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Law of Large Numbers for Wiener Measure with Density Having Two Large Deviation Minimizers

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Abstract. This paper discusses the situation that the large deviation rate functional has two distinct minimizers, for a model described by Wiener measures with certain densities involving a scaling. The motivation comes from the study of the so-called $\nabla\varphi$ interface model with weak self potentials. The pinned Wiener measures case was discussed by [3].

1. Introduction and results

In this paper, we are interested in the law of large numbers for a sequence of probability measures $\{\mu_N\}_{N=1,2,\dots}$ on the space $\mathcal{C} = C(I, \mathbf{R})$, $I = [0, 1]$, under the critical situation that the rate functional of the corresponding large deviation principle admits two minimizers. The sequence of probability measures $\{\mu_N\}_{N=1,2,\dots}$ is defined from the Wiener measures involving a proper scaling with densities determined by a class of potentials W. Such measures naturally arise as a continuous analog of the $\nabla\varphi$ interface model with weak self potentials in one dimension. The relation to the $\nabla\varphi$ interface model was stated in section 3 in [3]. The large deviation principle (LDP) is easily established for $\{\mu_N\}$ and the (unnormalized) rate functional is given by Σ^W, see (3) below. The purpose of the present paper is to prove the law of large numbers (LLN) for $\{\mu_N\}$ under the situation that Σ^W admits two minimizers $\bar{h}$ and $\hat{h}$. We shall specify the conditions for the potentials W, under which the limit points under μ_N are either $\bar{h}$ or $\hat{h}$ as $N \to \infty$.

We now formulate our problem more precisely. Let ν_0 be the law on the space $\mathcal{C}$ of the Brownian motion such that $x(0) = 0$. The canonical coordinate of $x \in \mathcal{C}$ is described by $x = \{x(t); t \in I\}$. For $a \in \mathbf{R}$, $x \in \mathcal{C}$ and $N = 1, 2, \dots$, we set

$$h^N(t) = \frac{1}{\sqrt{N}} x(t) + \bar{h}(t), \quad t \in I, \tag{1}$$

where $\bar{h}(t) \equiv a$. The law on $\mathcal{C}$ of h^N with x distributed under ν_0 is denoted by ν_N. Let $W = W(r)$ be a (measurable) function on $\mathbf{R}$ satisfying the condition:

$$\text{There exists } A > 0 \text{ such that } \lim_{r\to\infty} W(r) = 0, \ \lim_{r\to-\infty} W(r) = -A$$

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$$\text{and} \quad -A \le W(r) \le 0 \quad \text{for every } r \in \mathbf{R}. \tag{W.1}$$

We consider the distribution, indeed a finite volume Gibbs measure, μ_N on $\mathcal{C}$ defined by

$$\mu_N(dh) = Z_N^{-1} \exp\left\{-N \int_I W(Nh(t))dt\right\} \nu_N(dh), \tag{2}$$

where Z_N is the normalizing constant. Under μ_N, as $N \to \infty$, negative h has an advantage since the density factor becomes larger if it takes negative values. This causes a competition, especially when $a > 0$, between the effect of the potential W pushing h to the negative side and the boundary condition $a > 0$ keeping h at the positive side.

The large deviation principle (LDP) holds for μ_N on $\mathcal{C}$ as $N \to \infty$ under the uniform topology. The speed is N and its (unnormalized) rate functional is given by

$$\Sigma^W(h) = \frac{1}{2} \int_I \dot{h}^2(t)dt - A|\{t \in I; h(t) \le 0\}|, \tag{3}$$

for $h \in H^1_{a,F}(I)$, i.e., for absolutely continuous h with derivatives $\dot{h}(t) = dh/dt \in L^2(I)$ satisfying $h(0) = a$, where $|\cdot|$ stands for the Lebesgue measure. For more precise formulation, cf. [4], [6] and Theorem 6.4 in [2] for a discrete model. The LDP immediately implies the concentration property for μ_N:

$$\lim_{N\to\infty} \mu_N(\mathrm{dist}_\infty(h, \mathcal{H}^W) \le \delta) = 1$$

for every $\delta > 0$, where $\mathcal{H}^W = \{h^*;\ \text{minimizers of } \Sigma^W\}$ and dist_∞ denotes the distance under the uniform norm $\|\cdot\|_\infty$. In particular, if Σ^W has a unique minimizer h^*, then the law of large numbers (LLN) holds under μ_N:

$$\lim_{N\to\infty} \mu_N(\|h - h^*\|_\infty \le \delta) = 1 \tag{4}$$

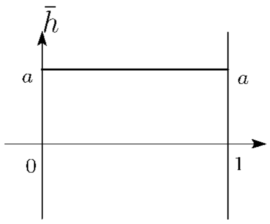
for every $\delta > 0$.

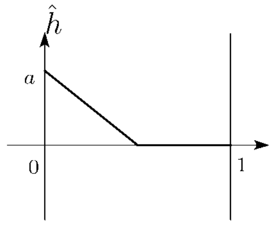
We consider the structure of $\mathcal{H}^W$. It is easy to see that $\mathcal{H}^W = \{\bar{h}\}$ when $a \le 0$. We now assume that $a > 0$. Let $\hat{h}$ be the curve composed of two straight line segments connecting three points $(0, a)$, $P(T, 0)$ and $(1, 0)$ in this order. The angles at the corner P is equal to $\theta \in [0, \pi/2]$, which is determined by the Young's relation (free boundary condition): $\tan\theta = \sqrt{2A}$. More precisely saying, if $0 < a \le \sqrt{2A}$ we have $T = a/\sqrt{2A}$, and

$$\hat{h}(t) = \begin{cases} a - \sqrt{2A}t, & t \in I_1 = [0, T], \\ 0, & t \in I_2 = [T, 1]. \end{cases}$$

Moreover, we can see that $\mathcal{H}^W = \{\bar{h}\}$ when $a > \sqrt{2A}$. Then, $\{\bar{h}, \hat{h}\}$ is the set of all critical points of Σ^W (cf. Section 6.3 in [2]), and this implies that $\mathcal{H}^W \subset \{\bar{h}, \hat{h}\}$.

This paper is concerned with the case where both $\bar{h}$ and $\hat{h}$ are minimizers of Σ^W, i.e. $\Sigma^W(\bar{h}) = \Sigma^W(\hat{h})$; note that $\Sigma^W(\bar{h}) = 0$ and $\Sigma^W(\hat{h}) = a(1 + \sqrt{2A})/2 - A$. In fact, in the





following, we always assume the conditions (W.1) and

$$a > 0 \quad \text{and} \quad \Sigma^W(\bar{h}) = \Sigma^W(\hat{h})\,. \tag{W.2}$$

If the condition (W.2) holds, we have $a = \sqrt{2A}/2$ and $T = 1/2$.

We are now in a position to state our main results.

THEOREM 1 (Concentration on $\bar{h}$). *In addition to the conditions* (W.1) *and* (W.2), *if*

$$W(r) = 0 \quad \textit{for all } r \geq K \tag{W.3}$$

is fulfilled for some $K \in \mathbf{R}$, *then* (4) *holds with* $h^* = \bar{h}$.

THEOREM 2 (Concentration on $\hat{h}$). *In addition to* (W.1) *and* (W.2), *if the following three conditions*

$$^\exists\lambda_1, \alpha_1 > 0 \textit{ such that } W(r) \sim -\lambda_1 r^{-\alpha_1} \ (\textit{i.e. the ratio tends to } 1) \textit{ as } r \to \infty \tag{W.4}$$

$$^\exists\lambda_2, \alpha_2 > 0 \textit{ such that } W(r) \leq -A + \lambda_2 |r|^{-\alpha_2} \textit{ as } r \to -\infty \tag{W.5}$$

$$0 < \alpha_1 < \min\{\alpha_2/(\alpha_2+1), \alpha_2/2\} \quad \textit{and} \quad \int_{I_1} \hat{h}(t)^{-\alpha_1} dt > \int_I \bar{h}(t)^{-\alpha_1} dt \tag{W.6}$$

are fulfilled, *then* (4) *holds with* $h^* = \hat{h}$.

The rate functional Σ^W of the LDP is determined only from the limit values $W(\pm\infty)$, but for Theorems 1 and 2 we need more delicate information on the asymptotic properties of W as $r \to \pm\infty$ to control the next order. Let us try to explain the roles of the above conditions in a rather intuitive way. The condition (W.3) (with $K = 0$) means that W is large at least for $r \geq 0$ so that the force pushing the interface (or the Brownian path) downward is weak and not enough to push it down to the level of $\hat{h}$. On the other hand, since the values of $Nh(t)$ in (2) are very large for t close to 0, compared with (W.3), the interface is pushed downward because of the condition (W.4) and, once it reaches near the level 0, the condition (W.5) forces it to stay there. This makes the interface reach the level of $\hat{h}$. The second condition in (W.6) is fulfilled if $1/2 < \alpha_1 < 1$, and such α_1, which simultaneously satisfies the first condition in (W.6), exists if $\alpha_2 > 1$.

The same kind of problem is discussed for weakly pinned Gaussian random walks in [1]. In one dimension, they proved the coexistence of $\bar{h}$ and $\hat{h}$ under the free boundary condition at the right edge and the concentration on $\hat{h}$ under the Dirichlet boundary condition at the right edge. The problem for the pinned Wiener measures with our densities is discussed by [3].

Section 2 gives the proofs of Theorems 1 and 2.

2. Proofs of results

We consider the following quantity:

$$\lim_{N\to\infty} \frac{\mu_N(\|h-\hat{h}\|_\infty \le \delta)}{\mu_N(\|h-\bar{h}\|_\infty \le \delta)} \tag{5}$$

for arbitrary small $\delta > 0$.

2.1. Proof of Theorem 1. If the limit of (5) is equal to 0, then (4) holds with $h^* = \bar{h}$. In view of the scaling, we may assume $K = 0$ in the condition (W.3) without loss of generality. Introduce the first hitting time $0 \le \tau \le 1$ of $h^N(t)$ to 0 on the event $\Omega_0 = \{h^N \text{ hits } 0\}$ by $\tau = \inf\{t \in I; h^N(t) = 0\}$. Then, from the condition (W.3) with $K = 0$, the strong Markov property of $h^N(t)$ under ν_N shows that

$$\begin{aligned} &Z_N\mu_N(\|h-\hat{h}\|_\infty \le \delta) \\ &\quad\le \int_{S\ge T-c} E^{\nu_0^S}\Big[\exp\Big\{-N\int_S^1 W(\sqrt{N}x(s))ds\Big\}\Big]\nu_N(\tau\in dS) \\ &\qquad + \nu_N(\Omega_0^c, \|h-\hat{h}\|_\infty \le \delta)\,, \end{aligned}$$

where ν_0^S (more generally ν_α^S) is the law on the space $C([S,1],\mathbf{R})$ of the Brownian motion such that $x(S) = 0$ (or $x(S) = \alpha$) and $c = \delta/\sqrt{2A}$ arises from the condition $\|h-\hat{h}\|_\infty \le \delta$. However, in the first term, the conditions (W.1) and (W.3) with $K = 0$ imply that

$$-N\int_S^1 W(\sqrt{N}x(s))ds \le ANX^{S,1}\,,$$

where $X^{S,1} = |\{s \in [S,1]; x(s) < 0\}|$ is the occupation time of x on the negative side. Since $X^{S,1} = (1-S)X^{0,1}$ in law and $\nu_0(X^{0,1} \in ds) = 1/\{\pi\sqrt{s(1-s)}\}ds$ (see Proposition 4.11 in [5], p. 273), we obtain that

$$E^{\nu_0^S}\Big[\exp\Big\{-N\int_S^1 W(\sqrt{N}x(s))ds\Big\}\Big] \le \int_I \frac{e^{AN(1-S)s}}{\pi\sqrt{s(1-s)}}ds\,.$$

Simple calculation yields that

$$\begin{aligned} \int_I \frac{e^{AN(1-S)s}}{\pi\sqrt{s(1-s)}}ds &= \frac{2}{\pi}\int_0^{\pi/2} e^{AN(1-S)/2}\cosh\Big(\frac{AN(1-S)}{2}\sin\theta\Big)d\theta \\ &\le \frac{2}{\pi}\int_0^{\pi/2} e^{AN(1-S)(1+\sin\theta)/2}d\theta\,. \end{aligned}$$

Then, by Laplace's method, we have

$$\int_I \frac{e^{AN(1-S)s}}{\pi\sqrt{s(1-s)}}ds \le \frac{2}{\sqrt{A(1-S)\pi}}\frac{1}{\sqrt{N}}e^{AN(1-S)},$$

for sufficiently large N, see [7] .

On the other hand, the distribution of τ under ν_N is given by

$$\nu_N(\tau \in dS) = \frac{a\sqrt{N}}{\sqrt{2\pi S^3}}e^{-\frac{a^2N}{2S}}dS\,,$$

for $0 < S < 1$, see (6.3) in [5], p. 80.

Combining these all facts, for N large enough, we have

$$Z_N\mu_N(\|h-\hat{h}\|_\infty \le \delta) \le \frac{2a}{\sqrt{2A}\pi}\int_{S\ge T-c}\frac{e^{-Nf(S)}}{\sqrt{S^3(1-S)}}dS + \nu_N(\|h-\hat{h}\|_\infty \le \delta)\,, \tag{6}$$

where

$$f(S) = \frac{a^2}{2S} - A(1-S)\,.$$

Since $f(S) = \Sigma^W(\hat{h}_S) - \Sigma^W(\hat{h})$ for the curve $\hat{h}_S$ defined similarly to $\hat{h}$ with T replaced by S, we see that $f(S) \ge 0$ and f attains its minimal value 0 at $S = T(= 1/2)$. Furthermore, by the condition (W.2), it behaves near T as

$$f(S) = \frac{2a^2}{S}\left(S-\frac{1}{2}\right)^2 \sim 4a^2\left(S-\frac{1}{2}\right)^2.$$

This proves that the first term in the right hand side of (6) behaves as $O(1/\sqrt{N})$ as $N\to\infty$. Therefore, for every $0 < \delta < \|\bar{h}-\hat{h}\|_\infty$, by noting that $\nu_N(\|h-\hat{h}\|_\infty \le \delta) \le e^{-CN}$ for some $C > 0$ (since the LDP holds for ν_N with speed N and the rate functional $\Sigma^0(h)$, which is defined by $A \equiv 0$ in (3)), we have that

$$\lim_{N\to\infty} Z_N\mu_N(\|h-\hat{h}\|_\infty \le \delta) = 0\,.$$

On the other hand, the condition (W.3) implies for every $0 < \delta < (a \wedge b)$ that

$$\lim_{N\to\infty} Z_N\mu_N(\|h-\bar{h}\|_\infty \le \delta) = \lim_{N\to\infty} \nu_0(\|x\|_\infty \le \sqrt{N}\delta) = 1\,.$$

Thus, the proof of Theorem 1 is concluded.

2.2. Proof of Theorem 2. We prove the limit of (5) is equal to ∞. From the definition (2) of μ_N and by recalling (1), we have

$$Z_N\mu_N(\|h-\hat{h}\|_\infty \le \delta)$$
$$= E^{\nu_0}\left[\exp\left\{-N\int_I W(\sqrt{N}x(t)+N\bar{h}(t))dt\right\},\ \|x+\sqrt{N}(\bar{h}-\hat{h})\|_\infty \le \sqrt{N}\delta\right]$$

$$= E^{\nu_0}[\exp\{\hat{F}_N(x)\}, \|x\|_\infty \le \sqrt{N}\delta],$$

where

$$\hat{F}_N(x) = -N\int_I W(\sqrt{N}x(t) + N\hat{h}(t))dt + \sqrt{N}\int_I (\dot{\bar{h}} - \dot{\hat{h}})(t)dx(t) - \frac{N}{2}\int_I (\dot{\bar{h}} - \dot{\hat{h}})^2(t)dt\,.$$

The third line follows by means of the Cameron-Martin formula for ν_0 transforming $x + \sqrt{N}(\bar{h} - \hat{h})$ into x. However, since $\dot{\bar{h}}(t) \equiv 0$ and $\int_I \dot{\hat{h}}(t)dt = \hat{h}(1) - \hat{h}(0) = -a$, we have

$$\frac{1}{2}\int_I (\dot{\bar{h}} - \dot{\hat{h}})^2(t)dt = AT\,,$$

by the condition (W.2). Moreover, since $\dot{\hat{h}} = -\sqrt{2A}$ on I_1° and 0 on I_2°,

$$\int_I (\dot{\bar{h}} - \dot{\hat{h}})(t)dx(t) = \sqrt{2A}(x(T) - x(0)) = \sqrt{2A}x(T)\,,$$

recall that $x(0) = 0$ under ν_0. Therefore, we can rewrite $\hat{F}_N(x)$ as

$$\begin{aligned}\hat{F}_N(x) &= -N\int_{I_1} W(\sqrt{N}x(t) + N\hat{h}(t))dt + \sqrt{2AN}x(T) - N\int_{I_2}\{W(\sqrt{N}x(t)) + A\}dt\\ &=: F_N^{(1)}(x) + F_N^{(2)}(x) + F_N^{(3)}(x)\,.\end{aligned}$$

To give a lower bound on $F_N^{(1)}$, we consider subinterval $\tilde{I}_1 = [0, T - \sqrt{2/A}\,\delta]$ of I_1. Then, since $\hat{h} \ge 2\delta$ on $\tilde{I}_1$, on the event $\mathcal{A}_1 = \{\|x\|_\infty \le \sqrt{N}\delta\}$, we have for $t \in \tilde{I}_1$,

$$\sqrt{N}x(t) + N\hat{h}(t) \ge -N\delta + N\hat{h}(t) \ge N\delta \to \infty \quad (\text{as } N \to \infty)\,,$$

and also $\sqrt{N}x(t) + N\hat{h}(t) \le N(\hat{h}(t) + \delta)$. Accordingly, by the condition (W.4), for every sufficiently small $\varepsilon > 0$, the integrand of $F_N^{(1)}$ times $-N$ is bounded from below as

$$-NW(\sqrt{N}x(t) + N\hat{h}(t)) \ge (\lambda_1 - \varepsilon)N^{1-\alpha_1}(\hat{h}(t) + \delta)^{-\alpha_1}\,,$$

which implies, by recalling $-W \ge 0$, that

$$F_N^{(1)} \ge (\lambda_1 - \varepsilon)N^{1-\alpha_1}\int_{\tilde{I}_1}(\hat{h}(t) + \delta)^{-\alpha_1}dt =: (\lambda_1 - \varepsilon)C_1(\delta)N^{1-\alpha_1}\,,$$

on $\mathcal{A}_1$ for sufficiently large N.

To give lower bounds on $F_N^{(2)}$ and $F_N^{(3)}$, we introduce two more events

$$\begin{aligned}&\mathcal{A}_2 = \{x(T) \ge 0\}\,,\\ &\mathcal{A}_3 = \{x(t) \le -N^{-\kappa} \text{ for all } t \in \tilde{I}_2 := [T + N^{-\frac{1}{2}-\kappa}, 1]\}\,,\end{aligned}$$

where $0 < \kappa < 1/2$ will be chosen later. Then, obviously $F_N^{(2)} \ge 0$ on $\mathcal{A}_2$. If $x \in \mathcal{A}_3$, noting that $-W(r) - A \ge -A$ for all $r \in \mathbf{R}$, we have from (W.5)

$$F_N^{(3)} \geq -AN^{\frac{1}{2}-\kappa} + N\int_{\tilde{I}_2}\{-W(\sqrt{N}x(t)) - A\}dt$$
$$\geq -AN^{\frac{1}{2}-\kappa} - \lambda_2 N^{1-\alpha_2(\frac{1}{2}-\kappa)}|\tilde{I}_2|,$$

for sufficiently large N. These estimates on $F_N^{(1)}$, $F_N^{(2)}$ and $F_N^{(3)}$ are summarized into

$$\hat{F}_N \geq (\lambda_1 - \varepsilon)C_1(\delta)N^{1-\alpha_1} - AN^{\frac{1}{2}-\kappa} - \lambda_2 N^{1-\alpha_2(\frac{1}{2}-\kappa)}|\tilde{I}_2| \tag{7}$$

on $\mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3$ for sufficiently large N.

The next lemma gives a lower bound on the probability $\nu_0(\mathcal{A}_2 \cap \mathcal{A}_3)$.

LEMMA 1. *There exists $C > 0$ such that*

$$\nu_0(\mathcal{A}_2 \cap \mathcal{A}_3) \geq CN^{-\frac{1}{4}-\frac{3}{2}\kappa}\exp\{-18N^{\frac{1}{2}-\kappa}\}.$$

PROOF. Consider an auxiliary event

$$\mathcal{A}_4 = \{-3N^{-\kappa} \leq x(T + N^{-\frac{1}{2}-\kappa}) \leq -2N^{-\kappa}\}.$$

Then, by the Markov property, we have

$$\nu_0(\mathcal{A}_2 \cap \mathcal{A}_3) \geq \nu_0(\mathcal{A}_2 \cap \mathcal{A}_3 \cap \mathcal{A}_4)$$
$$= E^{\nu_0}\big[\nu_{0,\alpha}^{0,T+N^{-\frac{1}{2}-\kappa}}(x(T) \geq 0) \cdot \nu_\alpha^{T+N^{-\frac{1}{2}-\kappa}}(x(t) \leq -N^{-\kappa}, {}^\forall t \in \tilde{I}_2), \mathcal{A}_4\big],$$

where $\alpha = x(T + N^{-\frac{1}{2}-\kappa})$ and $\nu_{0,\alpha}^{0,T+N^{-\frac{1}{2}-\kappa}}$ is the law on the space $C([0, T + N^{-\frac{1}{2}-\kappa}], \mathbf{R})$ of the Brownian bridge such that $x(0) = 0$, $x(T + N^{-\frac{1}{2}-\kappa}) = \alpha$. However,

$$\nu_{0,\alpha}^{0,T+N^{-\frac{1}{2}-\kappa}}(x(T) \geq 0) \geq C_1 N^{\frac{\kappa}{2}-\frac{1}{4}}\exp\{-18N^{\frac{1}{2}-\kappa}\} - C_2 N^{-\frac{1}{2}}\exp\{-2TN\},$$

for sufficiently large N with $C_1, C_2 > 0$, see the proof of Lemma 2.2 in [3]. On $\mathcal{A}_4$, we have

$$\nu_\alpha^{T+N^{-\frac{1}{2}-\kappa}}(x(t) \leq -N^{-\kappa},\ {}^\forall t \in \tilde{I}_2) \geq P_0\big(\max_{t\in I}|B(t)| \leq \bar{t}^{-1/2}N^{-\kappa}\big) \geq C_3 N^{-\kappa},$$

where $\bar{t} = 1 - T - N^{-\frac{1}{2}-\kappa}$ and $C_3 > 0$. Therefore, we obtain

$$\nu_0(\mathcal{A}_2 \cap \mathcal{A}_3) \geq C_4 N^{\frac{\kappa}{2}-\frac{1}{4}} \cdot N^{-\kappa} \cdot \exp\{-18N^{\frac{1}{2}-\kappa}\} \cdot \nu_0(\mathcal{A}_4),$$

for sufficiently large N with $C_4 > 0$. However, we obtain $\nu_0(\mathcal{A}_4) \geq N^{-\kappa}$, see the proof of Lemma 2.2 in [3]. This completes the proof of the lemma. □

Since Lemma 1 shows

$$\nu_0(\mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3) \geq \nu_0(\mathcal{A}_2 \cap \mathcal{A}_3) - \nu_0(\mathcal{A}_1^c)$$
$$\geq \nu_0(\mathcal{A}_2 \cap \mathcal{A}_3) - e^{-\delta^2 N/4} \geq \exp\{-20N^{\frac{1}{2}-\kappa}\},$$

for sufficiently large N (recall $\frac{1}{2} - \kappa < 1$), we have from (7)

$$
\begin{aligned}
&Z_N \mu_N(\|h - \hat{h}\|_\infty \le \delta) \qquad (8)\\
&\quad \ge \exp\{(\lambda_1 - \varepsilon)C_1(\delta)N^{1-\alpha_1} - AN^{\frac{1}{2}-\kappa} - \lambda_2 N^{1-\alpha_2(\frac{1}{2}-\kappa)}|\tilde{I}_2| - 20N^{\frac{1}{2}-\kappa}\}\\
&\quad \ge \exp\{(\lambda_1 - 2\varepsilon)C_1(\delta)N^{1-\alpha_1}\},
\end{aligned}
$$

for sufficiently large N if $1 - \alpha_1 > 0$ (i.e. $\alpha_1 < 1$), $\frac{1}{2} - \kappa < 1 - \alpha_1$ (i.e. $\kappa > \alpha_1 - \frac{1}{2}$) and $1 - \alpha_2(\frac{1}{2} - \kappa) < 1 - \alpha_1$ (i.e. $\kappa < \frac{1}{2} - \frac{\alpha_1}{\alpha_2}$). One can choose such $\kappa : \alpha_1 - \frac{1}{2} < \kappa < \frac{1}{2} - \frac{\alpha_1}{\alpha_2}$ under the first condition in (W.6), which implies that $\alpha_1(1 + \frac{1}{\alpha_2}) < 1$ and $\frac{1}{2} - \frac{\alpha_1}{\alpha_2} > 0$.

On the other hand, we have

$$Z_N \mu_N(\|h - \bar{h}\|_\infty \le \delta) = E^{\nu_0}[\exp\{\bar{F}_N(x)\}, \|x\|_\infty \le \sqrt{N}\delta], \qquad (9)$$

where

$$\bar{F}_N(x) = -N \int_I W(\sqrt{N}x(t) + N\bar{h}(t))dt.$$

However, since $\sqrt{N}x(t) + N\bar{h}(t) \ge N(\bar{h}(t) - \delta)$ on the event $\mathcal{A}_1$, the condition (W.4) shows

$$\bar{F}_N \le (\lambda_1 + \varepsilon)N^{1-\alpha_1} \int_I (\bar{h}(t) - \delta)^{-\alpha_1} dt =: (\lambda_1 + \varepsilon)C_2(\delta)N^{1-\alpha_1}. \qquad (10)$$

Comparing (8) and (9) with (10), since $(\lambda_1 - 2\varepsilon)C_1(\delta) > (\lambda_1 + \varepsilon)C_2(\delta)$ for sufficiently small δ and $\varepsilon > 0$ by the second condition in (W.6), the proof of Theorem 2 is concluded.

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