+q^7)$
12	$q^6(-2+2q+q^5+q^6+q^7)$
13	$q^6(2-6q+q^6+3q^7+4q^8+3q^9+q^{10})$
14	$q^7(-1+q)(2+2q+2q^2+2q^3+2q^4+2q^5+q^6)$
15	$q^7(2-2q+q^7+2q^8+2q^9+q^{10})$
16	$q^8(2+q^8+2q^9+2q^{10}+q^{11})$
17	q^{20}

$i \backslash j$	26 or 27
1	1
2	$q(1+2q+2q^2)$
3	$q^3(1+3q+q^2+q^3)$
4	$q^4(1+6q+3q^2+3q^3+q^4)$
5	$q^5(1+q+q^2)$
6	$q^5(-2+q+q^2+2q^3+2q^4+q^5)$
7	$q^5(-2+q+q^2+2q^3+2q^4+q^5)$
8	$q^5(-1+q+q^2+q^3+q^4)$
9	$q^5(2-4q+2q^2+q^3+3q^4+4q^5+3q^6+q^7)$
10	$q^6(1-2q+q^4+2q^5+2q^6+q^7)$
11	$q^6(1-2q+q^4+2q^5+2q^6+q^7)$
12	$q^6(1-q+q^5+q^6+q^7)$
13	$q^6(-1+3q+q^6+3q^7+4q^8+3q^9+q^{10})$
14	$q^7(1+q^6+q^7)$
15	$q^7(-1+q+q^7+2q^8+2q^9+q^{10})$
16	$q^8(-1+q^8+2q^9+2q^{10}+q^{11})$
17	q^{20}

B: $p=3$

TABLE 5. The values at unipotent elements of the almost characters of G not obtainable from the Green functions and whose restrictions at G_{uni} is not 0

	$B_2[1]$	$B_2[\epsilon]$	$B_2[\epsilon']$	$B_2[\epsilon'']$	$B_2[r]$	$F_4[i]$	$F_4[-i]$	$F_4^{I}[1]$	$F_4^{II}[1]$	$F_4[-1]$
x_0	.	.	.	.	.	.	.	.	.	.
x_1	.	.	.	.	.	.	.	.	.	.
x_2	.	.	.	.	.	.	.	.	.	.
x_3	.	.	.	.	.	.	.	.	.	.
x_4	.	.	.	.	.	.	.	.	.	.
x_5	.	.	.	.	.	.	.	.	.	.
x_6	.	.	.	.	.	.	.	.	.	.
x_7	.	.	.	.	.	.	.	.	.	.
x_8	.	.	.	.	.	.	.	.	.	.
x_9	q^5	q^9	q^7	q^7	$q^6\phi_4$	.	.	.	.	.
x_{10}	$-q^5$	$-q^9$	$-q^7$	$-q^7$	$-q^6\phi_4$	.	.	.	.	.
x_{11}	.	.	.	.	.	.	.	.	.	.
x_{12}	.	.	.	.	.	.	.	.	.	.
x_{13}	.	.	.	.	.	.	.	.	.	.
x_{14}	.	.	.	.	.	.	.	.	.	.
x_{15}	.	.	.	.	.	.	.	.	.	.
x_{16}	.	.	.	.	.	.	.	.	.	.
x_{17}	.	.	.	.	.	.	.	.	q^6	.
x_{18}	.	.	.	.	.	.	.	.	$-q^6$	.
x_{19}	.	.	.	.	.	.	.	.	q^6	.
x_{20}	q^2	.	.	q^4	q^3	.	.	.	.	.
x_{21}	$-q^2$	.	.	$-q^4$	$-q^3$	.	.	.	.	.
x_{22}	q^2	.	q^4	.	q^3	.	.	.	.	.
x_{23}	$-q^2$	.	$-q^4$	.	$-q^3$	.	.	.	.	.
x_{24}	$2q^2$	.	.	.	$2q^3$	.	.	q^4	.	.
x_{25}	.	.	.	.	.	.	.	$-q^4$	.	.
x_{26}	$-2q^2$	.	.	.	$-2q^3$	.	.	q^4	.	.
x_{27}	.	.	.	.	.	.	.	$-q^4$	.	.
x_{28}	.	.	.	.	.	.	.	q^4	.	.
x_{29}	.	.	.	.	.	.	.	.	.	q^3
x_{30}	.	.	.	.	.	.	.	.	.	$-q^3$
x_{31}	q	.	.	.	.	q^2	q^2	.	.	.
x_{32}	$-q$	.	.	.	.	iq^2	$-iq^2$	.	.	.
x_{33}	q	.	.	.	.	$-q^2$	$-q^2$	.	.	.
x_{34}	$-q$	.	.	.	.	$-iq^2$	iq^2	.	.	.

A: $p = 2$

	$F_4^{II}[1]$	$F_4[\theta]$	$F_4[\theta^2]$
x_0	.	.	.
x_1	.	.	.
x_2	.	.	.
x_3	.	.	.
x_4	.	.	.
x_5	.	.	.
x_6	.	.	.
x_7	.	.	.
x_8	.	.	.
x_9	.	.	.
x_{10}	.	.	.
x_{11}	.	.	.
x_{12}	.	.	.
x_{13}	.	.	.
x_{14}	q^6	.	.
x_{15}	q^6	.	.
x_{16}	$-q^6$	.	.
x_{17}	$-q^6$	.	.
x_{18}	q^6	.	.
x_{19}	.	.	.
x_{20}	.	.	.
x_{21}	.	.	.
x_{22}	.	.	.
x_{23}	.	.	.
x_{24}	.	.	.
x_{25}	.	q^2	q^2
x_{26}	.	θq^2	$\theta^2 q^2$
x_{27}	.	$\theta^2 q^2$	θq^2

B: $p = 3$

TABLE 6.A. The unipotent character table of G ($p=2$) (1/5)

	$[1_1]$	$[1_4]$	$[9_1]$	$[9_4]$	$[8_1]$	$[8_2]$	$[8_3]$	$[8_4]$	$[4_2]$	$[2_1]$	$[2_3]$	$B_2[1]$
x_0	1	q^{24}	$q^2\phi_3^2\phi_6^2\phi_{12}$	$q^{10}\phi_3^2\phi_6^2\phi_{12}$	$q^3\phi_4^2\phi_8\phi_{12}$	$q^9\phi_4^2\phi_8\phi_{12}$	$q^3\phi_4^2\phi_8\phi_{12}$	$q^9\phi_4^2\phi_8\phi_{12}$	$q\phi_2^2\phi_6^2\phi_8/2$	$q\phi_4\phi_8\phi_{12}/2$	$q\phi_4\phi_8\phi_{12}/2$	$q\phi_1^2\phi_3^2\phi_8/2$
x_1	1	·	$q^2\phi_3\phi_6\phi_8$	$q^{10}\phi_3\phi_6$	$q^3\phi_4\phi_8$	$q^9\phi_4\phi_8$	$q^3\phi_4(1+q^4+q^6)$	$q^9\phi_4$	$q\phi_2\phi_6(1+q^3+q^4)/2$	$q(1+q^4+q^6-q^7)/2$	$q(1+q^4+q^6+q^7)/2$	$q\phi_1\phi_3(-1+q^3-q^4)/2$
x_2	1	·	$q^2\phi_3\phi_6\phi_8$	$q^{10}\phi_3\phi_6$	$q^3\phi_4(1+q^4+q^6)$	$q^9\phi_4$	$q^3\phi_4\phi_8$	$q^9\phi_4\phi_8$	$q\phi_2\phi_6(1+q^3+q^4)/2$	$q(1+q^4+q^6+q^7)/2$	$q(1+q^4+q^6-q^7)/2$	$q\phi_1\phi_3(-1+q^3-q^4)/2$
x_3	1	·	$q^2(1+q^2+2q^4+q^6)$	$q^{10}\phi_4$	$q^3\phi_4\phi_8$	$q^9\phi_4$	$q^3\phi_4\phi_8$	$q^9\phi_4$	$q(1+2q^3+q^4+q^6)/2$	$q(1+q^4+q^6)/2$	$q(1+q^4+q^6)/2$	$q(1-2q^3+q^4+q^6)/2$
x_4	1	·	$q^2(1+q^2+2q^4)$	q^{10}	$q^3\phi_3\phi_6$	q^9	$q^3\phi_3\phi_6$	q^9	$q\phi_2(1-q+q^2+q^3)/2$	$q\phi_8/2$	$q\phi_8/2$	$q\phi_1(-1-q-q^2+q^3)/2$
x_5	1	·	$q^2\phi_3\phi_6$	·	$q^3(1+q^2+q^3+q^4)$	·	q^3	q^9	$q(1+q^3+2q^4)/2$	$q(1+q^3+2q^4)/2$	$-q\phi_1\phi_3/2$	$-q\phi_1\phi_3/2$
x_6	1	·	$q^2\phi_3\phi_6$	·	$q^3(1+q^2-q^3+q^4)$	·	q^3	q^9	$q\phi_2\phi_6/2$	$q\phi_2\phi_6/2$	$q(1-q^3+2q^4)/2$	$q(1-q^3+2q^4)/2$
x_7	1	·	$q^2\phi_3\phi_6$	·	q^3	q^9	$q^3(1+q^2+q^3+q^4)$	·	$q(1+q^3+2q^4)/2$	$-q\phi_1\phi_3/2$	$q(1+q^3+2q^4)/2$	$-q\phi_1\phi_3/2$
x_8	1	·	$q^2\phi_3\phi_6$	·	q^3	q^9	$q^3(1+q^2-q^3+q^4)$	·	$q\phi_2\phi_6/2$	$q(1-q^3+2q^4)/2$	$q\phi_2\phi_6/2$	$q(1-q^3+2q^4)/2$
x_9	1	·	$q^2\phi_3\phi_6$	·	$q^3\phi_4$	·	$q^3\phi_4$	·	$q(1+q^2+2q^3+q^4)/2$	$q(1+q^2-q^4)/2$	$q(1+q^2-q^4)/2$	$q(1+q^2-2q^3+q^4)/2$
x_{10}	1	·	$q^2\phi_3\phi_6$	·	$q^3\phi_4$	·	$q^3\phi_4$	·	$q(1+q^2+2q^3-q^4)/2$	$q\phi_3\phi_6/2$	$q\phi_3\phi_6/2$	$q(1+q^2-2q^3-q^4)/2$
x_{11}	1	·	$q^2\phi_4$	·	$q^3\phi_4$	·	q^3	·	$q\phi_2\phi_6/2$	$q\phi_2\phi_6/2$	$-q\phi_1\phi_3/2$	$-q\phi_1\phi_3/2$
x_{12}	1	·	$q^2\phi_4$	·	q^3	·	$q^3\phi_4$	·	$q\phi_2\phi_6/2$	$-q\phi_1\phi_3/2$	$q\phi_2\phi_6/2$	$-q\phi_1\phi_3/2$
x_{13}	1	·	$q^2\phi_4$	·	$q^3\phi_4$	·	q^3	·	$q(1+q^2+q^3)/2$	$q(1+q^2+q^3)/2$	$q(1+q^2-q^3)/2$	$q(1+q^2-q^3)/2$
x_{14}	1	·	$q^2\phi_4$	·	q^3	·	$q^3\phi_4$	·	$q(1+q^2+q^3)/2$	$q(1+q^2-q^3)/2$	$q(1+q^2+q^3)/2$	$q(1+q^2-q^3)/2$
x_{15}	1	·	$q^2\phi_4$	·	q^3	·	q^3	·	$q/2$	$q/2$	$q/2$	$q/2$
x_{16}	1	·	$q^2\phi_4$	·	q^3	·	q^3	·	$q\phi_4/2$	$q\phi_4/2$	$q\phi_4/2$	$q\phi_4/2$
x_{17}	1	·	q^2	·	q^3	·	q^3	·	$q(1+2q^2)/2$	$q(1+2q^2)/2$	$q(1+2q^2)/2$	$q(1+2q^2)/2$
x_{18}	1	·	q^2	·	q^3	·	q^3	·	$q/2$	$q/2$	$q/2$	$q/2$
x_{19}	1	·	q^2	·	q^3	·	q^3	·	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$
x_{20}	1	·	q^2	·	q^3	·	·	·	$q(1+2q)/2$	$q/2$	$q(1-2q)/2$	$q/2$
x_{21}	1	·	q^2	·	q^3	·	·	·	$q/2$	$q(1+2q)/2$	$q/2$	$q(1-2q)/2$
x_{22}	1	·	q^2	·	·	·	q^3	·	$q(1+2q)/2$	$q(1-2q)/2$	$q/2$	$q/2$
x_{23}	1	·	q^2	·	·	·	q^3	·	$q/2$	$q/2$	$q(1+2q)/2$	$q(1-2q)/2$
x_{24}	1	·	q^2	·	·	·	·	·	$q(1+4q)/2$	$q(1-2q)/2$	$q(1-2q)/2$	$q/2$
x_{25}	1	·	q^2	·	·	·	·	·	$q/2$	$q(1+2q)/2$	$q(1-2q)/2$	$q/2$
x_{26}	1	·	q^2	·	·	·	·	·	$q/2$	$q(1+2q)/2$	$q(1+2q)/2$	$q(1-4q)/2$
x_{27}	1	·	q^2	·	·	·	·	·	$q/2$	$q(1-2q)/2$	$q(1+2q)/2$	$q/2$
x_{28}	1	·	q^2	·	·	·	·	·	$q(1-2q)/2$	$q/2$	$q/2$	$q(1+2q)/2$
x_{29}	1	·	·	·	·	·	·	·	$q/2$	$q/2$	$q/2$	$q/2$
x_{30}	1	·	·	·	·	·	·	·	$q/2$	$q/2$	$q/2$	$q/2$
x_{31}	1	·	·	·	·	·	·	·	$q/2$	$-q/2$	$-q/2$	$q/2$
x_{32}	1	·	·	·	·	·	·	·	$-q/2$	$q/2$	$q/2$	$-q/2$
x_{33}	1	·	·	·	·	·	·	·	$q/2$	$-q/2$	$-q/2$	$q/2$
x_{34}	1	·	·	·	·	·	·	·	$-q/2$	$q/2$	$q/2$	$-q/2$

TABLE 6.A. The unipotent character table of G ($p=2$) (2/5)

	$[4_5]$	$[2_4]$	$[2_2]$	$B_2[\epsilon]$	$[12_1]$	$[9_3]$	$[1_3]$
x_0	$q^{13}\phi_2^2\phi_6^2\phi_8/2$	$q^{13}\phi_4\phi_8\phi_{12}/2$	$q^{13}\phi_4\phi_8\phi_{12}/2$	$q^{13}\phi_1^2\phi_3^2\phi_8/2$	$q^4\phi_2^4\phi_3^2\phi_8\phi_{12}/24$	$q^4\phi_3^2\phi_4^2\phi_8\phi_{12}/8$	$q^4\phi_4^2\phi_6^2\phi_8\phi_{12}/8$
x_1	$q^{13}\phi_2\phi_6/2$	$-q^{13}\phi_1\phi_3/2$	$q^{13}\phi_2\phi_6/2$	$-q^{13}\phi_1\phi_3/2$	$q^4\phi_2^3\phi_3(1+2q+q^2-q^3+3q^5)/24$	$q^4\phi_3\phi_4(1+q+q^2+q^4+q^5+3q^6)/8$	$q^4\phi_4\phi_6(1-q+q^2+q^4-q^5+3q^6)/8$
x_2	$q^{13}\phi_2\phi_6/2$	$q^{13}\phi_2\phi_6/2$	$-q^{13}\phi_1\phi_3/2$	$-q^{13}\phi_1\phi_3/2$	$q^4\phi_2^3\phi_3(1+2q+q^2-q^3+3q^5)/24$	$q^4\phi_2\phi_3\phi_4\phi_6(1+q+q^2-q^3)/8$	$q^4\phi_1\phi_3\phi_4\phi_6(-1+q-q^2-q^3)/8$
x_3	$q^{13}/2$	$q^{13}/2$	$q^{13}/2$	$q^{13}/2$	$q^4\phi_2^3(1+3q+4q^2+2q^3+2q^5)/24$	$q^4\phi_4(1+2q+3q^2+2q^3+2q^4+2q^5+2q^6)/8$	$q^4\phi_4(1-2q+3q^2-2q^3+2q^4-2q^5+2q^6)/8$
x_4	.	.	.	.	$q^4\phi_2^2(1+2q)(1+2q+3q^2)/24$	$q^4\phi_2\phi_4(1+q+2q^2)/8$	$q^4\phi_1\phi_4(-1+q-2q^2)/8$
x_5	.	.	.	.	$q^4\phi_2^2\phi_3(1+3q)/24$	$-q^4\phi_1\phi_2^2\phi_3/8$	$-q^4\phi_1^3\phi_3/8$
x_6	.	.	.	.	$q^4\phi_2^2(1+4q+4q^2-3q^3)/24$	$q^4\phi_2^3\phi_6/8$	$q^4\phi_1^2\phi_2\phi_6/8$
x_7	.	.	.	.	$q^4\phi_2^2\phi_3(1+3q)/24$	$q^4\phi_2\phi_3(1+3q^2)/8$	$q^4(1-2q+5q^2-q^3+2q^4+3q^5)/8$
x_8	.	.	.	.	$q^4\phi_2^2(1+4q+4q^2-3q^3)/24$	$q^4(1+2q+5q^2+q^3+2q^4-3q^5)/8$	$-q^4\phi_1\phi_6(1+3q^2)/8$
x_9	$q^9/2$	$-q^9/2$	$-q^9/2$	$q^9/2$	$q^4{\phi_2}^2(1+3q^2)/8$	$q^4\phi_4(3+2q-3q^2)/8$	$q^4\phi_4(3-2q-3q^2)/8$
x_{10}	$-q^9/2$	$q^9/2$	$q^9/2$	$-q^9/2$	$-q^4\phi_1{\phi_2}^3/8$	$q^4\phi_4(3+2q+q^2)/8$	$q^4\phi_4(3-2q+q^2)/8$
x_{11}	.	.	.	.	$q^4\phi_2(1+2q)(1+3q)/24$	$-q^4\phi_1\phi_2(1+2q)/8$	$q^4\phi_1\phi_2(-1+2q)/8$
x_{12}	.	.	.	.	$q^4\phi_2(1+2q)(1+3q)/24$	$q^4\phi_2(1+q+2q^2)/8$	$q^4\phi_1(-1+q-2q^2)/8$
x_{13}	.	.	.	.	$q^4\phi_2^2/8$	$q^4\phi_2(3-q)/8$	$-q^4\phi_1(3+q)/8$
x_{14}	.	.	.	.	$q^4\phi_2^2/8$	$q^4(3+2q+3q^2)/8$	$q^4(3-2q+3q^2)/8$
x_{15}	.	.	.	.	$q^4(1+2q)(1+4q)/24$	$q^4(1+2q)/8$	$q^4(1-2q)/8$
x_{16}	.	.	.	.	$q^4(1+2q)/8$	$q^4(3+2q)/8$	$q^4(3-2q)/8$
x_{17}	.	.	.	.	$q^4(5+q^2)/24$	$q^4(5+q^2)/8$	$q^4(5+q^2)/8$
x_{18}	.	.	.	.	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{19}	.	.	.	.	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{20}	.	.	.	.	$q^3\phi_2/4$	$q^3\phi_1/4$	$-q^3\phi_2/4$
x_{21}	.	.	.	.	$-q^3\phi_2/4$	$-q^3\phi_1/4$	$q^3\phi_2/4$
x_{22}	.	.	.	.	$q^3\phi_2/4$	$-q^3\phi_2/4$	$q^3\phi_1/4$
x_{23}	.	.	.	.	$-q^3\phi_2/4$	$q^3\phi_2/4$	$-q^3\phi_1/4$
x_{24}	.	.	.	.	$q^3(4+q)/8$	$-q^3(4+q)/8$	$-q^3(4+q)/8$
x_{25}	.	.	.	.	$-q^4/8$	$q^4/8$	$q^4/8$
x_{26}	.	.	.	.	$q^3(-4+q)/8$	$q^3(4-q)/8$	$q^3(4-q)/8$
x_{27}	.	.	.	.	$-q^4/8$	$q^4/8$	$q^4/8$
x_{28}	.	.	.	.	$q^4/8$	$-q^4/8$	$-q^4/8$
x_{29}	.	.	.	.	$q^3/4$	$q^3/4$	$-q^3/4$
x_{30}	.	.	.	.	$-q^3/4$	$-q^3/4$	$q^3/4$
x_{31}	.	.	.	.	$q^2/2$	$-q^2/2$	$q^2/2$
x_{32}	.	.	.	.	.	.	.
x_{33}	.	.	.	.	$-q^2/2$	$q^2/2$	$-q^2/2$
x_{34}	.	.	.	.	.	.	.

TABLE 6.A. The unipotent character table of G ($p=2$) (3/5)

	$F_4^{II}[1]$	$[6_2]$	$[16_1]$	$B_2[\epsilon'']$	$[4_4]$
x_0	$q^4\phi_1^4\phi_6^2\phi_8\phi_{12}/24$	$q^4\phi_3^2\phi_4^2\phi_6^2\phi_8/12$	$q^4\phi_2^4\phi_4^2\phi_6^2\phi_{12}/4$	$q^4\phi_1^2\phi_3^2\phi_4\phi_8\phi_{12}/4$	$q^4\phi_2^2\phi_4\phi_6^2\phi_8\phi_{12}/4$
x_1	$-q^4\phi_1^3\phi_6(1-2q+q^2+q^3-3q^5)/24$	$q^4\phi_3\phi_4\phi_6(1+2q^2+3q^4)/12$	$q^4\phi_2^4\phi_4\phi_6^2/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-2q^6+q^7)/4$	$q^4\phi_2\phi_6(1+q^3+q^4+2q^6+q^7)/4$
x_2	$-q^4\phi_1^3\phi_6(1-2q+q^2+q^3-3q^5)/24$	$q^4\phi_3\phi_4\phi_6(1+2q^2+3q^4)/12$	$q^4\phi_2^4\phi_4\phi_6^2/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-q^7)/4$	$q^4\phi_2\phi_6(1+q^3+q^4-q^7)/4$
x_3	$q^4\phi_1^3(-1+3q-4q^2+2q^3+2q^5)/24$	$q^4\phi_4(1+3q^2+6q^4+2q^6)/12$	$q^4\phi_2^2\phi_4(1+2q^3)/4$	$q^4(1-2q^3+q^4+2q^6)/4$	$q^4(1+2q^3+q^4+2q^6)/4$
x_4	$q^4\phi_1^2(-1+2q)(-1+2q-3q^2)/24$	$q^4(1+4q^2+7q^4)/12$	$q^4\phi_2^3(1-q+2q^2)/4$	$q^4\phi_1(-1-q-q^2+q^3)/4$	$q^4\phi_2(1-q+q^2+q^3)/4$
x_5	$q^4\phi_1^2(1-4q+4q^2+3q^3)/24$	$q^4\phi_2\phi_3(1-2q+3q^2)/12$	$q^4\phi_2(1+q+3q^3-q^4)/4$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$
x_6	$-q^4\phi_1^2\phi_6(-1+3q)/24$	$-q^4\phi_1\phi_6(1+2q+3q^2)/12$	$q^4\phi_2^3\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$
x_7	$q^4\phi_1^2(1-4q+4q^2+3q^3)/24$	$q^4\phi_2\phi_3(1-2q+3q^2)/12$	$q^4\phi_2(1+q+3q^3-q^4)/4$	$-q^4\phi_1\phi_3\phi_4/4$	$q^4(1+q^2+3q^3+4q^4-q^5)/4$
x_8	$-q^4\phi_1^2\phi_6(-1+3q)/24$	$-q^4\phi_1\phi_6(1+2q+3q^2)/12$	$q^4\phi_2^3\phi_6/4$	$q^4(1+q^2-3q^3+4q^4+q^5)/4$	$q^4\phi_2\phi_4\phi_6/4$
x_9	$q^4\phi_1^2(1+3q^2)/8$	$q^4\phi_4(1+3q^2)/4$	$q^4\phi_2^2\phi_4/4$	$q^4(1+2q^3-q^4)/4$	$q^4(1-2q^3-q^4)/4$
x_{10}	$-q^4\phi_1^3\phi_2/8$	$-q^4\phi_1\phi_2\phi_4/4$	$q^4\phi_2^2\phi_4/4$	$q^4(1-2q^3-q^4)/4$	$q^4(1+2q^3-q^4)/4$
x_{11}	$q^4\phi_1(1-2q)(-1+3q)/24$	$-q^4\phi_1\phi_2/12$	$q^4\phi_2^2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{12}	$q^4\phi_1(1-2q)(-1+3q)/24$	$-q^4\phi_1\phi_2/12$	$q^4\phi_2^2/4$	$-q^4\phi_1(1+q+2q^2)/4$	$q^4\phi_2(1-q+2q^2)/4$
x_{13}	$q^4\phi_1^2/8$	$q^4\phi_4/4$	$q^4\phi_2^2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{14}	$q^4\phi_1^2/8$	$q^4\phi_4/4$	$q^4\phi_2^2/4$	$q^4\phi_4/4$	$q^4\phi_4/4$
x_{15}	$q^4(1-4q)(1-2q)/24$	$q^4(1-2q)(1+2q)/12$	$q^4(1+2q)/4$	$q^4/4$	$q^4/4$
x_{16}	$q^4(1-2q)/8$	$q^4/4$	$q^4(1+2q)/4$	$q^4/4$	$q^4/4$
x_{17}	$q^4(5+q^2)/24$	$q^4(5+q^2)/12$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{18}	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/12$	$q^4\phi_4/4$	$q^4\phi_4/4$	$q^4\phi_4/4$
x_{19}	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/12$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{20}	$-q^3\phi_1/4$	$q^3/2$	.	$q^4/2$	$-q^4/2$
x_{21}	$q^3\phi_1/4$	$-q^3/2$	.	$-q^4/2$	$q^4/2$
x_{22}	$-q^3\phi_1/4$	$q^3/2$	.	.	.
x_{23}	$q^3\phi_1/4$	$-q^3/2$	.	.	.
x_{24}	$q^3(4+q)/8$	$q^3(4+q)/4$	$-q^4/4$	$q^4/4$	$q^4/4$
x_{25}	$-q^4/8$	$-q^4/4$	$q^4/4$	$-q^4/4$	$-q^4/4$
x_{26}	$q^3(-4+q)/8$	$q^3(-4+q)/4$	$-q^4/4$	$q^4/4$	$q^4/4$
x_{27}	$-q^4/8$	$-q^4/4$	$q^4/4$	$-q^4/4$	$-q^4/4$
x_{28}	$q^4/8$	$q^4/4$	$-q^4/4$	$q^4/4$	$q^4/4$
x_{29}	$-q^3/4$	.	$-q^3/2$	.	.
x_{30}	$q^3/4$	.	$q^3/2$	.	.
x_{31}	$-q^2/2$	.	.	.	.
x_{32}	.	.	.	.	.
x_{33}	$q^2/2$	.	.	.	.
x_{34}	.	.	.	.	.

TABLE 6.A. The unipotent character table of G ($p=2$) (4/5)

	$F_4[-1]$	$[9_2]$	$[4_1]$	$[1_2]$	$F_4^{\text{I}}[1]$
x_0	$q^4\phi_1^4\phi_3^2\phi_4^2\phi_{12}/4$	$q^4\phi_3^2\phi_4^2\phi_8\phi_{12}/8$	$q^4\phi_2^4\phi_6^2\phi_8\phi_{12}/8$	$q^4\phi_4^2\phi_6^2\phi_8\phi_{12}/8$	$q^4\phi_1^4\phi_3^2\phi_8\phi_{12}/8$
x_1	$q^4\phi_1^4\phi_3^2\phi_4/4$	$q^4\phi_2\phi_3\phi_4\phi_6(1+q+q^2-q^3)/8$	$q^4\phi_2^3\phi_6(1-q^2+q^3+2q^4-q^5)/8$	$q^4\phi_1\phi_3\phi_4\phi_6(-1+q-q^2-q^3)/8$	$q^4\phi_1^3\phi_3(-1+q^2+q^3-2q^4-q^5)/8$
x_2	$q^4\phi_1^4\phi_3^2\phi_4/4$	$q^4\phi_3\phi_4(1+q+q^2+q^4+q^5+3q^6)/8$	$q^4\phi_2^3\phi_6(1-q^2+q^3+2q^4-q^5)/8$	$q^4\phi_4\phi_6(1-q+q^2+q^4-q^5+3q^6)/8$	$q^4\phi_1^3\phi_3(-1+q^2+q^3-2q^4-q^5)/8$
x_3	$q^4\phi_1^2\phi_4(1-2q^3)/4$	$q^4\phi_4(1+2q+3q^2+2q^3+2q^4+2q^5+2q^6)/8$	$q^4\phi_2^2(1-q^2+2q^3+2q^4-2q^5+2q^6)/8$	$q^4\phi_4(1-2q+3q^2-2q^3+2q^4-2q^5+2q^6)/8$	$q^4\phi_1^2(1-q^2-2q^3+2q^4+2q^5+2q^6)/8$
x_4	$-q^4\phi_1^3(1+q+2q^2)/4$	$q^4\phi_2\phi_4(1+q+2q^2)/8$	$q^4\phi_2^2(1-q^2+2q^3)/8$	$q^4\phi_1\phi_4(-1+q-2q^2)/8$	$q^4\phi_1^2(1-q^2-2q^3)/8$
x_5	$-q^4\phi_1^3\phi_3/4$	$q^4\phi_2\phi_3(1+3q^2)/8$	$-q^4\phi_1\phi_2^2\phi_3/8$	$q^4(1-2q+5q^2-q^3+2q^4+3q^5)/8$	$-q^4\phi_1^3\phi_3/8$
x_6	$q^4\phi_1(-1+q+3q^3+q^4)/4$	$q^4(1+2q+5q^2+q^3+2q^4-3q^5)/8$	$q^4\phi_2^3\phi_6/8$	$-q^4\phi_1\phi_6(1+3q^2)/8$	$q^4\phi_1^2\phi_2\phi_6/8$
x_7	$-q^4\phi_1^3\phi_3/4$	$-q^4\phi_1\phi_2^2\phi_3/8$	$-q^4\phi_1\phi_2^2\phi_3/8$	$-q^4\phi_1^3\phi_3/8$	$-q^4\phi_1^3\phi_3/8$
x_8	$q^4\phi_1(-1+q+3q^3+q^4)/4$	$q^4\phi_2^3\phi_6/8$	$q^4\phi_2^3\phi_6/8$	$q^4\phi_1^2\phi_2\phi_6/8$	$q^4\phi_1^2\phi_2\phi_6/8$
x_9	$q^4\phi_1^2\phi_4/4$	$q^4\phi_4(3+2q-3q^2)/8$	$q^4\phi_1\phi_2^2(-3+q)/8$	$q^4\phi_4(3-2q-3q^2)/8$	$q^4\phi_1^2\phi_2(3+q)/8$
x_{10}	$q^4\phi_1^2\phi_4/4$	$q^4\phi_4(3+2q+q^2)/8$	$q^4{\phi_2}^2(3-4q+5q^2)/8$	$q^4\phi_4(3-2q+q^2)/8$	$q^4\phi_1^2(3+4q+5q^2)/8$
x_{11}	$q^4\phi_1^2/4$	$q^4\phi_2(1+q+2q^2)/8$	$-q^4\phi_1\phi_2(1+2q)/8$	$q^4\phi_1(-1+q-2q^2)/8$	$q^4\phi_1\phi_2(-1+2q)/8$
x_{12}	$q^4\phi_1^2/4$	$-q^4\phi_1\phi_2(1+2q)/8$	$-q^4\phi_1\phi_2(1+2q)/8$	$q^4\phi_1\phi_2(-1+2q)/8$	$q^4\phi_1\phi_2(-1+2q)/8$
x_{13}	$q^4\phi_1^2/4$	$q^4(3+2q+3q^2)/8$	$q^4\phi_2(3-q)/8$	$q^4(3-2q+3q^2)/8$	$-q^4\phi_1(3+q)/8$
x_{14}	$q^4\phi_1^2/4$	$q^4\phi_2(3-q)/8$	$q^4\phi_2(3-q)/8$	$-q^4\phi_1(3+q)/8$	$-q^4\phi_1(3+q)/8$
x_{15}	$q^4(1-2q)/4$	$q^4(1+2q)/8$	$q^4(1+2q)/8$	$q^4(1-2q)/8$	$q^4(1-2q)/8$
x_{16}	$q^4(1-2q)/4$	$q^4(3+2q)/8$	$q^4(3+2q)/8$	$q^4(3-2q)/8$	$q^4(3-2q)/8$
x_{17}	$-q^4\phi_1\phi_2/4$	$q^4(5+q^2)/8$	$q^4(5+q^2)/8$	$q^4(5+q^2)/8$	$q^4(5+q^2)/8$
x_{18}	$q^4\phi_4/4$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{19}	$-q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{20}	.	$-q^3\phi_2/4$	$-q^3\phi_2/4$	$q^3\phi_1/4$	$q^3\phi_1/4$
x_{21}	.	$q^3\phi_2/4$	$q^3\phi_2/4$	$-q^3\phi_1/4$	$-q^3\phi_1/4$
x_{22}	.	$q^3\phi_1/4$	$-q^3\phi_2/4$	$-q^3\phi_2/4$	$q^3\phi_1/4$
x_{23}	.	$-q^3\phi_1/4$	$q^3\phi_2/4$	$q^3\phi_2/4$	$-q^3\phi_1/4$
x_{24}	$-q^4/4$	$-q^3(4+q)/8$	$q^3(-4+3q)/8$	$-q^3(4+q)/8$	$q^3(-4+3q)/8$
x_{25}	$q^4/4$	$q^4/8$	$-3q^4/8$	$q^4/8$	$-3q^4/8$
x_{26}	$-q^4/4$	$q^3(4-q)/8$	$q^3(4+3q)/8$	$q^3(4-q)/8$	$q^3(4+3q)/8$
x_{27}	$q^4/4$	$q^4/8$	$-3q^4/8$	$q^4/8$	$-3q^4/8$
x_{28}	$-q^4/4$	$-q^4/8$	$3q^4/8$	$-q^4/8$	$3q^4/8$
x_{29}	$q^3/2$	$q^3/4$	$q^3/4$	$-q^3/4$	$-q^3/4$
x_{30}	$-q^3/2$	$-q^3/4$	$-q^3/4$	$q^3/4$	$q^3/4$
x_{31}	.	$-q^2/2$	$q^2/2$	$q^2/2$	$-q^2/2$
x_{32}	.	.	.	.	.
x_{33}	.	$q^2/2$	$-q^2/2$	$-q^2/2$	$q^2/2$
x_{34}	.	.	.	.	.

TABLE 6.A. The unipotent character table of G ($p=2$) (5/5)

	$B_2[r]$	$[6_1]$	$F_4[\theta], F_4[\theta^2]$	$[4_3]$	$B_2[\epsilon']$	$F_4[i]$	$F_4[-i]$
x_0	$q^4\phi_1^2\phi_2^2\phi_3^2\phi_6^2\phi_8/4$	$q^4\phi_3^2\phi_6^2\phi_8\phi_{12}/3$	$q^4\phi_1^4\phi_2^4\phi_4^2\phi_8/3$	$q^4\phi_2^2\phi_4\phi_6^2\phi_8\phi_{12}/4$	$q^4\phi_1^2\phi_3^2\phi_4\phi_8\phi_{12}/4$	$q^4\phi_1^4\phi_2^4\phi_3^2\phi_6^2/4$	$q^4\phi_1^4\phi_2^4\phi_3^2\phi_6^2/4$
x_1	$-q^4\phi_1\phi_2\phi_3\phi_6\phi_8/4$	$q^4\phi_3\phi_6(1+2q^4)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4\phi_2\phi_6(1+q^3+q^4-q^7)/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-q^7)/4$	$-q^4\phi_1^3\phi_2^3\phi_3\phi_6/4$	$-q^4\phi_1^3\phi_2^3\phi_3\phi_6/4$
x_2	$-q^4\phi_1\phi_2\phi_3\phi_6\phi_8/4$	$q^4\phi_3\phi_6(1+2q^4)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4\phi_2\phi_6(1+q^3+q^4+2q^6+q^7)/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-2q^6+q^7)/4$	$-q^4\phi_1^3\phi_2^3\phi_3\phi_6/4$	$-q^4\phi_1^3\phi_2^3\phi_3\phi_6/4$
x_3	$q^4(1+q^4+2q^8)/4$	$q^4\phi_4(1+3q^4-q^6)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4(1+2q^3+q^4+2q^6)/4$	$q^4(1-2q^3+q^4+2q^6)/4$	$q^4\phi_1^2\phi_2^2/4$	$q^4\phi_1^2\phi_2^2/4$
x_4	$-q^4\phi_1\phi_2\phi_4/4$	$q^4(1+q^2+4q^4)/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_2(1-q+q^2+q^3)/4$	$q^4\phi_1(-1-q-q^2+q^3)/4$	$q^4\phi_1^2\phi_2^2/4$	$q^4\phi_1^2\phi_2^2/4$
x_5	$-q^4\phi_1\phi_3\phi_4/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4(1+q^2+3q^3+4q^4-q^5)/4$	$-q^4\phi_1\phi_3\phi_4/4$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$
x_6	$q^4\phi_2\phi_4\phi_6/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_2\phi_4\phi_6/4$	$q^4(1+q^2-3q^3+4q^4+q^5)/4$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$
x_7	$-q^4\phi_1\phi_3\phi_4/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$
x_8	$q^4\phi_2\phi_4\phi_6/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$
x_9	$q^4(3+q^4)/4$	.	.	$q^4(1-2q^3-q^4)/4$	$q^4(1+2q^3-q^4)/4$	$q^4\phi_1^2\phi_2^2/4$	$q^4\phi_1^2\phi_2^2/4$
x_{10}	$q^4(3-4q^2-3q^4)/4$	.	.	$q^4(1+2q^3-q^4)/4$	$q^4(1-2q^3-q^4)/4$	$q^4\phi_1^2\phi_2^2/4$	$q^4\phi_1^2\phi_2^2/4$
x_{11}	$-q^4\phi_1\phi_2/4$	$q^4(1+2q^2)/3$	$-q^4\phi_1\phi_2/3$	$q^4\phi_2(1-q+2q^2)/4$	$-q^4\phi_1(1+q+2q^2)/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{12}	$-q^4\phi_1\phi_2/4$	$q^4(1+2q^2)/3$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{13}	$q^4(3-q^2)/4$	.	.	$q^4\phi_4/4$	$q^4\phi_4/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{14}	$q^4(3-q^2)/4$	.	.	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{15}	$q^4/4$	$q^4(1+2q^2)/3$	$-q^4\phi_1\phi_2/3$	$q^4/4$	$q^4/4$	$q^4/4$	$q^4/4$
x_{16}	$3q^4/4$	.	.	$q^4/4$	$q^4/4$	$q^4/4$	$q^4/4$
x_{17}	$q^4(5+q^2)/4$	$q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{18}	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$q^4\phi_4/4$	$q^4\phi_4/4$	$q^4\phi_4/4$	$q^4\phi_4/4$
x_{19}	$q^4\phi_1\phi_2/4$	$q^4(2+q^2)/3$	$q^4(2+q^2)/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{20}	$q^3/2$	.	.	.	.	.	.
x_{21}	$-q^3/2$	.	.	.	.	.	.
x_{22}	$q^3/2$	.	.	$-q^4/2$	$q^4/2$	.	.
x_{23}	$-q^3/2$	.	.	$q^4/2$	$-q^4/2$	.	.
x_{24}	$q^3(4-q)/4$	.	.	$q^4/4$	$q^4/4$	$-q^4/4$	$-q^4/4$
x_{25}	$q^4/4$	.	.	$-q^4/4$	$-q^4/4$	$q^4/4$	$q^4/4$
x_{26}	$-q^3(4+q)/4$	.	.	$q^4/4$	$q^4/4$	$-q^4/4$	$-q^4/4$
x_{27}	$q^4/4$	.	.	$-q^4/4$	$-q^4/4$	$q^4/4$	$q^4/4$
x_{28}	$-q^4/4$	.	.	$q^4/4$	$q^4/4$	$-q^4/4$	$-q^4/4$
x_{29}	.	.	.	.	.	.	.
x_{30}	.	.	.	.	.	.	.
x_{31}	.	.	.	.	.	.	.
x_{32}	.	.	.	.	.	iq^2	$-iq^2$
x_{33}	.	.	.	.	.	.	.
x_{34}	.	.	.	.	.	$-iq^2$	iq^2

TABLE 6.B. The unipotent character table of G ($p=3$) (1/5)

	$[1_1]$	$[1_4]$	$[9_1]$	$[9_4]$	$[8_1]$	$[8_2]$	$[8_3]$	$[8_4]$	$[4_2]$	$[2_3]$	$[2_1]$	$B_2[1]$
x_0	1	q^{24}	$q^2\phi_3^2\phi_6^2\phi_{12}$	$q^{10}\phi_3^2\phi_6^2\phi_{12}$	$q^3\phi_4^2\phi_8\phi_{12}$	$q^9\phi_4^2\phi_8\phi_{12}$	$q^3\phi_4^2\phi_8\phi_{12}$	$q^9\phi_4^2\phi_8\phi_{12}$	$q\phi_2^2\phi_6^2\phi_8/2$	$q\phi_4\phi_8\phi_{12}/2$	$q\phi_4\phi_8\phi_{12}/2$	$q\phi_1^2\phi_3^2\phi_8/2$
x_1	1	.	$q^2\phi_3\phi_6\phi_8$	$q^{10}\phi_3\phi_6$	$q^3\phi_4(1+q^4+q^6)$	$q^9\phi_4$	$q^3\phi_4\phi_8$	$q^9\phi_4\phi_8$	$q\phi_2\phi_6(1+q^3+q^4)/2$	$q(1+q^4+q^6-q^7)/2$	$q(1+q^4+q^6+q^7)/2$	$q\phi_1\phi_3(-1+q^3-q^4)/2$
x_2	1	.	$q^2\phi_3(1-q+q^2+q^4)$	$q^{10}\phi_3$	$q^3\phi_4\phi_8$	$q^9\phi_2\phi_4$	$q^3\phi_4(1+q^3+q^4)$	$q^9\phi_4$	$q(1+2q^3+2q^4+q^6)/2$	$q(1+2q^4+q^6)/2$	$q\phi_4\phi_{12}/2$	$q\phi_1^2\phi_3^2/2$
x_3	1	.	$q^2\phi_6(1+q+q^2+q^4)$	$q^{10}\phi_6$	$q^3\phi_4\phi_8$	$-q^9\phi_1\phi_4$	$q^3\phi_4(1-q^3+q^4)$	$q^9\phi_4$	$q\phi_2^2\phi_6^2/2$	$q\phi_4\phi_{12}/2$	$q(1+2q^4+q^6)/2$	$q(1-2q^3+2q^4+q^6)/2$
x_4	1	.	$q^2(1+q^2+2q^4)$	q^{10}	$q^3\phi_3\phi_6$	q^9	$q^3\phi_3\phi_6$	q^9	$q\phi_2(1-q+q^2+q^3)/2$	$q\phi_8/2$	$q\phi_8/2$	$q\phi_1(-1-q-q^2+q^3)/2$
x_5	1	.	$q^2\phi_3\phi_6$	.	$q^3(1+q^2+q^3+q^4)$	.	q^3	q^9	$q(1+q^3+2q^4)/2$	$-q\phi_1\phi_3/2$	$q(1+q^3+2q^4)/2$	$-q\phi_1\phi_3/2$
x_6	1	.	$q^2\phi_3\phi_6$	.	$q^3(1+q^2-q^3+q^4)$	.	q^3	q^9	$q\phi_2\phi_6/2$	$q(1-q^3+2q^4)/2$	$q\phi_2\phi_6/2$	$q(1-q^3+2q^4)/2$
x_7	1	.	$q^2\phi_3\phi_6$	.	q^3	q^9	$q^3(2+q^2+q^4)$	.	$q\phi_2^2\phi_6/2$	$q\phi_2^2\phi_6/2$	$q\phi_1^2\phi_3/2$	$q\phi_1^2\phi_3/2$
x_8	1	.	$q^2\phi_4$	.	$q^3\phi_4$	.	q^3	.	$q\phi_2\phi_6/2$	$-q\phi_1\phi_3/2$	$q\phi_2\phi_6/2$	$-q\phi_1\phi_3/2$
x_9	1	.	$q^2(1+2q^2)$	.	$q^3\phi_4$	.	$2q^3$	.	$q\phi_2\phi_4/2$	$q(1+q+q^2-q^3)/2$	$q(1-q+q^2+q^3)/2$	$-q\phi_1\phi_4/2$
x_{10}	1	.	q^2	.	$q^3\phi_4$	.	.	.	$q(1-q+q^2+q^3)/2$	$-q\phi_1\phi_4/2$	$q\phi_2\phi_4/2$	$q(1+q+q^2-q^3)/2$
x_{11}	1	.	$q^2\phi_4$	.	q^3	.	$2q^3$	.	$q\phi_2/2$	$q\phi_2/2$	$-q\phi_1/2$	$-q\phi_1/2$
x_{12}	1	.	$q^2\phi_4$	.	q^3	.	$3q^3$	.	$q\phi_2^2/2$	$q\phi_2^2/2$	$q\phi_1^2/2$	$q\phi_1^2/2$
x_{13}	1	.	$q^2\phi_4$	.	q^3	.	q^3	.	$q\phi_4/2$	$q\phi_4/2$	$q\phi_4/2$	$q\phi_4/2$
x_{14}	1	.	q^2	.	q^3	.	$4q^3$	.	$q\phi_2(1+2q)/2$	$q\phi_2(1+2q)/2$	$q\phi_1(-1+2q)/2$	$q\phi_1(-1+2q)/2$
x_{15}	1	.	q^2	.	q^3	.	.	.	$q(1-q+2q^2)/2$	$q(1-q+2q^2)/2$	$q(1+q+2q^2)/2$	$q(1+q+2q^2)/2$
x_{16}	1	.	q^2	.	q^3	.	$2q^3$	.	$q\phi_2/2$	$q\phi_2/2$	$-q\phi_1/2$	$-q\phi_1/2$
x_{17}	1	.	q^2	.	q^3	.	.	.	$-q\phi_1/2$	$-q\phi_1/2$	$q\phi_2/2$	$q\phi_2/2$
x_{18}	1	.	q^2	.	q^3	.	q^3	.	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$	$-q\phi_1\phi_2/2$
x_{19}	1	.	q^2	.	q^3	.	.	.	$q\phi_2/2$	$-q\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$
x_{20}	1	.	q^2	.	.	.	q^3	.	$q\phi_2/2$	$q\phi_2/2$	$-q\phi_1/2$	$-q\phi_1/2$
x_{21}	1	.	q^2	.	.	.	.	.	$q\phi_2/2$	$-q\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$
x_{22}	1	.	q^2	.	.	.	.	.	$-q\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$	$q\phi_2/2$
x_{23}	1	.	.	.	.	.	.	.	q	q	.	.
x_{24}	1	.	.	.	.	.	.	.	.	.	q	q
x_{25}	1	.	.	.	.	.	.	.	.	.	.	.
x_{26}	1	.	.	.	.	.	.	.	.	.	.	.
x_{27}	1	.	.	.	.	.	.	.	.	.	.	.

TABLE 6.B. The unipotent character table of G ($p=3$) (2/5)

	$[4_5]$	$[2_4]$	$[2_2]$	$B_2[\epsilon]$	$[12_1]$	$[9_3]$	$[1_3]$
x_0	$q^{13}\phi_2^2\phi_6^2\phi_8/2$	$q^{13}\phi_4\phi_8\phi_{12}/2$	$q^{13}\phi_4\phi_8\phi_{12}/2$	$q^{13}\phi_1^2\phi_3^2\phi_8/2$	$q^4\phi_2^4\phi_3^2\phi_8\phi_{12}/24$	$q^4\phi_3^2\phi_4^2\phi_8\phi_{12}/8$	$q^4\phi_4^2\phi_6^2\phi_8\phi_{12}/8$
x_1	$q^{13}\phi_2\phi_6/2$	$q^{13}\phi_2\phi_6/2$	$-q^{13}\phi_1\phi_3/2$	$-q^{13}\phi_1\phi_3/2$	$q^4\phi_2^3\phi_3(1+2q+q^2-q^3+3q^5)/24$	$q^4\phi_2\phi_3\phi_4\phi_6(1+q+q^2-q^3)/8$	$q^4\phi_1\phi_3\phi_4\phi_6(-1+q-q^2-q^3)/8$
x_2	q^{13}	.	q^{13}	.	$q^4\phi_2^3\phi_3\phi_4(1+2q)/24$	$q^4\phi_2^2\phi_3\phi_4(1-q+2q^2)/8$	$q^4\phi_4(1-2q+3q^2+q^3-2q^4+q^5+2q^6)/8$
x_3	.	q^{13}	.	q^{13}	$q^4\phi_2^3(1+3q+4q^2-q^3-3q^4+2q^5)/24$	$q^4\phi_4(1+2q+3q^2-q^3-2q^4-q^5+2q^6)/8$	$q^4\phi_1^2\phi_4\phi_6(1+q+2q^2)/8$
x_4	.	.	.	.	$q^4\phi_2^2(1+2q)(1+2q+3q^2)/24$	$q^4\phi_2\phi_4(1+q+2q^2)/8$	$q^4\phi_1\phi_4(-1+q-2q^2)/8$
x_5	.	.	.	.	$q^4\phi_2^2\phi_3(1+3q)/24$	$-q^4\phi_1\phi_2^2\phi_3/8$	$-q^4\phi_1^3\phi_3/8$
x_6	.	.	.	.	$q^4\phi_2^2(1+4q+4q^2-3q^3)/24$	$q^4\phi_2^3\phi_6/8$	$q^4\phi_1^2\phi_2\phi_6/8$
x_7	.	.	.	.	$q^4\phi_2^2\phi_3/6$	$q^4\phi_3\phi_4/2$	$q^4\phi_4\phi_6/2$
x_8	.	.	.	.	$q^4\phi_2(1+2q)(1+3q)/24$	$-q^4\phi_1\phi_2(1+2q)/8$	$q^4\phi_1\phi_2(-1+2q)/8$
x_9	.	.	.	.	$q^4\phi_2^2/4$	$q^4\phi_2(3-q)/4$	$-q^4\phi_1(3+q)/4$
x_{10}	.	.	.	.	.	.	.
x_{11}	.	.	.	.	$q^4\phi_2(1+2q)/6$	$q^4\phi_2/2$	$-q^4\phi_1/2$
x_{12}	.	.	.	.	$q^4\phi_2/2$	$q^4(3+q)/2$	$q^4(3-q)/2$
x_{13}	.	.	.	.	.	.	.
x_{14}	.	.	.	.	$q^4(23+q^2)/24$	$q^4(23+q^2)/8$	$q^4(23+q^2)/8$
x_{15}	.	.	.	.	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{16}	.	.	.	.	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{17}	.	.	.	.	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{18}	.	.	.	.	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{19}	.	.	.	.	.	.	.
x_{20}	.	.	.	.	.	.	.
x_{21}	.	.	.	.	.	.	.
x_{22}	.	.	.	.	.	.	.
x_{23}	.	.	.	.	.	.	.
x_{24}	.	.	.	.	.	.	.
x_{25}	.	.	.	.	$2q^2/3$	.	.
x_{26}	.	.	.	.	$-q^2/3$	.	.
x_{27}	.	.	.	.	$-q^2/3$	.	.

TABLE 6.B. The unipotent character table of G ($p=3$) (3/5)

	$F_4^{II}[1]$	$[6_2]$	$[16_1]$	$B_2[\epsilon'']$	$[4_4]$
x_0	$q^4\phi_1^4\phi_6^2\phi_8\phi_{12}/24$	$q^4\phi_3^2\phi_4^2\phi_6^2\phi_8/12$	$q^4\phi_2^4\phi_4^2\phi_6^2\phi_{12}/4$	$q^4\phi_1^2\phi_3^2\phi_4\phi_8\phi_{12}/4$	$q^4\phi_2^2\phi_4\phi_6^2\phi_8\phi_{12}/4$
x_1	$-q^4\phi_1^3\phi_6(1-2q+q^2+q^3-3q^5)/24$	$q^4\phi_3\phi_4\phi_6(1+2q^2+3q^4)/12$	$q^4\phi_2^4\phi_4\phi_6^2/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-q^7)/4$	$q^4\phi_2\phi_6(1+q^3+q^4-q^7)/4$
x_2	$q^4\phi_1^3(-1+3q-4q^2-q^3+3q^4+2q^5)/24$	$q^4\phi_3\phi_4(1-q+3q^2+q^3+2q^4)/12$	$q^4\phi_2^2\phi_4(1+3q^3)/4$	$q^4\phi_1^2\phi_2\phi_3\phi_4/4$	$q^4(1+3q^3+3q^4+4q^6+q^7)/4$
x_3	$q^4\phi_1^3\phi_4\phi_6(-1+2q)/24$	$q^4\phi_4\phi_6(1+q+3q^2-q^3+2q^4)/12$	$q^4\phi_2^3\phi_4\phi_6/4$	$q^4(1-3q^3+3q^4+4q^6-q^7)/4$	$-q^4\phi_1\phi_2^2\phi_6\phi_4/4$
x_4	$q^4\phi_1^2(-1+2q)(-1+2q-3q^2)/24$	$q^4(1+4q^2+7q^4)/12$	$q^4\phi_2^3(1-q+2q^2)/4$	$q^4\phi_1(-1-q-q^2+q^3)/4$	$q^4\phi_2(1-q+q^2+q^3)/4$
x_5	$q^4\phi_1^2(1-4q+4q^2+3q^3)/24$	$q^4\phi_2\phi_3(1-2q+3q^2)/12$	$q^4\phi_2(1+q+3q^3-q^4)/4$	$q^4\phi_1^2\phi_2\phi_3/4$	$q^4\phi_1^2\phi_2\phi_3/4$
x_6	$-q^4\phi_1^2\phi_6(-1+3q)/24$	$-q^4\phi_1\phi_6(1+2q+3q^2)/12$	$q^4\phi_2^3\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$	$-q^4\phi_1\phi_2^2\phi_6/4$
x_7	$q^4\phi_1^2\phi_6/6$	$q^4\phi_3\phi_6/3$	$q^4\phi_2^2\phi_6/2$	$q^4\phi_1^2\phi_3/2$	$q^4\phi_2^2\phi_6/2$
x_8	$q^4\phi_1(1-2q)(-1+3q)/24$	$-q^4\phi_1\phi_2/12$	$q^4\phi_2^2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_9	$q^4\phi_1^2/4$	$q^4\phi_4/2$	$q^4\phi_2^2/2$	$-q^4\phi_1\phi_2/2$	$-q^4\phi_1\phi_2/2$
x_{10}	.	.	.	.	.
x_{11}	$q^4\phi_1(-1+2q)/6$	$-q^4\phi_1\phi_2/3$	$q^4\phi_2/2$	$-q^4\phi_1/2$	$q^4\phi_2/2$
x_{12}	$-q^4\phi_1/2$	q^4	$q^4\phi_2/2$	$-q^4\phi_1/2$	$q^4\phi_2/2$
x_{13}	.	.	$q^4\phi_2/2$	$q^4\phi_2/2$	$-q^4\phi_1/2$
x_{14}	$q^4(23+q^2)/24$	$q^4(23+q^2)/12$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{15}	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/12$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{16}	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/12$	$q^4(3+q^2)/4$	$q^4(3+q^2)/4$	$q^4(3+q^2)/4$
x_{17}	$-q^4\phi_1\phi_2/24$	$-q^4\phi_1\phi_2/12$	$q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/4$
x_{18}	$q^4\phi_1\phi_2/24$	$q^4\phi_1\phi_2/12$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{19}	.	.	.	.	.
x_{20}	.	.	.	.	.
x_{21}	.	.	.	.	.
x_{22}	.	.	.	.	.
x_{23}	.	.	.	.	.
x_{24}	.	.	.	.	.
x_{25}	$2q^2/3$	$-2q^2/3$	.	.	.
x_{26}	$-q^2/3$	$q^2/3$	.	.	.
x_{27}	$-q^2/3$	$q^2/3$	.	.	.

TABLE 6.B. The unipotent character table of G ($p=3$) (4/5)

	$F_4[-1]$	$[9_2]$	$[4_1]$	$[1_2]$	$F_4^{I}[1]$
x_0	$q^4\phi_1^4\phi_3^2\phi_4^2\phi_{12}/4$	$q^4\phi_3^2\phi_4^2\phi_8\phi_{12}/8$	$q^4\phi_2^4\phi_6^2\phi_8\phi_{12}/8$	$q^4\phi_4^2\phi_6^2\phi_8\phi_{12}/8$	$q^4\phi_1^4\phi_3^2\phi_8\phi_{12}/8$
x_1	$q^4\phi_1^4\phi_3^2\phi_4/4$	$q^4\phi_3\phi_4(1+q+q^2+q^4+q^5+3q^6)/8$	$q^4\phi_2^3\phi_6(1-q^2+q^3+2q^4-q^5)/8$	$q^4\phi_4\phi_6(1-q+q^2+q^4-q^5+3q^6)/8$	$q^4\phi_1^3\phi_3(-1+q^2+q^3-2q^4-q^5)/8$
x_2	$-q^4\phi_1^3\phi_3\phi_4/4$	$q^4\phi_3\phi_4(1+q+q^2-q^3+2q^4)/8$	$q^4\phi_2^2(1-q^2+q^3+4q^4-3q^5+2q^6)/8$	$q^4\phi_1^2\phi_3\phi_4(1-q+2q^2)/8$	$q^4\phi_1^3\phi_3(-1+q^2+2q^3)/8$
x_3	$-q^4\phi_1^2\phi_4(-1+3q^3)/4$	$q^4\phi_2^2\phi_4\phi_6(1+q+2q^2)/8$	$q^4\phi_2^3\phi_6(1-q^2+2q^3)/8$	$q^4\phi_4\phi_6(1-q+q^2+q^3+2q^4)/8$	$q^4\phi_1^2(1-q^2-q^3+4q^4+3q^5+2q^6)/8$
x_4	$-q^4\phi_1^3(1+q+2q^2)/4$	$q^4\phi_2\phi_4(1+q+2q^2)/8$	$q^4\phi_2^2(1-q^2+2q^3)/8$	$q^4\phi_1\phi_4(-1+q-2q^2)/8$	$q^4\phi_1^2(1-q^2-2q^3)/8$
x_5	$-q^4\phi_1^3\phi_3/4$	$q^4\phi_2\phi_3(1+3q^2)/8$	$-q^4\phi_1\phi_2^2\phi_3/8$	$q^4(1-2q+5q^2-q^3+2q^4+3q^5)/8$	$-q^4\phi_1^3\phi_3/8$
x_6	$q^4\phi_1(-1+q+3q^3+q^4)/4$	$q^4(1+2q+5q^2+q^3+2q^4-3q^5)/8$	$q^4\phi_2^3\phi_6/8$	$-q^4\phi_1\phi_6(1+3q^2)/8$	$q^4\phi_1^2\phi_2\phi_6/8$
x_7	$q^4\phi_1^2\phi_3/2$	.	.	.	.
x_8	$q^4\phi_1^2/4$	$q^4\phi_2(1+q+2q^2)/8$	$-q^4\phi_1\phi_2(1+2q)/8$	$q^4\phi_1(-1+q-2q^2)/8$	$q^4\phi_1\phi_2(-1+2q)/8$
x_9	$q^4\phi_1^2/2$	$q^4\phi_2^2/4$	$q^4\phi_2^2/4$	$q^4\phi_1^2/4$	$q^4\phi_1^2/4$
x_{10}	.	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$
x_{11}	$-q^4\phi_1/2$	.	.	.	.
x_{12}	$-q^4\phi_1/2$	.	.	.	.
x_{13}	$-q^4\phi_1/2$	$q^4\phi_2/2$	$q^4\phi_2/2$	$-q^4\phi_1/2$	$-q^4\phi_1/2$
x_{14}	$-q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{15}	$-q^4\phi_1\phi_2/4$	$q^4(7+q^2)/8$	$q^4(7+q^2)/8$	$q^4(7+q^2)/8$	$q^4(7+q^2)/8$
x_{16}	$q^4(3+q^2)/4$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{17}	$q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$	$-q^4\phi_1\phi_2/8$
x_{18}	$-q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$	$q^4\phi_1\phi_2/8$
x_{19}	.	.	.	.	.
x_{20}	.	.	.	.	.
x_{21}	.	.	.	.	.
x_{22}	.	.	.	.	.
x_{23}	.	.	.	.	.
x_{24}	.	.	.	.	.
x_{25}	.	.	.	.	.
x_{26}	.	.	.	.	.
x_{27}	.	.	.	.	.

TABLE 6.B. The unipotent character table of G ($p=3$) (5/5)

	$B_2[r]$	$[6_1]$	$F_4[\theta]$	$F_4[\theta^2]$	$[4_3]$	$B_2[\epsilon']$	$F_4[i], F_4[-i]$
x_0	$q^4\phi_1^2\phi_2^2\phi_3^2\phi_6^2\phi_8/4$	$q^4\phi_3^2\phi_6^2\phi_8\phi_{12}/3$	$q^4\phi_1^4\phi_2^4\phi_4^2\phi_8/3$	$q^4\phi_1^4\phi_2^4\phi_4^2\phi_8/3$	$q^4\phi_2^2\phi_4\phi_6^2\phi_8\phi_{12}/4$	$q^4\phi_1^2\phi_3^2\phi_4\phi_8\phi_{12}/4$	$q^4\phi_1^4\phi_2^4\phi_3^2\phi_6^2/4$
x_1	$-q^4\phi_1\phi_2\phi_3\phi_6\phi_8/4$	$q^4\phi_3\phi_6(1+2q^4)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4\phi_2\phi_6(1+q^3+q^4+2q^6+q^7)/4$	$q^4\phi_1\phi_3(-1+q^3-q^4-2q^6+q^7)/4$	$-q^4\phi_1^3\phi_2^3\phi_3\phi_6/4$
x_2	$q^4\phi_1^2\phi_3(1+q+q^2+q^3+2q^4)/4$	$q^4\phi_3(1-q+q^2+2q^4+q^5-q^6)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4\phi_4(1-q^2+q^3+2q^4-q^5)/4$	$q^4\phi_1^2\phi_3(1+q+q^2-q^3)/4$	$-q^4\phi_1^3\phi_2^2\phi_3/4$
x_3	$q^4\phi_2^2\phi_6(1-q+q^2-q^3+2q^4)/4$	$q^4\phi_6(1+q+q^2+2q^4-q^5-q^6)/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$-q^4\phi_1^3\phi_2^3\phi_4/3$	$q^4\phi_2^2\phi_6(1-q+q^2+q^3)/4$	$q^4\phi_4(1-q^2-q^3+2q^4+q^5)/4$	$q^4\phi_1^2\phi_2^3\phi_6/4$
x_4	$-q^4\phi_1\phi_2\phi_4/4$	$q^4(1+q^2+4q^4)/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_2(1-q+q^2+q^3)/4$	$q^4\phi_1(-1-q-q^2+q^3)/4$	$q^4\phi_1^2\phi_2^2/4$
x_5	$-q^4\phi_1\phi_3\phi_4/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4(1+q^2+3q^3+4q^4-q^5)/4$	$-q^4\phi_1\phi_3\phi_4/4$	$q^4\phi_1^2\phi_2\phi_3/4$
x_6	$q^4\phi_2\phi_4\phi_6/4$	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_2\phi_4\phi_6/4$	$q^4(1+q^2-3q^3+4q^4+q^5)/4$	$-q^4\phi_1\phi_2^2\phi_6/4$
x_7	.	$q^4\phi_3\phi_6/3$	$q^4\phi_1^2\phi_2^2/3$	$q^4\phi_1^2\phi_2^2/3$	.	.	.
x_8	$-q^4\phi_1\phi_2/4$	$q^4(1+2q^2)/3$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$q^4\phi_2(1-q+2q^2)/4$	$-q^4\phi_1(1+q+2q^2)/4$	$-q^4\phi_1\phi_2/4$
x_9	$-q^4\phi_1\phi_2/2$	.	.	.	.	.	.
x_{10}	q^4	.	.	.	$q^4\phi_4/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$
x_{11}	.	$q^4(1+2q^2)/3$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	.	.	.
x_{12}	.	.	.	.	.	.	.
x_{13}	q^4	.	.	.	.	.	.
x_{14}	$q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{15}	$q^4(7+q^2)/4$	$q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{16}	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/4$	$q^4\phi_1\phi_2/4$
x_{17}	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$-q^4\phi_1\phi_2/3$	$q^4(3+q^2)/4$	$q^4(3+q^2)/4$	$q^4(3+q^2)/4$
x_{18}	$q^4\phi_1\phi_2/4$	$q^4(2+q^2)/3$	$q^4(2+q^2)/3$	$q^4(2+q^2)/3$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$	$-q^4\phi_1\phi_2/4$
x_{19}	.	.	.	.	.	.	.
x_{20}	.	.	.	.	.	.	.
x_{21}	.	.	.	.	.	.	.
x_{22}	.	.	.	.	.	.	.
x_{23}	.	.	.	.	.	.	.
x_{24}	.	.	.	.	.	.	.
x_{25}	.	$-2q^2/3$	$q^2/3$	$q^2/3$	.	.	.
x_{26}	.	$q^2/3$	$(1+3\theta)q^2/3$	$(1+3\theta^2)q^2/3$	.	.	.
x_{27}	.	$q^2/3$	$(1+3\theta^2)q^2/3$	$(1+3\theta)q^2/3$	.	.	.

TABLE 7. The table $g_{cc'}$

	z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}
x_0	$\phi_2\phi_3\phi_4\phi_6\phi_8\phi_{12}$	.	.	.	.	.	.	.	.	.	.	.
x_1	$q^4+\phi_2\phi_4\phi_8$	$q^5\phi_2\phi_3\phi_6$	.	.	.	.	.	.	.	.	.	.
x_2	$\phi_2\phi_3\phi_6$	.	$q^4\phi_2\phi_3\phi_4\phi_6$	.	.	.	.	.	.	.	.	.
x_3	ϕ_5	q^5	$q^4\phi_2\phi_4$	$q^6\phi_2\phi_4$	.	.	.	.	.	.	.	.
x_4	ϕ_2	$q^2\phi_2$	$q^2(1+q+2q^2+q^3)$	$q^2\phi_1\phi_2\phi_3$	$q^6\phi_2$	.	.	.	.	.	.	.
x_5	.	.	$\phi_3\phi_4$	$q^2\phi_1\phi_3$	.	$q^4\phi_2\phi_3$	.	.	.	.	.	.
x_6	.	.	$\phi_2^2\phi_6$	$q^2\phi_2\phi_6$	.	.	$q^4\phi_2\phi_6$	.	.	.	.	.
x_7	1	$2q\phi_3$	.	$q\phi_1\phi_2\phi_3$	$q^3\phi_1\phi_3$	.	.	.	.	.	.	.
x_8	1	.	.	$q\phi_2^2\phi_6$	$q^3\phi_2\phi_6$	.	.	.	.	.	.	.
x_9	.	1	$\phi_2\phi_4$	.	$\phi_1\phi_2\phi_4$	.	.	$q^3\phi_2\phi_4$	.	.	.	.
x_{10}	.	1	$\phi_2\phi_4$	.	$\phi_1\phi_2\phi_4$	.	.	.	$q^3\phi_2\phi_4$	.	.	.
x_{11}	.	.	ϕ_2	.	$q^2\phi_2$	$q^3\phi_2(1+2q)/2$	$q^3\phi_1/2$	.	.	.	.	.
x_{12}	.	ϕ_2	1	$\phi_1\phi_2$	$q^3\phi_2$	.	.	.	.	.	.	.
x_{13}	.	.	ϕ_2	1	$\phi_1\phi_2$	$q^3\phi_2$	.	$q^3/2$	$q^3/2$	$q^4\phi_2$	.	.
x_{14}	.	1	1	$q\phi_2$	$\phi_1\phi_3$	.	.	$q^3\phi_2/2$	$q^3\phi_2/2$	.	.	.
x_{15}	.	.	1	q	$q^2\phi_2$	$q^4/2$	$q^4/2$	.	.	.	.	.
x_{16}	.	.	1	ϕ_2	$\phi_1\phi_2$	$q^3/2$	$q^3/2$	$q^3/2$	$q^3/2$	q^4	.	.
x_{17}	.	.	.	3	$-2+q$	$q\phi_2/2$	$q\phi_1/2$	.	.	$3q^3$	.	.
x_{18}	.	.	.	1	q	$q\phi_2/2$	$q\phi_1/2$	.	.	q^3	.	.
x_{19}	.	.	.	.	ϕ_2	$q\phi_2/2$	$q\phi_1/2$	.	.	.	.	.
x_{20}	.	.	.	.	.	ϕ_2	.	1	.	$\phi_1\phi_2$	$q^2\phi_2$	.
x_{21}	.	.	.	.	.	ϕ_2	.	.	1	$\phi_1\phi_2$	.	$q^2\phi_2$
x_{22}	.	.	.	.	1	.	.	ϕ_2	.	$\phi_1\phi_2$	.	.
x_{23}	.	.	.	.	1	.	.	.	ϕ_2	$\phi_1\phi_2$	.	.
x_{24}	.	.	.	.	.	1	.	2	.	$-2+q$	$2q^2$	.
x_{25}	.	.	.	.	.	1	.	.	.	q	q^2	q^2
x_{26}	.	.	.	.	.	1	.	.	2	$-2+q$	.	$2q^2$
x_{27}	.	.	.	.	.	.	1	1	1	$-2+q$	.	.
x_{28}	.	.	.	.	.	.	1	.	.	q	.	.
x_{29}	.	.	.	.	.	.	.	.	.	1	$q/2$	$q/2$
x_{30}	.	.	.	.	.	.	.	.	.	1	$q/2$	$q/2$
x_{31}	.	.	.	.	.	.	.	.	.	.	1	.
x_{32}	.	.	.	.	.	.	.	.	.	.	.	1
x_{33}	.	.	.	.	.	.	.	.	.	.	1	.
x_{34}	.	.	.	.	.	.	.	.	.	.	.	1

TABLE 8. The values at unipotent elements of the almost characters of $Sp(8, 2^n)$ grouped according to families (1/2)

	$\binom{4}{\phi}$	$\binom{13}{1}$	$\binom{12}{2}$	$\binom{013}{13}$	$\binom{01234}{1234}$	$\binom{0124}{123}$	$\binom{0123}{124}$	$\binom{1234}{012}$	$\binom{01234}{12}$	$\binom{023}{12}$	$\binom{012}{23}$	$\binom{123}{02}$	$\binom{0123}{2}$
u_0	1	$q^3\phi_4^2\phi_8$	$q^4\phi_3\phi_6\phi_8$	$q^5\phi_4^2\phi_8$	q^{16}	$q^9\phi_4\phi_8$	$q^{10}\phi_3\phi_6$	q^{12}	.	$q^6\phi_3\phi_6\phi_8$	$q^8\phi_8$	$q^7\phi_4\phi_8$	.
u_1	1	$q^3\phi_3\phi_4\phi_6$	$q^4\phi_3\phi_6$	$q^5\phi_4$	.	q^9	.	q^{12}	.	$q^6\phi_3\phi_6$	.	$q^7\phi_3\phi_6$	.
u_2	1	$q^3\phi_4^2$	$q^4\phi_3\phi_6$	$q^5\phi_4^2$	.	q^9	q^{10}	.	.	$q^6\phi_4$	q^8	q^7	.
u_3	1	$q^3\phi_4^2$	$q^4\phi_4$	$q^5\phi_4$	.	q^9	.	.	.	$q^6\phi_4$	.	q^7	.
u_4	1	$q^3\phi_4$	q^4	$q^5\phi_4$	.	.	.	.	.	q^6	q^8	.	.
u_5	1	$q^3(1+2q^2)$	$q^4\phi_4$	q^5	.	.	.	.	.	q^6	.	q^7	.
u_6	1	$q^3\phi_4$	.	.	.	.	.	.	q^7	.	.	.	q^5
u_7	1	$q^3\phi_4$	.	.	.	.	.	.	$-q^7$	.	.	.	$-q^5$
u_8	1	$q^3\phi_4$	q^4	q^5	.	.	.	.	.	q^6	.	.	.
u_9	1	q^3	q^4	q^5	.	.	.	.	.	.	.	.	.
u_{10}	1	q^3	q^4	q^5	.	.	.	.	.	.	.	.	.
u_{11}	1	q^3	.	.	.	.	.	.	.	.	.	.	.
u_{12}	1	q^3	q^4	.	.	.	.	.	.	.	.	.	.
u_{13}	1	q^3	.	.	.	.	.	.	.	.	.	.	q^5
u_{14}	1	q^3	.	.	.	.	.	.	.	.	.	.	$-q^5$
u_{15}	1	q^3	.	.	.	.	.	.	.	.	.	.	.
u_{16}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{17}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{18}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{19}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{20}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{21}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{22}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{23}	1	.	.	.	.	.	.	.	.	.	.	.	.
u_{24}	1	.	.	.	.	.	.	.	.	.	.	.	.

TABLE 8. The values at unipotent elements of the almost characters of $Sp(8, 2^n)$ grouped according to families (2/2)

	$\binom{014}{12}$	$\binom{012}{14}$	$\binom{124}{01}$	$\binom{0124}{1}$	$\binom{03}{2}$	$\binom{02}{3}$	$\binom{23}{0}$	$\binom{023}{-}$	$\binom{04}{1}$	$\binom{01}{4}$	$\binom{14}{0}$	$\binom{014}{-}$
u_0	$q^4\phi_3\phi_6\phi_8$	$q^6\phi_3\phi_6$	$q^6\phi_3\phi_6$	.	$q^2\phi_3\phi_6\phi_8$	$q^3\phi_4\phi_8$	$q^4\phi_8$	.	$q\phi_4\phi_8$	q^4	$q^2\phi_3\phi_6$	.
u_1	$q^4\phi_3\phi_6$	.	$q^6\phi_3\phi_6$	.	$q^2\phi_3\phi_6$	q^3	$q^4\phi_8$	.	$q\phi_3\phi_6$	.	$q^2\phi_3\phi_6$	.
u_2	$q^4\phi_3\phi_6$	$q^6\phi_4$	q^6	.	$q^2(1+q^2+2q^4)$	$q^3\phi_3\phi_6$	q^4	.	$q\phi_3\phi_6$	q^4	$q^2\phi_4$	.
u_3	$q^4\phi_3\phi_6$	.	q^6	.	$q^2\phi_3\phi_6$	q^3	q^4	.	$q\phi_3\phi_6$	.	$q^2\phi_4$	.
u_4	q^4	q^6	.	.	$q^2\phi_3\phi_6$	$q^3\phi_4$	q^4	.	$q\phi_4$	q^4	q^2	.
u_5	q^4	.	q^6	.	$q^2\phi_4$	q^3	q^4	.	$q\phi_4$	.	$q^2\phi_4$	.
u_6	q^4	.	q^6	$q^4\phi_4$	q^2	.	q^4	q^5	$q\phi_4$	.	$q^2\phi_4$	q^3
u_7	q^4	.	q^6	$-q^4\phi_4$	q^2	.	q^4	$-q^5$	$q\phi_4$	.	$q^2\phi_4$	$-q^3$
u_8	q^4	.	.	.	$q^2\phi_4$	q^3	q^4	.	$q\phi_4$	.	q^2	.
u_9	q^4	q^5	.	.	$q^2\phi_4$	$q^3\phi_2$	.	.	$q\phi_4$	q^3	q^2	.
u_{10}	q^4	$-q^5$	.	.	$q^2\phi_4$	$-q^3\phi_1$	.	.	$q\phi_4$	$-q^3$	q^2	.
u_{11}	q^4	.	.	.	q^2	.	.	.	$q\phi_4$	.	q^2	.
u_{12}	.	.	.	.	q^2	q^3	.	.	q	.	q^2	.
u_{13}	.	.	.	q^4	q^2	.	q^4	.	q	.	q^2	q^3
u_{14}	.	.	.	$-q^4$	q^2	.	q^4	.	q	.	q^2	$-q^3$
u_{15}	.	.	.	.	q^2	.	.	.	q	.	q^2	.
u_{16}	.	.	.	.	q^2	q^3	.	.	q	q^2	.	.
u_{17}	.	.	.	q^3	.	.	.	q^2	q	.	q^2	q^2
u_{18}	.	.	.	$-q^3$	.	.	.	$-q^2$	q	.	q^2	$-q^2$
u_{19}	.	.	.	.	q^2	.	.	.	q	q^2	.	.
u_{20}	.	.	.	.	q^2	.	.	.	q	$-q^2$	.	.
u_{21}	.	.	.	.	.	.	.	q^2	q	.	.	.
u_{22}	.	.	.	.	.	.	.	$-q^2$	q	.	.	.
u_{23}	.	.	.	.	.	.	.	.	.	.	.	q
u_{24}	.	.	.	.	.	.	.	.	.	.	.	$-q$

TABLE 9. The values at unipotent elements of the unipotent characters of $Sp(8, 2^n)$ that belong to the 4-element families (1/2)

	$\binom{0124}{123}$	$\binom{0123}{124}$	$\binom{1234}{012}$	$\binom{01234}{12}$	$\binom{023}{12}$	$\binom{012}{23}$	$\binom{123}{02}$	$\binom{0123}{2}$	$\binom{014}{12}$	$\binom{012}{14}$	$\binom{124}{01}$	$\binom{0124}{1}$
u_0	$q^9\phi_2^2\phi_4\phi_6/2$	$q^9\phi_3\phi_8/2$	$q^9\phi_6\phi_8/2$	$q^9\phi_1^2\phi_3\phi_4/2$	$q^6\phi_3\phi_4\phi_8/2$	$q^6\phi_4\phi_6\phi_8/2$	$q^6\phi_2^2\phi_6\phi_8/2$	$q^6\phi_1^2\phi_3\phi_8/2$	$q^4\phi_3\phi_4^2\phi_6/2$	$q^4\phi_3\phi_6\phi_8/2$	$q^4\phi_3\phi_6\phi_8/2$	$q^4\phi_1^2\phi_2^2\phi_3\phi_6/2$
u_1	$q^9\phi_2\phi_6/2$	$-q^9\phi_1\phi_3/2$	$q^9\phi_2\phi_6/2$	$-q^9\phi_1\phi_3/2$	$q^6\phi_2\phi_3\phi_6/2$	$-q^6\phi_1\phi_3\phi_6/2$	$q^6\phi_2\phi_3\phi_6/2$	$-q^6\phi_1\phi_3\phi_6/2$	$q^4\phi_3\phi_4\phi_6/2$	$-q^4\phi_1\phi_2\phi_3\phi_6/2$	$q^4\phi_3\phi_4\phi_6/2$	$-q^4\phi_1\phi_2\phi_3\phi_6/2$
u_2	$q^9\phi_2/2$	$q^9\phi_2/2$	$-q^9\phi_1/2$	$-q^9\phi_1/2$	$q^6(1+q+2q^2)/2$	$q^6(1-q+2q^2)/2$	$q^6\phi_2/2$	$-q^6\phi_1/2$	$q^4\phi_4(1+2q^2)/2$	$q^4(1+q^2+2q^4)/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$
u_3	$q^9/2$	$q^9/2$	$q^9/2$	$q^9/2$	$q^6\phi_3/2$	$q^6\phi_6/2$	$q^6\phi_3/2$	$q^6\phi_6/2$	$q^4\phi_4^2/2$	$q^4\phi_8/2$	$q^4\phi_4^2/2$	$q^4\phi_8/2$
u_4	·	·	·	·	$q^6\phi_4/2$	$q^6\phi_4/2$	$-q^6\phi_1\phi_2/2$	$-q^6\phi_1\phi_2/2$	$q^4\phi_4/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$	$-q^4\phi_1\phi_2/2$
u_5	·	·	·	·	$q^6\phi_2/2$	$-q^6\phi_1/2$	$q^6\phi_2/2$	$-q^6\phi_1/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$	$q^4\phi_4/2$	$-q^4\phi_1\phi_2/2$
u_6	$q^7/2$	$-q^7/2$	$-q^7/2$	$q^7/2$	$q^5/2$	$-q^5/2$	$-q^5/2$	$q^5/2$	$q^4\phi_4$	$-q^6$	·	q^4
u_7	$-q^7/2$	$q^7/2$	$q^7/2$	$-q^7/2$	$-q^5/2$	$q^5/2$	$q^5/2$	$-q^5/2$	·	q^4	$q^4\phi_4$	$-q^6$
u_8	·	·	·	·	$q^6/2$	$q^6/2$	$q^6/2$	$q^6/2$	$q^4/2$	$q^4/2$	$q^4/2$	$q^4/2$
u_9	·	·	·	·	·	·	·	·	$q^4\phi_2/2$	$q^4\phi_2/2$	$-q^4\phi_1/2$	$-q^4\phi_1/2$
u_{10}	·	·	·	·	·	·	·	·	$-q^4\phi_1/2$	$-q^4\phi_1/2$	$q^4\phi_2/2$	$q^4\phi_2/2$
u_{11}	·	·	·	·	·	·	·	·	$q^4/2$	$q^4/2$	$q^4/2$	$q^4/2$
u_{12}	·	·	·	·	·	·	·	·	·	·	·	·
u_{13}	·	·	·	·	$q^5/2$	$-q^5/2$	$-q^5/2$	$q^5/2$	$q^4/2$	$-q^4/2$	$-q^4/2$	$q^4/2$
u_{14}	·	·	·	·	$-q^5/2$	$q^5/2$	$q^5/2$	$-q^5/2$	$-q^4/2$	$q^4/2$	$q^4/2$	$-q^4/2$
u_{15}	·	·	·	·	·	·	·	·	·	·	·	·
u_{16}	·	·	·	·	·	·	·	·	·	·	·	·
u_{17}	·	·	·	·	·	·	·	·	$q^3/2$	$-q^3/2$	$-q^3/2$	$q^3/2$
u_{18}	·	·	·	·	·	·	·	·	$-q^3/2$	$q^3/2$	$q^3/2$	$-q^3/2$
u_{19}	·	·	·	·	·	·	·	·	·	·	·	·
u_{20}	·	·	·	·	·	·	·	·	·	·	·	·
u_{21}	·	·	·	·	·	·	·	·	·	·	·	·
u_{22}	·	·	·	·	·	·	·	·	·	·	·	·
u_{23}	·	·	·	·	·	·	·	·	·	·	·	·
u_{24}	·	·	·	·	·	·	·	·	·	·	·	·

TABLE 9. The values at unipotent elements of the unipotent characters of $Sp(8, 2^n)$ that belong to the 4-element families (2/2)

	$\binom{03}{2}$	$\binom{02}{3}$	$\binom{23}{0}$	$\binom{023}{-}$	$\binom{04}{1}$	$\binom{01}{4}$	$\binom{14}{0}$	$\binom{014}{-}$
u_0	$q^2\phi_3\phi_4\phi_8/2$	$q^2\phi_2^2\phi_6\phi_8/2$	$q^2\phi_4\phi_6\phi_8/2$	$q^2\phi_1^2\phi_3\phi_8/2$	$q\phi_2^2\phi_4\phi_6/2$	$q\phi_6\phi_8/2$	$q\phi_3\phi_8/2$	$q\phi_1^2\phi_3\phi_4/2$
u_1	$q^2\phi_3(1+q^2-q^3+q^4)/2$	$q^2\phi_2\phi_6(1+q-q^3)/2$	$q^2\phi_6(1+q^2+q^3+q^4)/2$	$q^2\phi_1\phi_3(-1+q-q^3)/2$	$q\phi_2\phi_3\phi_6/2$	$-q\phi_1\phi_3\phi_6/2$	$q\phi_2\phi_3\phi_6/2$	$-q\phi_1\phi_3\phi_6/2$
u_2	$q^2(\phi_2\phi_3\phi_6+q^2\phi_4)/2$	$q^2\phi_2(1+q^3+q^4)/2$	$q^2(-\phi_1\phi_3\phi_6+q^2\phi_4)/2$	$q^2\phi_1(-1+q^3-q^4)/2$	$q\phi_2(1+q^2+q^3)/2$	$q(1-q+q^2+q^4)/2$	$q(1+q+q^2+q^4)/2$	$q\phi_1(-1-q^2+q^3)/2$
u_3	$q^2(1+q+2q^2+q^4)/2$	$q^2(1+q+q^4)/2$	$q^2(1-q+2q^2+q^4)/2$	$q^2(1-q+q^4)/2$	$q\phi_5/2$	$q\phi_{10}/2$	$q\phi_5/2$	$q\phi_{10}/2$
u_4	$q^2\phi_3\phi_4/2$	$q^2\phi_2^2\phi_6/2$	$q^2\phi_4\phi_6/2$	$q^2\phi_1^2\phi_3/2$	$q\phi_2\phi_4/2$	$q(1-q+q^2+q^3)/2$	$q(1+q+q^2-q^3)/2$	$-q\phi_1\phi_4/2$
u_5	$q^2(1+q+2q^2)/2$	$q^2\phi_2/2$	$q^2(1-q+2q^2)/2$	$-q^2\phi_1/2$	$q\phi_2\phi_4/2$	$-q\phi_1\phi_4/2$	$q\phi_2\phi_4/2$	$-q\phi_1\phi_4/2$
u_6	$q^2(1+q^2+q^3)/2$	$q^2(1-q^2-q^3)/2$	$q^2(1+q^2-q^3)/2$	$q^2(1-q^2+q^3)/2$	$q(1+q+2q^2+q^3)/2$	$q(1-q-q^3)/2$	$q(1+q+q^3)/2$	$q(1-q+2q^2-q^3)/2$
u_7	$q^2(1+q^2-q^3)/2$	$q^2(1-q^2+q^3)/2$	$q^2(1+q^2+q^3)/2$	$q^2(1-q^2-q^3)/2$	$q(1+q+q^3)/2$	$q(1-q+2q^2-q^3)/2$	$q(1+q+2q^2+q^3)/2$	$q(1-q-q^3)/2$
u_8	$q^2(1+q+2q^2)/2$	$q^2\phi_2/2$	$q^2(1-q+2q^2)/2$	$-q^2\phi_1/2$	$q\phi_3/2$	$q\phi_6/2$	$q\phi_3/2$	$q\phi_6/2$
u_9	$q^2(1+q+2q^2)/2$	$q^2(1+q+2q^2)/2$	$-q^2\phi_1/2$	$-q^2\phi_1/2$	$q(1+q+2q^2)/2$	$q(1-q+2q^2)/2$	$q\phi_2/2$	$-q\phi_1/2$
u_{10}	$q^2\phi_2/2$	$q^2\phi_2/2$	$q^2(1-q+2q^2)/2$	$q^2(1-q+2q^2)/2$	$q\phi_2/2$	$-q\phi_1/2$	$q(1+q+2q^2)/2$	$q(1-q+2q^2)/2$
u_{11}	$q^2/2$	$q^2/2$	$q^2/2$	$q^2/2$	$q\phi_3/2$	$q\phi_6/2$	$q\phi_3/2$	$q\phi_6/2$
u_{12}	$q^2\phi_2/2$	$q^2\phi_2/2$	$-q^2\phi_1/2$	$-q^2\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$
u_{13}	$q^2\phi_4/2$	$-q^2\phi_1\phi_2/2$	$q^2\phi_4/2$	$-q^2\phi_1\phi_2/2$	$q\phi_3/2$	$q(1-q-q^2)/2$	$q(1+q-q^2)/2$	$q\phi_6/2$
u_{14}	$q^2\phi_4/2$	$-q^2\phi_1\phi_2/2$	$q^2\phi_4/2$	$-q^2\phi_1\phi_2/2$	$q(1+q-q^2)/2$	$q\phi_6/2$	$q\phi_3/2$	$q(1-q-q^2)/2$
u_{15}	$q^2/2$	$q^2/2$	$q^2/2$	$q^2/2$	$q\phi_2/2$	$-q\phi_1/2$	$q\phi_2/2$	$-q\phi_1/2$
u_{16}	$q^2\phi_2/2$	$q^2\phi_2/2$	$-q^2\phi_1/2$	$-q^2\phi_1/2$	$q\phi_2/2$	$q\phi_2/2$	$-q\phi_1/2$	$-q\phi_1/2$
u_{17}	$q^2/2$	$-q^2/2$	$-q^2/2$	$q^2/2$	$q(1+2q)/2$	$q(1-2q)/2$	$q/2$	$q/2$
u_{18}	$-q^2/2$	$q^2/2$	$q^2/2$	$-q^2/2$	$q/2$	$q/2$	$q(1+2q)/2$	$q(1-2q)/2$
u_{19}	$q^2/2$	$q^2/2$	$q^2/2$	$q^2/2$	$q\phi_2/2$	$q\phi_2/2$	$-q\phi_1/2$	$-q\phi_1/2$
u_{20}	$q^2/2$	$q^2/2$	$q^2/2$	$q^2/2$	$-q\phi_1/2$	$-q\phi_1/2$	$q\phi_2/2$	$q\phi_2/2$
u_{21}	$q^2/2$	$-q^2/2$	$-q^2/2$	$q^2/2$	$q/2$	$q/2$	$q/2$	$q/2$
u_{22}	$-q^2/2$	$q^2/2$	$q^2/2$	$-q^2/2$	$q/2$	$q/2$	$q/2$	$q/2$
u_{23}	.	.	.	.	$q/2$	$-q/2$	$-q/2$	$q/2$
u_{24}	.	.	.	.	$-q/2$	$q/2$	$q/2$	$-q/2$

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