On an Infinite Family of Elliptic Curves with Rank ≥ 14 over Q

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(Communicated by Shokichi IYANAGA, M. J. A., Feb. 12, 1997)

In [2], Nagao constructed an example $\mathscr{E}$ of elliptic curve over $\boldsymbol{Q}(t)$ with rank ≥ 13. In this paper, we show in utilizing $\mathscr{E}$ and a method introduced in our previous paper [3] that there are infinitely many elliptic curves with rank ≥ 14 over $\boldsymbol{Q}$.

The curve given in [2] was

$$\mathscr{E}: y^2 = (9s^2 + 211950)x^4 + (-2700s^2 - 63901710)x^3 + (-18s^4 + 396150s^2 + 6706476489)x^2 + (2700s^4 - 29575350s^2 - 284435346600)x + 9s^6 - 159200s^4 + 891699592s^2 + 4156297690000$$

where $s = (-t^2 + 23550)/(2t)$. As was shown in [2], there are following 13 points on $\mathscr{E}$.

$P_1 = (s + 148, 662s^2 + 66873s + 1868944)$,
$P_2 = (s + 116, -554s^2 - 39687s - 191632)$,
$P_3 = (s + 104, -526s^2 - 28497s + 163372)$,
$P_4 = (s + 57, 508s^2 - 19332s - 368809)$,
$P_5 = (s + 25, 580s^2 - 49116s + 566825)$,
$P_6 = (s, -670s^2 + 69759s - 2038700)$,
$P_7 = (-s + 148, -662s^2 + 66873s - 1868944)$,
$P_8 = (-s + 116, 554s^2 - 39687s + 191632)$,
$P_9 = (-s + 104, 526s^2 - 28497s - 163372)$,
$P_{10} = (-s + 57, -508s^2 - 19332s + 368809)$,
$P_{11} = (-s + 25, -580s^2 - 49116s - 566825)$,
$P_{12} = (-s, 670s^2 + 69759s + 2038700)$,
$P_{13} = ((s + 703)/15, (-224s^3 - 844s^2 + 900484s + 2161725)/75)$.

Next, let us consider the following elliptic curve:

$$C: q^2 = p(p - 13728)(p + 80472).$$

$(-27456, -7742592)$ is on C, and it is easy to see that this point is of infinite order in the Mordell-Weil group of C, so that C has positive rank.

Let $\boldsymbol{Q}(C)$ be the function field of C. Now we consider $\mathscr{E}$ over $\boldsymbol{Q}(C)$, like in [3], by specializing $t = q/(2p)$.

Then we have the point $P_{14} = (x_{14}, y_{14})$ on $\mathscr{E}$, where

$$x_{14} = (-1104719616 - p^2 + 708q)/(4q)$$
$$y_{14} = (2404198697051119288832 - 12282065003400192p + 1177306772832p^2 + 11117812p^3 + 197p^4 - 108850233203712q - 98532p^2q)/(4q^2).$$

Theorem 1. *$P_1, \ldots, P_{14}$ are independent points.*

Proof. This is shown by specializing $(p, q) = (-27456, -7742592)$. Let $R_1, \ldots, R_{14}$ be the rational points obtained from $P_1, \ldots, P_{14}$ by the above specialization. By using calculation system PARI, we see that the determinant of the matrix $(\langle R_i, R_j \rangle)$ $(1 \leq i, j \leq 14)$ associated to the canonical height is 2344685535688581.87. Since this determinant is non-zero, we see that $R_1, \ldots, R_{14}$ are independent points.

So we see that $P_1, \ldots, P_{14}$ are independent.

Q.E.D.

Now by the theorem 20.3 in [1], specializing (p, q) to elements of the Mordell-Weil group of C, we have

Theorem 2. *There are infinitely many elliptic curves over $\boldsymbol{Q}$ with rank ≥ 14.*

References

[1] J. H. Silverman: The arithmetic of elliptic curves. Graduate Texts in Math., vol. 106, Springer-Verlag, New York (1986).

[2] K. Nagao: An example of elliptic curve over $\boldsymbol{Q}(T)$ with rank ≥ 13. Proc. Japan Acad., **70A**, 152–153 (1994).

[3] S. Kihara: On the rank of the elliptic curve $y^2 = x^3 + k$. II. Proc. Japan Acad., **72A**, 228–229 (1996).