# 63. Studies on Holonomic Quantum Fields. XV

## Double Scaling Limit of One Dimensional XY Model

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The aim of this article is to show that the double scaling limit ([6] [7] [8]) of the one dimensional $XY$ chain can be handled in the framework of monodromy preserving deformation theory (cf. [1] [2] [3]).

We wish to express our gratitude to Profs. B. M. McCoy and C. A. Tracy, who have urged us to study the topics of our present notes XV, XVI from deformation theoretical viewpoint. In particular we are grateful to Prof. McCoy for handing us related references, including the thesis of Vaidya [8].

1. The one-dimensional spin $\frac{1}{2}$ $XY$ model is described by the Hamiltonian

(1) $$H_M=-\frac{1}{4}\sum_{m=0}^{M-1}((1+\gamma)\sigma_m^x\sigma_{m+1}^x+(1-\gamma)\sigma_m^y\sigma_{m+1}^y+2h\sigma_m^z)$$

$$\sigma_m^\alpha=I_2\otimes\cdots\otimes\overset{m}{\sigma^\alpha}\otimes\cdots\otimes I_2 \qquad (\alpha=x,y,z)$$

where $\sigma^x=\begin{pmatrix} & 1\\ 1 & \end{pmatrix}$, $\sigma^y=\begin{pmatrix} & -i\\ i & \end{pmatrix}$, $\sigma^z=\begin{pmatrix}1 & \\ & -1\end{pmatrix}$.

In the sequel we shall be concerned with the double scaling limit of the model (1), defined as follows ([6]):

(2) $$m,n\to\infty\,;\ \varepsilon=\sqrt{1-h^2}\to 0,\ \gamma\to 0$$

$$\text{keeping } g=\gamma/\varepsilon>0,\ a=m\varepsilon,\ t=\frac{n}{2}\varepsilon^2 \text{ fixed.}$$

The result is quite similar to the scaling limit of the Ising model, except that the characteristic dispersion relation $\omega(p)=\sqrt{p^2+m^2}$ for the latter is now replaced by ([6])

(3) $$\omega(p)=\sqrt{(p^2+\mu^2)(p^2+\mu^{*2})}$$

where $\mu=g+\sqrt{g^2-1}$ $(g\geqq 1)$, $=g+i\sqrt{1-g^2}$ $(0<g\leqq 1)$, $\mu^*=\mu^{-1}$. Denote by $\psi^\dagger(p)$, $\psi(p)$ the creation-annihilation operators of free fermion such that $[\psi^\dagger(p),\psi(p')]_+=2\pi\delta(p-p')$, and set

(4) $$\hat{\psi}_\pm(p,t)=\psi^\dagger(-p)e^{it\omega(p)}\pm\psi(p)e^{-it\omega(p)}$$

(5) $$\psi^{(\pm,n)}(a,t)=\int_{-\infty}^{+\infty}\frac{dp}{2\pi}\sqrt{\omega(p)}^{\pm 1}\hat{\psi}_\pm(p,t)e^{iap}p^n=\left(\frac{1}{i}\frac{\partial}{\partial a}\right)^n\psi^{(\pm)}(a,t)$$

$$(n=0,1,2,\cdots)$$

(6) $$\hat{\psi}_{\pm}(p)=\hat{\psi}_{\pm}(p,0), \qquad \psi^{(\pm)}(a,t)=\psi^{(\pm,0)}(a,t).$$

Then we find that, in the notation of [4],

(7) $$\bar{\sigma}^{x}_{mn}=\sqrt{2\varepsilon}g^{1/4}\varphi(a,t)+O(\varepsilon^{3/2})$$
$$\bar{\sigma}^{y}_{mn}=\sqrt{2\varepsilon}g^{1/4}\tilde{\varphi}^{(-)}(a,t)+O(\varepsilon^{3/2})$$

(8) $$\varphi(a,t)=:e^{\rho(a,t)/2}:$$
$$\tilde{\varphi}^{(\pm)}(a,t)=:\psi^{(\pm,1)}(a,t)\psi^{(\pm)}(a,t)e^{\rho(a,t)/2}:,$$

with

(9) $$\rho(a,t)=\iint_{-\infty}^{+\infty}\frac{dp}{2\pi}\frac{dp'}{2\pi}(\psi^{\dagger}(p)\ \psi(p))\begin{pmatrix}R^{--}_{at}(p,p') & R^{-+}_{at}(p,p')\\ R^{+-}_{at}(p,p') & R^{++}_{at}(p,p')\end{pmatrix}\begin{pmatrix}\psi^{\dagger}(p')\\ \psi(p')\end{pmatrix}$$
$$R^{\sigma\sigma'}_{at}(p,p')=\frac{1}{\sqrt{\omega\omega'}}\frac{-i(-\sigma'\omega+\sigma\omega')}{\sigma p+\sigma' p'-i0}e^{ia(\sigma p+\sigma' p')-it(\sigma\omega+\sigma'\omega')}$$
$$(\sigma,\sigma'=\pm\ ;\ \omega=\omega(p),\ \omega'=\omega(p')).$$

Just as in the case of the Ising model, $\varphi(a,t)$ satisfies a peculiar equal-time commutation relation with the free fields. Namely let

(10) $$\psi_{\pm}(x,t)=\int_{-\infty}^{+\infty}\frac{dp}{2\pi}\sqrt{a(p)}^{\mp 1}\hat{\psi}_{\pm}(p,t)e^{ixp}$$

where

(11) $$\sqrt{a(p)}=\sqrt{b(p)\cdot b(-p)^{-1}} \qquad b(p)=\sqrt{(p+i\mu)(p+i\mu^{*})}^{-1},$$

with $\sqrt{a(0)}=1$, $b(0)=-i$. Then we have

(12) $$\varphi(a,t)\psi_{\pm}(x,t)=\varepsilon(x-a)\psi_{\pm}(x,t)\varphi(a,t).$$

In what follows we shall restrict ourselves to the case of $t=0$. The operator $\varphi(a,0)$ is abbreviated to $\varphi(a)$.

We shall give formal asymptotic expansions for the commutators $[\hat{\psi}_{\pm}(p),\varphi(a)]$ as $p\to\pm\infty$.

(13) $$[\hat{\psi}_{\pm}(p),\varphi(a)]=2i\sqrt{\omega}^{\pm 1}e^{-iap}\frac{1}{p}\left(\varphi^{(\mp)}(a)-\frac{i}{p}\frac{\partial}{\partial a}\varphi^{(\mp)}(a)+O\left(\frac{1}{p^2}\right)\right)$$

where

(14) $$\varphi^{(\pm)}(a)=:\psi^{(\pm)}(a)e^{\rho(a,0)/2}:.$$

We have also

(15) $$[\hat{\psi}_{\pm}(p),\varphi^{(\pm)}(a)]_{+}=\pm 2\sqrt{\omega}^{\pm 1}e^{-iap}\left(\varphi(a)-\frac{i}{p}\frac{\partial}{\partial a}\varphi(a)+O\left(\frac{1}{p^2}\right)\right),$$

(16) $$[\hat{\psi}_{\pm}(p),\varphi^{(\mp)}(a)]_{+}=2i\sqrt{\omega}^{\pm 1}e^{-iap}\frac{1}{p^2}\left(\tilde{\varphi}^{(\mp)}(a)-\frac{i}{p}\frac{\partial}{\partial a}\tilde{\varphi}^{(\mp)}(a)+O\left(\frac{1}{p^2}\right)\right).$$

**2.** Let $W(\Lambda)$ be an orthogonal space spanned by creation operators $\psi^{\dagger(j)}(p)$ and annihilation operators $\psi^{(j)}(p)$ ($j=1,\cdots,2n$; $p\in\boldsymbol{R}$) with the following table of inner product:

$$\langle\psi^{(j)}(p),\psi^{(j')}(p')\rangle=0, \qquad \langle\psi^{\dagger(j)}(p),\psi^{\dagger(j')}(p')\rangle=0,$$
$$\langle\psi^{(j)}(p),\psi^{\dagger(j')}(p')\rangle=\begin{cases}0 & 1\leqq j,j'\leqq n\\ \delta_{jj'+n}2\pi\delta(p-p') & n+1\leqq j\leqq 2n,\ 1\leqq j'\leqq n\\ \delta_{j+nj'}2\pi\delta(p-p') & 1\leqq j\leqq n,\ n+1\leqq j'\leqq 2n\\ \lambda_{j-nj'-n}2\pi\delta(p-p') & n+1\leqq j,j'\leqq 2n.\end{cases}$$

Here $\Lambda=(\lambda_{jj'})_{j,j'=1,\cdots,n}$ is a symmetric $n\times n$ matrix such that $\lambda_{jj}=0$ (cf. p. 253 in I [5] where $\lambda_{jj}=1$). We define operators $\varphi^{(j)}(a)$ (resp. $\varphi^{(j)(\pm)}(a)$) by (8) (resp. (14)) with $\psi^{\dagger}(p)$, $\psi(p)$ replaced by $\psi^{\dagger(j)}(p)$, $\psi^{(j)}(p)$ and with $t=0$. For $a_1,\cdots,a_n\in \boldsymbol{R}$ satisfying $a_1<\cdots<a_n$, we abbreviate $\varphi^{(j+n)}(a_j)$ (resp. $\varphi^{(j+n)(\pm)}(a_j)$) to $\varphi_j$ (resp. $\varphi_j^{(\pm)}$), and introduce the following main objects in this note.

$$\tau_n(a_1,\cdots,a_n;\Lambda)=\langle\varphi_1\cdots\varphi_n\rangle_{W(\Lambda)}, \tag{17}$$

$$w_{jj'}^{(1)\varepsilon+}(p;a_1,\cdots,a_n;\Lambda)=\sqrt{\omega(p)}\langle\psi^{(j')}(p)\varphi_1\cdots\varphi_j^{(\varepsilon)}\cdots\varphi_n\rangle_{W(\Lambda)}/\tau_n \tag{18}$$

$$w_{jj'}^{(1)\varepsilon-}(p;a_1,\cdots,a_n;\Lambda)=\sqrt{\omega(p)}\langle\varphi_1\cdots\varphi_j^{(\varepsilon)}\cdots\varphi_n\psi^{\dagger(j')}(-p)\rangle_{W(\Lambda)}/\tau_n \tag{19}$$

$$(\varepsilon=\pm,\ j,j'=1,\cdots,n).$$

Here we mean by $\langle\quad\rangle_{W(\Lambda)}$ the vacuum expectation value with respect to creation and annihilation operators $\psi^{\dagger(j)}(p)$ and $\psi^{(j)}(p)$.

Denote by $R_j$ $(j=1,\cdots,n)$, $K$ and ${}^tK$ the integral operators with kernels

$$R_j(p,p')\frac{dp'}{2\pi}=\frac{1}{\sqrt{\omega}\sqrt{\omega'}}\begin{pmatrix}\omega-\omega' & -\omega-\omega'\\ \omega+\omega' & -\omega+\omega'\end{pmatrix}\frac{1}{2\pi}\,\frac{-ie^{ia_j(p+p')}}{p+p'-i0}dp', \tag{20}$$

$$K(p,p')\frac{dp'}{2\pi}=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\delta(p+p')dp', \tag{21}$$

$${}^tK(p,p')\frac{dp'}{2\pi}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\delta(p+p')dp', \tag{22}$$

respectively. We set

$$R=\begin{bmatrix}R_1 & & & \\ & R_2 & & \\ & & \ddots & \\ & & & R_n\end{bmatrix},\qquad A(\Lambda)=\begin{bmatrix}0 & \lambda_{12}K & \cdots & \lambda_{1n}K\\ -\lambda_{21}{}^tK & 0 & \cdots & \lambda_{2n}K\\ \vdots & \vdots & & \vdots\\ -\lambda_{n1}{}^tK & -\lambda_{n2}{}^tK & \cdots & 0\end{bmatrix}.$$

Then $2n\times 2n$ matrix $W=(\sqrt{\omega(p)}^{-1}w_{jj'}^{(1)\varepsilon\varepsilon'}(-p))_{\substack{\varepsilon,\varepsilon'=\pm\\ j,j'=1,\cdots,n}}$ satisfies an integral equation,

$$(1-RA(\Lambda)){}^tW={}^tW|_{\Lambda=0} \tag{23}$$

where $W|_{\Lambda=0}=(\delta_{jj'}(\delta_{1\varepsilon}+\delta_{-1\varepsilon}\varepsilon')\sqrt{\omega}^{\varepsilon}e^{ia_{j'}p})_{\substack{\varepsilon,\varepsilon'=\pm\\ j,j'=1,\cdots,n}}$. Hence $w_{jj'}^{(1)\varepsilon\varepsilon'}(p)$ is expressed as the following Neumann series.

$$\begin{aligned} w_{jj'}^{(1)\varepsilon\varepsilon'}(p)=&\ \delta_{jj'}(\delta_{1\varepsilon}\omega+\delta_{-1\varepsilon}\varepsilon')e^{-ia_{j'}p}\\ &+\sum_{l=1}^{\infty}\sum_{j_2,\cdots,j_l=1}^{n}\int\frac{dp_1}{2\pi i\omega_1}\cdots\int\frac{dp_l}{2\pi i\omega_l}\lambda_{j_1j_2}\lambda_{j_2j_3}\cdots\lambda_{j_lj_{l+1}}\\ &\times e^{-ia_{j_1}p+i(a_{j_1}-a_{j_2})p_1+\cdots+i(a_{j_l}-a_{j_{l+1}})p_l}\\ &\times(\omega-\varepsilon'\varepsilon_{j_1j_2}\omega_1)(\omega_1+\varepsilon_{j_1j_2j_3}\omega_2)\cdots(\omega_{l-1}+\varepsilon_{j_{l-1}j_lj_{l+1}}\omega_l)(\delta_{1\varepsilon}\omega_l-\delta_{-1\varepsilon}\varepsilon_{j_lj_{l+1}})\\ &\times\frac{1}{p-p_1+i0}\,\frac{1}{p_1-p_2+i0}\cdots\frac{1}{p_{l-1}-p_l+i0}\qquad(j_1=j',\ j_{l+1}=j),\end{aligned} \tag{24}$$

where $\omega_l=\omega(p_l)$, $\varepsilon_{jj'j''}=\varepsilon_{jj'}\varepsilon_{j'j''}$ and $\varepsilon_{jj'}=1$ (if $j>j'$), 0 (if $j=j'$), $-1$ (if $j<j'$). A similar expression for $\tau_n$ reads as follows.

$$\log\tau_n=-\sum_{l=2}^{\infty}\frac{1}{2l}\sum_{j_1,\cdots,j_l=1}^{n}\int\frac{dp_1}{2\pi i\omega_1}\cdots\int\frac{dp_l}{2\pi i\omega_l} \tag{25}$$

$$\times \lambda_{j_1 j_2} \cdots \lambda_{j_l j_1} e^{i(a_{j_1}-a_{j_2})p_1+\cdots+i(a_{j_l}-a_{j_1})p_l}$$
$$\times (\omega_1+\varepsilon_{j_1 j_2 j_3}\omega_2)\cdots(\omega_l+\varepsilon_{j_l j_1 j_2}\omega_1)$$
$$\times \frac{1}{p_1-p_2+i0}\cdots\frac{1}{p_l-p_1+i0}.$$

3. We define $w_{jj'}^{(k)\varepsilon\varepsilon'}(p)$ $(k=2,3)$ to be the right hand side of (24) with $p-p_1+i0$ replaced by $p-p_1-i0$ $(k=2)$ or by $p-p_1-i\varepsilon_{j'j_2}0$ $(k=3)$. This amounts to setting

(26) $$w_{jj'}^{(2)\varepsilon\varepsilon'}(p) = w_{jj'}^{(1)\varepsilon\varepsilon'}(p) + \sum_{j_2<j'} \lambda_{j'j_2}(1-\varepsilon')w_{jj_2}^{(1)\varepsilon\varepsilon'} + \sum_{j_2>j'} \lambda_{j'j_2}(1+\varepsilon')w_{jj_2}^{(1)\varepsilon\varepsilon'},$$

(27) $$w_{jj'}^{(3)\varepsilon\varepsilon'}(p) = w_{jj'}^{(1)\varepsilon\varepsilon'}(p) + \sum_{j_2<j'} \lambda_{j'j_2}(1-\varepsilon')w_{jj_2}^{(1)\varepsilon\varepsilon'}.$$

Define $2n\times 2n$ matrices $Y^{(k)}=Y^{(k)}(p)$ by

(28) $$Y_{jj'}^{(k)\varepsilon\varepsilon'} = \frac{1}{2}(w_{jj'}^{(k)\varepsilon+} + \varepsilon' w_{jj'}^{(k)\varepsilon-}).$$

Then we have

(29) $$Y^{(k)}(p) = \hat{Y}^{(k)}(p)\omega(p)^{2L}e^{A_\infty p},$$

where $L=\operatorname{diag}\left(\frac{1}{2}, \cdots, \frac{1}{2}, 0, \cdots, 0\right)$, $A_\infty=\operatorname{diag}(-ia_1, \cdots, -ia_n, -ia_1, \cdots, -ia_n)$ and

(30) $$\hat{Y}_{jj'}^{(k)\varepsilon\varepsilon'}(p) = \delta_{jj'}\delta_{\varepsilon\varepsilon'} + \sum_{l=1}^{\infty} \sum_{j_2,\cdots,j_l=1}^{n} \int \frac{dp_1}{2\pi i\omega_1} \cdots \int \frac{dp_l}{2\pi i\omega_l} \lambda_{j_1 j_2}\lambda_{j_2 j_3}\cdots\lambda_{j_l j_{l+1}}$$
$$\times e^{i(a_{j_1}-a_{j_2})p_1+\cdots+i(a_{j_l}-a_{j_{l+1}})p_l}$$
$$\times (\delta_{1\varepsilon'}-\delta_{-1\varepsilon'}\varepsilon_{j_1 j_2}\omega_1)(\omega_1+\varepsilon_{j_1 j_2 j_3}\omega_2)\cdots(\omega_{l-1}+\varepsilon_{j_{l-1}j_l j_{l+1}}\omega_l)$$
$$\times (\delta_{1\varepsilon}\omega_l - \delta_{-1\varepsilon}\varepsilon_{j_l j_{l+1}})$$
$$\times \frac{1}{p-p_1+i\varepsilon_{j_1 j_2}^{(k)}0}\frac{1}{p_1-p_2+i0}\cdots\frac{1}{p_{l-1}-p_l+i0} \quad (j_1=j',\ j_{l+1}=j).$$

Here $\varepsilon_{j_1 j_2}^{(k)}=1$ $(k=1)$, $-1$ $(k=2)$, $-\varepsilon_{j'j_2}$ $(k=3)$. Each contour of integration in (30) can be deformed either into $C_+$ or into $C_-$ of Fig. 1. $\hat{Y}^{(1)}(p)$ (resp. $\hat{Y}^{(2)}(p)$) is holomorphic and invertible in the upper (resp. lower) half $p$-plane, while $\hat{Y}^{(3)}(p)$ is holomorphic and invertible outside $C_+$ and $C_-$.

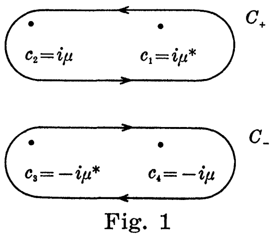


Fig. 1

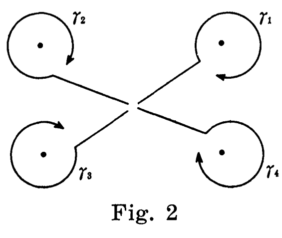


Fig. 2

For the inverse of $\hat{Y}^{(k)}(p)$, we have

(31) $$(\hat{Y}^{(k)-1})_{jj'}^{\varepsilon\varepsilon'}(p) = \delta_{jj'}\delta_{\varepsilon\varepsilon'} - \sum_{l=1}^{\infty} \sum_{j_2,\cdots,j_l=1}^{n} \int \frac{dp_1}{2\pi i\omega_1} \cdots \int \frac{dp_l}{2\pi i\omega_l} \lambda_{j_1 j_2}\lambda_{j_2 j_3}\cdots\lambda_{j_l j_{l+1}}$$
$$\times e^{i(a_{j_1}-a_{j_2})p_1+\cdots+i(a_{j_l}-a_j)p_l}$$

$$\times(\delta_{1\epsilon'}-\delta_{-1\epsilon'}\varepsilon_{j_1j_2}\omega_1)(\omega_1+\varepsilon_{j_1j_2j_3}\omega_2)\cdots(\omega_{l-1}+\varepsilon_{j_{l-1}j_lj_{l+1}}\omega_l)$$
$$\times(\delta_{1\epsilon}\omega_l-\delta_{-1\epsilon}\varepsilon_{j_lj_{l+1}})$$
$$\times\frac{1}{p_1-p_2+i0}\cdots\frac{1}{p_{l-1}-p_l+i0}\,\frac{1}{p-p_l+i\varepsilon^{(k)}_{j_lj_{l+1}}0}$$
$$(j_1=j',\ j_{l+1}=j).$$

Summing up, $Y^{(3)}(p)$ is a multi-valued holomorphic matrix with regular singularities at $p=c_s$ $(s=1,\cdots,4)$ and an irregular singularity of rank 1 at $p=\infty$. Denote by $\gamma_s$ the path encircling $c_s$ (Fig. 2), and by $\gamma_s Y^{(3)}(p)$ the analytic continuation of $Y^{(3)}(p)$ along $\gamma_s$. Then we have $\gamma_s Y^{(3)}(p)=Y^{(3)}(p)M_s$, with

(32) $$M_1=M_2=\frac{1}{2}\begin{pmatrix}\tilde{\Lambda}^{-1}+\tilde{\Lambda} & \tilde{\Lambda}^{-1}-\tilde{\Lambda}\\ -\tilde{\Lambda}^{-1}+\tilde{\Lambda} & -\tilde{\Lambda}^{-1}-\tilde{\Lambda}\end{pmatrix},$$

$$M_3=M_4=-\frac{1}{2}\begin{pmatrix}{}^t\tilde{\Lambda}^{-1}+{}^t\tilde{\Lambda} & {}^t\tilde{\Lambda}^{-1}-{}^t\tilde{\Lambda}\\ -{}^t\tilde{\Lambda}^{-1}+{}^t\tilde{\Lambda} & -{}^t\tilde{\Lambda}^{-1}-{}^t\tilde{\Lambda}\end{pmatrix},\qquad \tilde{\Lambda}=\begin{bmatrix}1 & 2\lambda_{12}\cdots 2\lambda_{1n}\\ 0 & 1\ \cdots 2\lambda_{2n}\\ \vdots & \vdots\qquad \vdots\\ 0 & 0\ \cdots\ 1\end{bmatrix}.$$

4. For $\omega=d\log\tau_n$ we have a general formula $-\frac{1}{2}\operatorname{trace}\sum_{l=2}^{\infty}\times(A(\Lambda)R)^{l-1}A(\Lambda)dR$ (see [5]). This yields an expansion in $\Lambda$ of $\omega$ by using the expressions for $A(\Lambda)$, $R$ and

(33) $$(dR_j)(p,p')=\frac{1}{\sqrt{\omega}\sqrt{\omega'}}\begin{pmatrix}\omega-\omega' & -\omega-\omega'\\ \omega+\omega' & -\omega+\omega'\end{pmatrix}e^{ia_j(p+p')}da_j$$
$$+\frac{1}{4\sqrt{\omega}\sqrt{\omega'}}\begin{pmatrix}\omega+\omega' & -\omega+\omega'\\ \omega-\omega' & -\omega-\omega'\end{pmatrix}e^{ia_j(p+p')}\sum_{s=1}^{4}\frac{dc_s}{(p-c_s)(p'+c_s)}.$$

Consider Taylor expansions: $\hat{Y}^{(1)}(p)=\sum_{m=0}^{\infty}\hat{Y}_{sm}(p-c_s)^m$ $(s=1,2)$, $\hat{Y}^{(2)}(p)=\sum_{m=0}^{\infty}\hat{Y}_{sm}(p-c_s)^m$ $(s=3,4)$ and $\hat{Y}^{(3)}(p)=\sum_{m=0}^{\infty}\hat{Y}_{\infty m}/p^m$. Expansions in $\Lambda$ of $\hat{Y}_{sm}$, $\hat{Y}_{\infty m}$ as well as of $\hat{Y}_{s0}^{-1}$ follow from (30) and (31). We note that $Y^{(3)}(p)$ is so normalized that $\hat{Y}_{\infty 0}=1$. We find

(34) $$\omega=\sum_{s=1}^{4}\frac{1}{2}\operatorname{trace}\hat{Y}_{s0}^{-1}\hat{Y}_{s1}L\,dc_s-\frac{1}{2}\operatorname{trace}\hat{Y}_{\infty 0}^{-1}\hat{Y}_{\infty 1}dA_\infty$$

by comparing the series expression for $\omega$ with that for the right hand side.

Setting $\Omega=dY^{(3)}\cdot Y^{(3)-1}$, we have

(35) $$\Omega=\sum_{s=1}^{4}A_s d\log(p-c_s)+d(pA_\infty)+\Theta$$

where $A_s=\hat{Y}_{s0}L\hat{Y}_{s0}^{-1}$, $\Theta=[\hat{Y}_{\infty 1},dA_\infty]$. The integrability condition for $\Omega$, $d\Omega-\Omega^2=0$, has been worked out by K. Ueno [9] in a more general situation. The resulting completely integrable deformation equations read

(36) $$dA_s=-\sum_{s'(\neq s)}[A_s,A_{s'}]d\log(c_s-c_{s'})-[A_s,d(c_sA_\infty)+\Theta],$$

(37) $$d\Theta=-[dA_\infty,\Gamma]+\Theta^2,$$

with $\Gamma=\sum_{s=1}^{4}A_s dc_s$. We also set $A_{\infty 1}=\sum_{s=1}^{4}A_s$ and $A_{\infty 2}=\sum_{s=1}^{4}A_s c_s$. Now (34) is rewritten as

(38) $$\omega=\frac{1}{2}\,\mathrm{trace}\Big(A_{\infty 2}dA_\infty+\frac{1}{2}\Theta A_{\infty 1}+\frac{1}{2}\sum_{s\neq s'}(A_sA_{s'}-L^2)d\log(c_s-c_{s'})+\Gamma A_\infty\Big).$$

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