3. K-theoretic Groups with Positioning Map and Equivariant Surgery*),**)

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1. Introduction. Since 1990 the authors have studied equivariant surgery on manifolds allowing half dimensional singular sets. The full detail of obtained theory [3] is rather complicated. The purpose of this article is to present an outline of the theory for use in Transformation Groups. We treat material here in a restrictive way comparing with [3] in order to make the paper easy reading. However we will describe the theory so far as one can have important geometric applications. The following two theorems are examples of such applications.

Theorem 1.1 ([3]). *A standard sphere S has a smooth, one fixed point action of some finite group if and only if the dimension of S is greater than* 5. *Moreover, if* $\dim S > 5$ *then S has such an exotic action of A_5 (the alternating group of degree* 5).

Background of this theorem is explained in [2], [4], and [10]. The corresponding assertion in the category of locally linear actions was proven in [4].

After [11], we denote by $\mathscr{G}_p^q$ the class of all finite groups G having series of normal subgroups $P \lhd H \lhd G$ such that $|P|$ is a power of p, H/P is cyclic, and $|G/H|$ is a power of q.

Theorem 1.2 ([8]). *A finite group G admits a smooth, one fixed point action on a standard sphere of some dimension if and only if $G \notin \mathscr{G}_p^q$ for any primes p and q.*

This theorem was proven by [12] under the hypothesis that G is abelian of odd order. It was also shown in [9] that any finite nonsolvable group G admits such exotic actions.

In the current paper G will be a finite group, R will be $\boldsymbol{Z}$ (the ring of integers), $\boldsymbol{Z}_{(p)}$ (the localization of $\boldsymbol{Z}$ at a prime p) or $\boldsymbol{Q}$ (the ring of rational numbers), and $A = R[G]$ will be the group ring of G with coefficients in R.

2. Grothendieck-Witt rings. Let Θ be a finite G-set. A G-map α from Θ to a finitely generated A-module M is called a *Θ-positioning map* of M. If Θ consists of a unique point then (M, α) is nothing but a pointed module.

In order to generalize the ordinary Grothendieck-Witt ring $GW(R, G)$ (cf. [1] or [6]), we introduce the category $\boldsymbol{H}_{G-inv}(R, \Theta)$ as follows. The

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objects in this category are all triples (M, B, α) consisting of

M a finitely generated R-projective, $R[G]$-module,

$B: M \times M \to R$ a G-invariant, symmetric, nonsingular hermitian form over R,

(hence the associated map $M \to M^{\#}$; $x \mapsto B(x, -)$ is bijective,) and

$\alpha: \Theta \to M$ a positioning map.

The morphisms $f: (M, B, \alpha) \to (M', B', \alpha')$ of this category are all $R[G]$-homomorphisms $f: M \to M'$ which preserve hermitian forms and positioning maps. That is, $B(x, y) = B'(f(x), f(y))$ and $\alpha'(\gamma) = f(\alpha(\gamma))$ for all $x, y \in M$ and $\gamma \in \Theta$. A positioning map α is said to be *totally isotropic* or *t-iso* for short (resp. *trivial* or *triv* for short) if $B(\alpha(\Theta), \alpha(\Theta)) = \{0\}$ (resp. $\alpha(\Theta) = \{0\}$). We denote by $\boldsymbol{H}_{G-inv}(R, \Theta)^{\%}$, % = t-iso, or triv, the full subcategory of $\boldsymbol{H}_{G-inv}(R, \Theta)$ consisting of all (M, B, α) such that α is %. Now let $KH_0(R, G, \Theta)$, $KH_0(R, G, \Theta)^{t-iso}$, and $KH_0(R, G)$ be the Grothendieck ring of the categories $\boldsymbol{H}_{G-inv}(R, \Theta)$, $\boldsymbol{H}_{G-inv}(R, \Theta)^{t-iso}$, and $\boldsymbol{H}_{G-inv}(R, \Theta)^{triv}$, respectively.

For $\boldsymbol{M} = (M, B, \alpha) \in \boldsymbol{H}_{G-inv}(R, \Theta)$, a submodule U of M is called a *weak Quillen submodule* of $\boldsymbol{M}$ if U is an R-direct, $R[G]$-submodule satisfying $U \subseteq U^{\perp}$ and $U \supset \alpha(\Theta)$, where

$$U^{\perp} = \{x \in M \mid B(x, y) = 0 \text{ for all } y \in U\}.$$

Such $(\boldsymbol{M}, U)$ is called a *weak Quillen pair.* $U^{\perp}/U$ has a hermitian form $B^{\perp}$ defined by

$$B^{\perp}([x], [y]) = B(x, y) \ (x, y \in U^{\perp})$$

if $(\boldsymbol{M}, U)$ is a weak Quillen pair. The Grothendieck-Witt group with positioning map $GW_0(R, G, \Theta)$ is defined by

$$GW_0(R, G, \Theta) = KH_0(R, G, \Theta)/\langle[\boldsymbol{M}] - [(U^{\perp}/U, B^{\perp}, triv)]\rangle,$$

where $(\boldsymbol{M}, U)$ runs over all weak Quillen pairs. Similarly, we obtain $GW_0(R, G, \Theta)^{t-iso}$ and $GW_0(R, G)$ from $KH_0(R, G, \Theta)^{t-iso}$ and $KH_0(R, G)$, respectively. Let $\boldsymbol{M}_1 = (M_1, B_1, \alpha_1)$ and $\boldsymbol{M}_2 = (M_2, B_2, \alpha_2) \in \boldsymbol{H}_{G-inv}(R, \Theta)$. Then the product $\boldsymbol{M}_1 \top \boldsymbol{M}_2 = (M_1 \otimes_R M_2, B_1 \top B_2, \alpha_1 \top \alpha_2)$ is defined by $B_1 \top B_2(x \otimes y, x' \otimes y') = B_1(x, x')B_2(y, y') (x, x' \in M_1$ and $y, y' \in M_2)$, and $\alpha_1 \top \alpha_2(\gamma) = \alpha_1(\gamma) \otimes \alpha_2(\gamma) (\gamma \in \Theta)$. The Grothendieck-Witt groups above are commutative rings. In particular, $GW_0(R, G, \Theta)$ has the identity element $[R, B, \alpha]$, where G acts trivially on R, $B(x, y) = xy$ $(x, y \in R)$ and $\alpha(\gamma) = 1 (\gamma \in \Theta)$.

Proposition 2.1. *The natural homomorphism* $GW_0(R, G) \xrightarrow{(1)} GW_0(R, G, \Theta)^{t-iso}$ *is bijective, and the natural homomorphism* $GW_0(R, G) \xrightarrow{(2)} GW_0(R, G, \Theta)$ *is injective. Moreover,* $GW_0(R, G, \Theta)/GW_0(R, G)$ *is a free* R*-module.*

Proof. There is a canonical retraction $GW_0(R, G, \Theta)^{t-iso} \to GW_0(R, G)$. Thus the homomorphism (1) is injective. The surjectivity of (1) is obtained from the fact that for (M, B, α) in $\boldsymbol{H}_{G-inv}(R, \Theta)^{t-iso}$, there is a weak Quillen submodule containing all $\alpha(\gamma) (\gamma \in \Theta)$. The injectivity of (2) follows from the exactness of the sequence

$$0 \to GW_0(R, G)^{t-iso} \to GW_0(R, G, \Theta) \to \bigoplus_{(x,y) \in \Theta \times \Theta} R,$$

where the last homomorphism is defined by

$$[M, B, \alpha] \mapsto (B(\alpha(x), \alpha(y))) \quad (x, y \in \Theta).$$

The last assertion in the proposition follows from the exact sequence above.

3. Special Grothendieck-Witt rings. Let S be a conjugation invariant subset of

$$G(2) = \{g \in G \mid g^2 = 1 \text{ and } g \neq 1\}.$$

S is regarded as a G-set via the conjugation G-action.

To an object (M, B, α) in $\boldsymbol{H}_{G-inv}(R, S)$, we associate a function $\nabla : M \to \mathrm{Map}(S, R/2)$ by

(3.1) $\nabla(x)(g) = [B(\alpha(g) - x, gx)] (\in R/2)$ for $x \in M$ and $g \in S$.

Let $\boldsymbol{SH}_{G-inv}(R, S)$ be the full subcategory of $\boldsymbol{H}_{G-inv}(R, S)$ consisting of all objects (M, B, α) having trivial ∇, i.e., $\nabla(x)(g) = 0$ in $R/2$ for all $x \in M$ and $g \in S$. Let $\boldsymbol{SH}_{G-inv}(R, S)^{\%}$ (% = t-iso or triv) be the full subcategory of $\boldsymbol{SH}_{G-inv}(R, S)$ consisting of all objects (M, B, α) with % positioning map α.

In a parallel way to the Grothendieck-Witt rings $GW_0(R, G, S)$, $GW_0(R, G, S)^{t-iso}$, and $GW_0(R, G)$, we obtain the *special Grothendieck-Witt groups*

$$SGW_0(R, G, S),\ SGW_0(R, G, S)^{t-iso}, \text{ and } SGW_0(R, G, S)^{triv}$$

from the categories

$$\boldsymbol{SH}_{G-inv}(R, S),\ \boldsymbol{SH}_{G-inv}(R, S)^{t-iso}, \text{ and } \boldsymbol{SH}_{G-inv}(R, S)^{triv}.$$

For example,

$$SGW_0(R, G, S)^{triv} = KH_0(R, G, S)^{triv} / \langle [\boldsymbol{M}] - [(U^{\perp}/U, B^{\perp}, triv)] \rangle,$$

where $(\boldsymbol{M}, U)$ runs over all weak Quillen pairs in $\boldsymbol{SH}_{G-inv}(R, S)^{triv}$.

Proposition 3.2. *The natural homomorphism* $SGW_0(R, G, S)^{triv} \xrightarrow{(1)} SGW_0(R, G, S)^{t-iso}$ *is surjective and the natural homomorphism* $SGW_0(R, G, S)^{t-iso} \xrightarrow{(2)} SGW_0(R, G, S)$ *is injective. Moreover,* $SGW_0(R, G, S)/SGW_0(R, G, S)^{t-iso}$ *is a free* R*-module.*

Proof. The proof is quite similar to that of Proposition 2.1. We omit the details.

Let $\boldsymbol{M}_1 = (M_1, B_1, \alpha_1) \in \boldsymbol{H}_{G-inv}(R, S)$ and $\boldsymbol{M}_2 = (M_2, B_2, \alpha_2) \in \boldsymbol{SH}_{G-inv}(R, S)$. Then the ∇ associated to the product $\boldsymbol{M}_1 \top \boldsymbol{M}_2$ is computed as follows.

$$(3.3)\quad \begin{aligned} \nabla(x_1 \otimes x_2)(g) &= [B(\alpha_1(g) \otimes \alpha_2(g) - x_1 \otimes x_2, gx_1 \otimes gx_2)] \\ &= [B_1(\alpha_1(g) - x_1, gx_1) B_2(\alpha_2(g), gx_2) \\ &\quad + B_1(x_1, gx_1) B_2(\alpha_2(g) - x_2, gx_2)] \\ &= [B_1(\alpha_1(g) - x_1, gx_1) B_2(\alpha_2(g), gx_2)]. \end{aligned}$$

By the formula (3.3), we can verify that the special Grothendieck-Witt groups are commutative rings. We note that the identity object of $\boldsymbol{H}_{G-inv}(R, S)$ belongs to $\boldsymbol{SH}_{G-inv}(R, S)$. Thus $SGW_0(R, G, S)$ is a commutative ring with identity element. However, $SGW_0(R, G, S)^{triv}$ must not have the identity element. The next proposition also follows from (3.3).

Proposition 3.4. $SGW_0(R, G, S)^{triv}$ *is a module over* $GW_0(R, G)$.

4. Induction properties. In order to argue induction properties of the special Grothendieck-Witt rings, we let $\Theta(H) = S \cap H$ for $H \leqq G$. It automatically holds that $\Theta(H) \cap \Theta(K) = \Theta(H \cap K)(H, K \leqq G)$ and $\Theta(\{1\}) = \emptyset$.

Proposition 4.1. *The correspondences* $H \mapsto GW_0(R, H, S \cap H)$ *and* $H \mapsto SGW_0(R, H, S \cap H)$ *are Green functors with identity element. The correspondences* $H \mapsto SGW_0(R, H, S \cap H)^{t-iso}$ *and* $H \mapsto SGW_0(R, H, S \cap H)^{triv}$ *are Green functors possibly without identity element. Moreover the last correspondence is a Green module over the Green functor* $H \mapsto GW_0(R, H)$.

(For the definition of a Green functor, see [1, p. 246] or [5, p. 165].)

Theorem 4.2. *Let* $\mathcal{H}$ *be a conjugation invariant family of subgroups of* G *satisfying* $\cup_{H \in \mathcal{H}}((S \cap H) \times (S \cap H)) = S \times S$. *Let* β *be an element in the Burnside ring* $\Omega(G)$ *such that* $\mathrm{Res}_H^G \beta = 1$ *in* $\Omega(H)$ *for all* $H \in \mathcal{H}$. *Further let* 1_Ω *be the identity element of* $\Omega(G)$. *Then* $(1_\Omega - \beta)^2 SGW_0(\boldsymbol{Z}, G, S) = 0$ (*resp.* $(1_\Omega - \beta)^{l+1} SGW_0(\boldsymbol{Z}, G, S) = 0$, *for a certain integer* $l \geqq 1$) *if* $\mathcal{H}$ *contains all* 2-*hyperelementary* (*resp. cyclic*) *subgroups of* G.

Proof. It is easy to check that $(1_\Omega - \beta) SGW_0(\boldsymbol{Z}, G, S) \subseteqq SGW_0(\boldsymbol{Z}, G, S)^{t-iso}$.

Suppose that $\mathcal{H}$ contains all cyclic subgroups of G. Let H be a 2-hyperelementary subgroup of G. By an elementary calculation, we can show that $\mathrm{Res}_H^G (1_\Omega - \beta)^l \in 4\Omega(H)$ for sufficiently large l. (This holds for all $l \geqq 2h + 2$ where h satisfies $|G| = 2^h m$ with m odd (see [8]).) Fix such $l \geqq 1$. Let ι be the identity element of $GW_0(\boldsymbol{Z}, G)$. By Theorems 1 and 3 of [6], $(1_\Omega - \beta)^l \iota = 0$. By Proposition 3.4, we get $(1_\Omega - \beta)^l SGW_0(\boldsymbol{Z}, G, S)^{triv} = 0$, and hence $(1_\Omega - \beta)^l SGW_0(\boldsymbol{Z}, G, S)^{t-iso} = 0$. Thus, we conclude $(1_\Omega - \beta)^{l+1} SGW_0(\boldsymbol{Z}, G, S) = 0$. We omit the proof for the case where $\mathcal{H}$ contains all 2-hyperelementary subgroups of G.

Corollary 4.3. *Let* $M : H \mapsto M(H)$ *be a Green module over the Green functor* $H \mapsto SGW_0(\boldsymbol{Z}, H, S \cap H)$. *Then,* $(1_\Omega - \beta)^2 x = 0$ (*resp.* $(1_\Omega - \beta)^{l+1} x = 0$) *for any* $x \in M(G)$ *if* $\mathcal{H}$ *contains all* 2-*hyperelementary* (*res. cyclic*) *subgroups of* G.

Corollary 4.4. *Let* $\mathcal{K}$ *be a lower closed family of subgroups of* G *containing all elements in* $\mathcal{H}$ *and all cyclic subgroups of* G. *Suppose* β *has the form* $\beta = \sum_{H \in \mathcal{K}} a(H) [G/H] \, (a(H) \in \boldsymbol{Z})$. *Then* $M(G) = \sum_{H \in \mathcal{K}} \mathrm{Ind}_H^G M(H)$, *and the restriction homomorphism* $\mathrm{Res} : M(G) \to \bigoplus_{H \in \mathcal{K}} M(H)$ *is injective.*

Remark. The induction results above can be sharpened by the (nontrivial) fact that the natural homomorphism $SGW_0(\boldsymbol{Z}, G, S) \to GW_0(\boldsymbol{Z}, G, S)$ is injective.

5. Surgery obstruction group. Let $\lambda = 1$ or -1 and let $w : G \to \{\pm 1\}$ be a homomorphism. Then $A = R[G]$ has the antiinvolution ‐ defined by $(\sum_{g \in G} a_g g)^- = \sum_{g \in G} w(g) a_g g^{-1}$ $(a_g \in R)$. Let Q and S be conjugation invariant subsets of $G(2)$ such that $g = -\lambda \bar{g}$ for all $g \in Q$, and $g = \lambda \bar{g}$ for all $g \in S$. We set $A_q = \{\sum a_g g \mid a_g \in R$ and $g \in G \setminus S\}$, $A_s = \{\sum a_g g \mid a_g \in R$ and $g \in S\}$, and $\Lambda = \Lambda(Q) = \langle x - \lambda \bar{x}, g \mid x \in A$ and $g \in Q \rangle_R$ as subsets of A.

A triple $\boldsymbol{M} = (M, \langle , \rangle, q)$ of a finitely generated A-module M, a λ-hermitian form $\langle , \rangle$ and a quadratic form $q : M \to A_q / \Lambda$ is called a *quadratic module* if the following conditions are fulfilled.

(Q1) $\langle , \rangle$ is biadditive.

(Q2) $\langle ax, by\rangle = \overline{b\langle x, y\rangle}\, \bar{a}$.
(Q3) $\langle x, y\rangle = \lambda\overline{\langle y, x\rangle}$.
(Q4) $q(gx) = gq(x)\bar{g}$ in $A_q/\Lambda = A/(\Lambda + A_s)$.
(Q5) $q(x+y) - q(x) - q(y) = \langle x, y\rangle$ in $A_q/\Lambda = A/(\Lambda + A_s)$.
(Q6) $\widetilde{q(x)} + \lambda\overline{\widetilde{q(x)}} = \langle x, x\rangle$ in $A_q = A/A_s$ where $\widetilde{q(x)}$ is a lifting of $q(x)$.

($x, y \in M$, $a, b \in A$, and $g \in G$.) Let $\boldsymbol{Q}(A) = \boldsymbol{Q}^\lambda(A, Q, S)$ be the category of all quadratic modules $\boldsymbol{M} = (M, \langle,\rangle, q)$ such that M is stably free over A and $\langle,\rangle$ is nonsingular. Let Θ be a finite G-set. Let $\boldsymbol{Q}(A, \Theta) = \boldsymbol{Q}^\lambda(A, Q, S, \Theta)$ be the category of all $\boldsymbol{M} = (M, \langle,\rangle, q, \alpha)$ where $(M, \langle,\rangle, q) \in \boldsymbol{Q}(A)$ and $\alpha : \Theta \to M$ is a positioning map. An A-direct summand L of $\boldsymbol{M}$ is called a *lagrangian* of $\boldsymbol{M}$ if L is free over A, $\langle L, L\rangle = 0$, $q(L) = 0$, $L = L^\perp$, and $\alpha(\Theta) \subset L$. If $\boldsymbol{M}$ has a lagrangian then $\boldsymbol{M}$ is called *null*. Define $KQ_0^\lambda(A, Q, S, \Theta)$ to be the Grothendieck group of the category $Q^\lambda(A, Q, S, \Theta)$ and set

$$WQ_0^\lambda(A, Q, S, \Theta) = KQ_0^\lambda(A, Q, S, \Theta)/\langle\text{null modules}\rangle.$$

In the remainder of this section we set $\Theta = S$. We associate a function $\nabla : M \to Map(S, R/2)$ to $\boldsymbol{M} = (M, \langle,\rangle, q, \alpha) \in \boldsymbol{Q}^\lambda(A, Q, S, S)$ by

$$\nabla(x)(g) = [\varepsilon(\langle\alpha(g) - x, gx\rangle)] \quad (x \in M, g \in S),$$

where $\varepsilon : A \to R$ is the homomorphism defined by $\varepsilon(\sum_{g\in G} a_g g) = a_1 (a_g \in R)$. Let $\boldsymbol{SQ}^\lambda(A, Q, S, S)$ be the category of all $\boldsymbol{M}$ having trivial ∇. Define $SKQ_0^\lambda(A, Q, S, S)$ to be the Grothendieck group of the category $\boldsymbol{SQ}^\lambda(A, Q, S, S)$ and set

$$SWQ_0^\lambda(A, Q, S, S) = SKQ_0^\lambda(A, Q, S, S)/\langle\text{null modules}\rangle.$$

Now let $n = 2k$ be an even integer $\geqq 6$, and $\lambda = (-1)^k$. Set

$$W_n(R, G, Q, S, S) = SWQ_0^\lambda(A, Q, S, S).$$

We can define $W_n(R, G, Q, S, \Theta)$ for more general Θ, but we omit such generalization for simplicity.

Proposition 5.1. *The correspondence* $H \mapsto W_n(R, H, Q\cap H, S\cap H, S\cap H)$ *is a Mackey functor, moreover a Green module over the Green functor* $H \mapsto SGW_0(\boldsymbol{Z}, H, S\cap H)$.

6. G-surgery theorem. Let $n = 2k$ be an even integer $\geqq 6$. Manifolds and group actions on them should be understood to be smooth. Let X be a G-manifold without boundary. Suppose the conditions (6.1)-(6.4).

(6.1) X is compact, connected, simply connected, oriented, and of dimension n.

(6.2) $\dim X^H \leqq k$ for any $H \leqq G$, $H \neq \{1\}$.

(6.3) If $\dim X^H = k$ $(H \leqq G)$ then $|H| = 2$ and X^H is connected and orientable so that each $g : X^H \to X^{gHg^{-1}}$ $(g \in G)$ is orientation preserving.

(6.4) $\dim(X^H \cap X^K) \leqq k - 2$ whenever $\dim X^H = k$ and $\dim X^K = k - 1$ $(H, K \leqq G)$.

The orientation homomorphism $w = w_X : G \to \{\pm 1\}$ is defined by setting $w(g) = 1$ (resp. -1) for $g \in G$ acting on X as an orientation preserving (resp. reversing) transformation. This defines an antiinvolution on $A = R[G]$. We set $\lambda = (-1)^k$, $Q = \{g \in G(2) \mid \dim X^g = k - 1\}$, and $S =$

$\{g \in G(2) \mid \dim X^g = k\}$. Denote by $T(X)$ the tangent bundle of X.

Let Y be another G-manifold satisfying (6.1)-(6.4) for X replaced by Y and let ξ be a G-vector bundle over Y.

Theorem 6.5. *Let $f : X \to Y$ be a degree one G-map and $b : T(X) \to f^*\xi$ a stable G-vector bundle isomorphism. Suppose the following conditions* (N1) *and* (N2).

(N1) *f is k-connected (i.e., $f_\# : \pi_i(X) \to \pi_i(Y)$ is surjective whenever $i \leqq k$).*

(N2) $K_k(f\,;R) = \mathrm{Ker}[f_* : H_k(X\,;R) \to H_k(Y\,;R)]$ *is stably $R[G]$-free.*

Then (f, b) determines a unique element $\sigma(f, b)$ in the group $W_n(R, G, Q, S, S)$ having the property: If $\sigma(f, b) = 0$ then (f, b) is converted, by G-surgery in the free part, to (f', b') such that X' satisfies (6.1) *for X replaced by X' and $f' : X' \to Y$ is a k-connected, R-homology equivalence, where $b' : T(X') \to f'^*\xi$. (In particular, $X'^H = X^H$ for any subgroup $H \neq \{1\}$.)*

Remark. If $f^H : X^H \to Y^H$ is an R-homology equivalence for any hyperelementary subgroup $H \neq \{1\}$ then the condition (N2) automatically follows from (N1).

References

[1] Bak, A.: K-Theory of Forms. Princeton Univ. Press, Princeton (1981).

[2] Bak, A., and Morimoto M.: Equivariant surgery and applications. Proc. Conf. of Topology in Hawaii 1991 (ed. Dovermann, K. H.). World Scientific Publ., Singapore, pp. 13–25 (1992).

[3] ——: Equivariant surgery on compact manifolds with half dimensional singular sets (1992) (preprint).

[4] Buchdahl, N. P., Kwasik, S., and Schultz, R.: One fixed point actions on low-dimensional spheres. Invent. Math., **102**, 633–662 (1990).

[5] tom Dieck, T.: Transformation Groups and Representation Theory. Lect. Notes in Math., vol. 766, Springer, Berlin, Heidelberg, New York (1979).

[6] Dress, A.: Induction and structure theorems for Grothendieck and Witt rings of orthogonal representations of finite groups. Bull. Amer. Math. Soc., **79**, 741–745 (1973).

[7] ——: Induction and structure theorems for orthogonal representations of finite groups. Ann. Math., **102**, 291–325 (1975).

[8] Laitinen, E., and Morimoto, M.: Finite groups with smooth one fixed point actions on spheres. Reports of the Dept. Math. Univ. Helsinki, Preprint, no. 25 (1993).

[9] Laitinen, E., Morimoto, M., and Pawałowski, K.: Smooth actions of finite nosolvable groups on spheres. ibid., no. 12 (1992).

[10] Morimoto, M.: Most standard spheres have smooth one fixed point actions of A_5 on spheres. K-Theory, **4**, 289–302 (1991).

[11] Oliver, R.: Fixed point sets of group actions on finite acyclic complexes. Comment. Math. Helv., **50**, 155–177 (1975).

[12] Petrie, T.: One fixed point actions on spheres. I. Adv. Math., **46**, 3–14 (1982).

[13] ——: ditto. II. ibid., **46**, 15–70 (1982).