27. *Studies on Holonomic Quantum Fields. XIII*

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This is a continuation of our preceding work [1] [2] on the theory of holonomic quantum fields in higher dimensional space-time. We shall deal here with the part corresponding to the deformation theory [3] [4] in the case of 2 space-time dimensions.

It has been pointed out previously [1] [5] that, in a most general setting, a Clifford group element g which induces a given rotation T is specified (up to a constant factor) by the following four operators:

$$(1)\qquad \begin{aligned} F_{++}&=Y_+^{-1}(-E_-)Y_+=-TE_-(E_++TE_-)^{-1}\\ F_{+-}&=Y_+^{-1}E_+Y_-=E_+(E_++TE_-)^{-1}T\\ F_{-+}&=Y_-^{-1}(-E_-)Y_+=-E_-(E_++TE_-)^{-1}\\ F_{--}&=Y_-^{-1}E_+Y_-=(E_++E_-T)^{-1}E_+.\end{aligned}$$

Moreover the vacuum expectation value $\langle g\otimes g^{-1}\rangle$ [1] is also expressible in terms of them (and the ones obtained by the replacement $T\mapsto T^{-1}$). Now we consider the specific case discussed in XII-§2 [2]; namely let T be a rotation in the space of free wave functions, defined as the multiplication by a matrix $M(\xi)$ on a spacelike hypersurface Γ. For simplicity we let $\Gamma=\{x^0=0\}$. Then the kernel functions $F_{\varepsilon\varepsilon'}(x,x')$ of $F_{\varepsilon\varepsilon'}$ $(\varepsilon,\varepsilon'=\pm)$ are analytically prolongable to the domain $\{\varepsilon x^s>0, \varepsilon' x'^s>0, x\neq x'\}$ $(x^0=-ix^s, x'^0=-ix'^s)$ of the Euclidean space $X^{\text{Euc}}=\boldsymbol{R}^s$. The resulting functions $F^{\text{Euc}}_{\varepsilon\varepsilon'}(x,x')$ are fundamental solutions of the Euclidean Dirac equation, and satisfy the boundary conditions

$$(2)\qquad \begin{aligned} F^{\text{Euc}}_{+\varepsilon'}(\xi,x')&=M(\xi)F^{\text{Euc}}_{-\varepsilon'}(\xi,x')\\ F^{\text{Euc}}_{\varepsilon+}(x,\xi')&=F^{\text{Euc}}_{\varepsilon-}(x,\xi')M(\xi')^{-1},\qquad \xi,\xi'\in\Gamma.\end{aligned}$$

In this sense they are solutions to a generalized Riemann-Hilbert problem. The purpose of this note is to characterize them by means of a variational formula of Hadamard's type [6] [7].

In §1 we formulate the Riemann-Hilbert problem for Euclidean Dirac equations, and state existence and uniqueness of the solution, assuming that $M(\xi)$ is close to 1. In §2 we give $M(\xi)$-preserving variational formulas for this solution $w(x,x')$ and its boundary values $w(x,\eta^+)$, $w(\xi^-,x')$ and $w(\xi^-,\eta^+)$, viewed as functionals of the boundary Γ. We also calculate their second variations, and state the complete integrability of the (first) variational equations. These equations, along with the integro-differential equations derived from the Euclidean covariance of $w(x, x')$, constitute natural generalizations of the extended holonomic system II-(12), (15) [3], (3.3.51)–(3.3.53) [4] in 2 dimensional space. In a coming note we shall show that the latter (as well as its massless version) is understood as a limiting case of our variational formulas.

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1. Let D^+ be a bounded domain of $X^{\mathrm{Euc}}=\boldsymbol{R}^s$ with real analytic boundary Γ. We set $D^-=X^{\mathrm{Euc}}-\overline{D^+}$. Let $M(\xi)$ be an $N\times N$ real analytic matrix defined on Γ. Let further γ^μ be $r\times r$ matrices satisfying $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2\delta^{\mu\nu}$ $(\mu,\nu=1,\cdots,s)$, and set $\partial\!\!\!/=\sum_{\mu=1}^s\gamma^\mu\partial_\mu$. We consider the following Riemann-Hilbert type problem for the Euclidean Dirac equation with positive mass m: Find a matrix $w(x, x')$ of size rN satisfying

(3) (i) $(-\partial\!\!\!/_x+m)w(x,x')=\delta^s(x-x')$ $\quad(x, x'\in X^{\mathrm{Euc}}-\Gamma)$

(ii) $|w(x,x')|=O(e^{-m|x|})$ $\quad(|x|\to\infty,\ x'$ fixed)

(iii) $w(\xi^+,x')=M(\xi)w(\xi^-,x')$ $\quad(\xi\in\Gamma,\ x'\notin\Gamma)$.

Here $w(\xi^\pm,x')=\lim_{D^\pm\ni x\to\xi}w(x,x')$, and $M(\xi)$ signifies $1_r\otimes M(\xi)$. Analogously we consider the "adjoint problem"

(3)′ (i)′ $w(x,x')(\overleftarrow{\partial\!\!\!/}_{x'}+m)=\delta^s(x-x')$ $\quad(x, x'\in X^{\mathrm{Euc}}-\Gamma)$

(ii)′ $|w(x,x')|=O(e^{-m|x'|})$ $\quad(x$ fixed, $|x'|\to\infty)$

(iii)′ $w(x,\xi'^+)=w(x,\xi'^-)M(\xi')^{-1}$ $\quad(x\notin\Gamma,\ \xi'\in\Gamma)$

where

$$w(x,x')\overleftarrow{\partial\!\!\!/}_{x'}=\sum_{\mu=1}^s\partial_\mu^{x'}w(x,x')\gamma^\mu \quad\text{and}\quad w(x,\xi'^\pm)=\lim_{D^\pm\ni x'\to\xi'}w(x,x').$$

Theorem 1. *Assume that* $\max_{\xi\in\Gamma}|M(\xi)-1_N|$ *is sufficiently small. Then the problems* (3) *and* (3)′ *admit unique solutions, which are in fact identical.*

We call this solution the Green's function for the Riemann-Hilbert problem (3), (3)′.

Uniqueness of the solution is easily seen by using the Green's formula. We sketch below the proof of existence. Let

$$S_{\mathrm{Euc}}(x)=(\partial\!\!\!/+m)\Delta_{\mathrm{Euc}}(x),\qquad \Delta_{\mathrm{Euc}}(x)=\frac{1}{2\pi}\left(\frac{m}{2\pi|x|}\right)^{s/2-1}K_{s/2-1}(m|x|),$$

denote a fundamental solution of the Euclidean Dirac equation, i.e. $(-\partial\!\!\!/+m)S_{\mathrm{Euc}}(x)=\delta^s(x)$. We seek for a solution of (3) in the form

$$w(x,x')=S_{\mathrm{Euc}}(x-x')+\int_\Gamma d\sigma(\xi)S_{\mathrm{Euc}}(x-\xi)\varkappa(\xi)u_{x'}(\xi). \tag{4}$$

Here $d\sigma(\xi)$ denotes the surface element of Γ, $\varkappa(\xi)=\sum_{\mu=1}^s\gamma^\mu n_\mu(\xi)$, and $n(\xi)=(n_1(\xi),\cdots,n_s(\xi))$ is the unit outer normal of Γ. Set

$$(E_\pm f)(\xi)=\pm\lim_{D^\pm\ni x\to\xi}\int_\Gamma d\sigma(\xi')S_{\mathrm{Euc}}(x-\xi')\varkappa(\xi')f(\xi'),$$

$(Mf)(\xi)=M(\xi)f(\xi)$. Then conditions (3)-(i)–(iii) hold if and only if

(5) $$(E_+ + ME_-)u_{x'}(\xi) = (M(\xi)-1)S_{\mathrm{Euc}}(\xi - x').$$

It is shown that (i) $E_\pm = 1 - E_\mp$ is a pseudo differential operator of order 0 on Γ, (ii) $E_+ + ME_- = 1 + (M-1)E_-$ is a bounded, invertible operator on $L^2(\Gamma; d\sigma)$, and (iii) $E_+ + ME_-$ is elliptic. Therefore (5) has a unique solution, which is real analytic on Γ. Problem (3)′ is treated similarly.

Remark. Analogous results hold for the massless case $m=0$. This time we impose the growth condition $O(1/|x|^{s-1})$ in place of (3)-(ii), (3)′-(ii)′.

2. For a fixed Γ the variation of $w(x, x')$ as a functional of M is given by

(6) $$\delta w(x, x') = \int_\Gamma d\sigma(\xi) w(x, \xi^+) \varkappa(\xi) \delta M(\xi) w(\xi^-, x').$$

$w(x, x')$ is characterized by (6) and the initial condition $w(x, x'; \Gamma, 1) = S_{\mathrm{Euc}}(x, x')$.

Next we vary Γ while preserving $M(\xi)$ in the sense of [2]. Namely given a vector field $\sum_{\mu=1}^{s} \rho^\mu(\xi)\partial_\mu$ we set $\Gamma^\rho = \{\xi^\rho = \xi + \rho(\xi) \mid \xi \in \Gamma\}$ and $M^\rho(\xi^\rho) = M(\xi)$. We denote by $w^\rho(x, x')$ the Green's function corresponding to (Γ^ρ, M^ρ) and by $\delta w^\rho(x, x')$ its variation as a functional of ρ. We abbreviate $\delta w^0(x, x')$ to $\delta w(x, x')$.

Theorem 2.

(7) $$\delta w(x, x') = \int_\Gamma d\sigma(\xi) \sum_{\mu=1}^{s} \delta\rho^\mu(\xi) w(x, \xi^+) \cdot (n_\mu \partial\!\!\!/ - \varkappa \partial_\mu) M(\xi) \cdot w(\xi^-, x').$$

For $\xi, \eta \in \Gamma$ we denote by $\delta' w(x, \eta^+)$, $'\delta w(\xi^-, x')$ and $'\delta' w(\xi^-, \eta^+)$ the variations at $\rho=0$ of $w^\rho(x, \eta^{\rho+})$, $w^\rho(\xi^{\rho-}, x')$ and $w^\rho(\xi^{\rho-}, \eta^{\rho+})$, respectively, as functionals of ρ. Then we have

Corollary 3.

(8) $$\delta' w(x, \eta^+) = \int_\Gamma d\sigma(\zeta) \sum_{\mu=1}^{s} \delta\rho^\mu(\zeta) w(x, \zeta) \cdot (n_\mu \partial\!\!\!/ - \varkappa \partial_\mu) M(\zeta) \cdot w(\zeta^-, \eta^+) + \sum_{\mu=1}^{s} \delta\rho^\mu(\eta) \partial_\mu^\eta w(x, \eta^+).$$

(9) $$'\delta w(\xi^-, x') = \int_\Gamma d\sigma(\zeta) \sum_{\mu=1}^{s} \delta\rho^\mu(\zeta) w(\xi^-, \zeta^+) \cdot (n_\mu \partial\!\!\!/ - \varkappa \partial_\mu) M(\zeta) \cdot w(\zeta^-, x') + \sum_{\mu=1}^{s} \delta\rho^\mu(\xi) \partial_\mu^\xi w(\xi^-, x').$$

(10) $$'\delta' w(\xi^-, \eta^+) = \int_\Gamma d\sigma(\zeta) \sum_{\mu=1}^{s} \delta\rho^\mu(\zeta) w(\xi^-, \zeta^+) \cdot (n_\mu \partial\!\!\!/ - \varkappa \partial_\mu) M(\zeta) \cdot w(\zeta^-, \eta^+) + \sum_{\mu=1}^{s} \delta\rho^\mu(\xi) \partial_\mu^\xi w(\xi^-, \eta^+) + \sum_{\mu=1}^{s} \delta\rho^\mu(\eta) \partial_\mu^\eta w(\xi^-, \eta^+).$$

We notice that by using the Euclidean Dirac equation $\partial_\mu^\eta w(x, \eta^+)$, $\partial_\mu^\xi w(\xi^-, x')$, $\partial_\mu^\xi w(\xi^-, \eta^+)$ and $\partial_\mu^\eta w(\xi^-, \eta^+)$ are rewritten in the forms containing only tangential derivatives. For example we have $\partial_\mu^\xi w(\xi^-, \eta^+) = (\partial_\mu^\xi - n_\mu(\xi)\varkappa(\xi)\partial\!\!\!/^\xi + m n_\mu(\xi)\varkappa(\xi)) w(\xi^-, \eta^+)$.

The second variation of $w^\rho(x, x')$ as a functional of ρ is defined by

(11) $$\delta^2 w^\rho(x,x')=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s \delta\rho^\mu(\zeta)\delta F^\rho_\mu(x,x';\zeta)$$

where $F^\rho_\mu(x,x';\zeta)$ is given by

(12) $$\delta w^\rho(x,x')=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s \delta\rho^\mu(\zeta)F^\rho_\mu(x,x';\zeta).$$

We abbreviate $\delta^2 w^0(x,x')$ to $\delta^2 w(x,x')$. $\delta'^2 w(x,\eta^+)$, $'\delta^2 w(\xi^-,x')$ and $'\delta'^2 w(\xi^-,\eta^+)$ are similarly defined.

We introduce the delta function $\delta(\xi,\eta)$ on Γ satisfying

$$\int_\Gamma d\sigma(\xi)\delta(\xi,\eta)=1.$$

Theorem 4.

(13) $$\begin{aligned}\delta^2 w(x,x')=&\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s\delta\rho^\mu(\zeta)\int_\Gamma d\sigma(\theta)\sum_{\nu=1}^s\delta\rho^\nu(\theta)\\
&\times\{w(x,\zeta)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\zeta)\cdot w(\zeta^-,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot w(\theta^-,x')\\
&+w(x,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot w(\theta^-,\zeta^+)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\zeta)\cdot w(\zeta^-,x')\\
&+w(x,\theta^+)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\theta)\cdot\partial^\zeta_{\nu t}\delta(\zeta,\theta)\cdot w(\theta^-,x')\\
&-w(x,\theta^+)(n_\mu\partial_\nu-n_\nu\partial_\mu)M(\theta)\cdot\mathscr{D}^\zeta_t\delta(\zeta,\theta)\cdot w(\theta^-,x')\\
&-w(x,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot\partial^\zeta_{\mu t}\delta(\zeta,\theta)\cdot w(\theta^-,x')\\
&+w(x,\theta^+)mn_\mu(\theta)n_\nu(\theta)[\mathscr{D}M(\theta),\varkappa(\theta)]w(\theta^-,x')\delta(\zeta,\theta)\\
&+w(x,\theta^+)(\overleftarrow{\partial}^\theta_\mu\varkappa-\overleftarrow{\mathscr{D}}^\theta n_\mu)\varkappa(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot w(\theta^-,x')\delta(\zeta,\theta)\\
&+w(x,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\varkappa(\varkappa\partial^\theta_\nu-n_\nu\mathscr{D}^\theta)w(\theta^-,x')\cdot\delta(\zeta,\theta)\}.\end{aligned}$$

(14) $$\begin{aligned}\delta'^2 w(x,\eta^+)=&\delta^2 w(x,\eta^+)+\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s\delta\rho^\mu(\zeta)\int_\Gamma d\sigma(\theta)\sum_{\nu=1}^s\delta\rho^\nu(\theta)\\
&\times\{w(x,\zeta^+)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\zeta)\cdot\partial^\eta_\nu w(\zeta^-,\eta^+)\delta(\eta,\theta)\\
&+w(x,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot\partial^\eta_\mu w(\theta^-,\eta^+)\delta(\eta,\zeta)\\
&+\partial^\eta_\mu\partial^\eta_\nu w(x,\eta^+)\cdot\delta(\eta,\zeta)\delta(\eta,\theta)\}.\end{aligned}$$

(15) $$\begin{aligned}'\delta^2 w(\xi^-,x')=&\delta^2 w(\xi^-,x')+\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s\delta\rho^\mu(\zeta)\int_\Gamma d\sigma(\theta)\sum_{\nu=1}^s\delta\rho^\nu(\theta)\\
&\times\{\partial^\xi_\mu w(\xi^-,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot w(\theta^-,x')\delta(\xi,\zeta)\\
&+\partial^\xi_\nu w(\xi^-,\zeta^+)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\zeta)\cdot w(\zeta^-,x')\delta(\xi,\theta)\\
&+\partial^\xi_\mu\partial^\xi_\nu w(\xi^-,x')\cdot\delta(\xi,\zeta)\delta(\xi,\theta)\}.\end{aligned}$$

(16) $$\begin{aligned}'\delta'^2 w(\xi^-,\eta^+)=&\delta'^2 w(\xi^-,\eta^+)+\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s\delta\rho^\mu(\zeta)\int_\Gamma d\sigma(\theta)\sum_{\nu=1}^s\delta\rho^\nu(\theta)\\
&\times\{\partial^\xi_\mu w(\xi^-,\theta^+)(n_\nu\mathscr{D}-\varkappa\partial_\nu)M(\theta)\cdot w(\theta^-,\eta^+)\delta(\xi,\zeta)\\
&+\partial^\xi_\nu w(\xi^-,\zeta^+)(n_\mu\mathscr{D}-\varkappa\partial_\mu)M(\zeta)\cdot w(\zeta^-,\eta^+)\delta(\xi,\theta)\\
&+\partial^\xi_\mu\partial^\xi_\nu w(\xi^-,\eta^+)\cdot\delta(\xi,\zeta)\delta(\xi,\theta)+\partial^\xi_\mu\partial^\eta_\nu w(\xi^-,\eta^+)\cdot\delta(\xi,\zeta)\delta(\eta,\theta)\\
&+\partial^\xi_\nu\partial^\eta_\mu w(\xi^-,\eta^+)\cdot\delta(\xi,\theta)\delta(\eta,\zeta)\}.\end{aligned}$$

Here $\partial_{\nu t}=\partial_\nu-n_\nu\sum_{\lambda=1}^s n_\lambda\partial_\lambda$ denotes the tangential component of ∂_ν, and $\mathscr{D}_t=\sum_{\mu=1}^s\gamma^\mu\partial_{\mu t}$.

A functional differential equation of the form

(17) $$\delta w(x,x')=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^s\delta\rho^\mu(\zeta)F_\mu(x,x';\zeta)$$

is said to be completely integrable if $F_{\mu\nu}(x,x';\zeta,\theta)=F_{\nu\mu}(x,x';\theta,\zeta)$ where $F_{\mu\nu}(x,x';\zeta,\theta)$ is given by

$$(18)\qquad \delta F^0_\mu(x,x';\zeta)=\int_\Gamma d\sigma(\theta)\sum_{\mu=1}^{s}\delta\rho^\nu(\theta)F_{\mu\nu}(x,x';\zeta,\theta).$$

In the course of proof of Theorem 4 we see that (7) is completely integrable and that (8), (9) and (10) are also completely integrable in a similar sense. Moreover the following systems of functional differential equations, (19)+(20)+(21)+(22) or (20)+(22) or (21)+(22), are completely integrable.

$$(19)\qquad \delta w_1(x,x')=\int_\Gamma d\sigma(\xi)\sum_{\mu=1}^{s}\delta\rho^\mu(\xi)w_2(x,\xi)\cdot(n_\mu\partial\!\!\!/-\varkappa\partial_\mu)M(\xi)\cdot w_3(\xi,x').$$

$$(20)\qquad \delta w_2(x,\eta)=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^{s}\delta\rho^\mu(\zeta)w_2(x,\zeta)\cdot(n_\mu\partial\!\!\!/-\varkappa\partial_\mu)M(\zeta)\cdot w_4(\zeta,\eta)$$
$$+\sum_{\mu=1}^{s}\delta\rho^\mu(\eta)\cdot w_2(x,\eta)(\overleftarrow{\partial}{}^\eta_\mu-\overleftarrow{\partial\!\!\!/}{}^\eta n_\mu\varkappa-mn_\mu\varkappa).$$

$$(21)\qquad \delta w_3(\xi,x')=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^{s}\delta\rho^\mu(\zeta)w_4(\xi,\zeta)\cdot(n_\mu\partial\!\!\!/-\varkappa\partial_\mu)M(\zeta)\cdot w_3(\zeta,x')$$
$$+\sum_{\mu=1}^{s}\delta\rho^\mu(\xi)\cdot(\partial^\xi_\mu-n_\mu\varkappa\partial\!\!\!/^\xi+mn_\mu\varkappa)w_3(\xi,x').$$

$$(22)\qquad \delta w_4(\xi,\eta)=\int_\Gamma d\sigma(\zeta)\sum_{\mu=1}^{s}\delta\rho^\mu(\zeta)w_4(\xi,\zeta)\cdot(n_\mu\partial\!\!\!/-\varkappa\partial_\mu)M(\zeta)\cdot w_4(\zeta,\eta)$$
$$+\sum_{\mu=1}^{s}\delta\rho^\mu(\eta)\cdot w_4(\xi,\eta)(\overleftarrow{\partial}{}^\eta_\mu-\overleftarrow{\partial\!\!\!/}{}^\eta n_\mu(\eta)\varkappa(\eta)-mn_\mu(\eta)\varkappa(\eta))$$
$$+\sum_{\mu=1}^{s}\delta\rho^\mu(\xi)\cdot(\partial^\xi_\mu-n_\mu(\xi)\varkappa(\xi)\partial\!\!\!/^\xi+mn_\mu(\xi)\varkappa(\xi))w_4(\xi,\eta).$$

The Euclidean covariance of $w(x,x')$ and the variational formula (7) implies the following integro-differential equations.

$$(23)\qquad \partial^x_\mu w(x,x')+\partial^{x'}w(x,x')$$
$$+\int_\Gamma d\sigma(\xi)w(x,\xi^+)\cdot(n_\mu\partial\!\!\!/-\varkappa\partial_\mu)M(\xi)\cdot w(\xi^-,x')=0,$$

$$(24)\qquad \left(x^\mu\partial^x_\nu-x^\nu\partial^x_\mu+\frac{1}{2}\gamma^{\mu\nu}\right)w(x,x')+w(x,x')\left(\overleftarrow{\partial}{}^{x'}_\nu x'^\mu-\overleftarrow{\partial}{}^{x'}_\mu x'^\nu-\frac{1}{2}\gamma^{\mu\nu}\right)$$
$$+\int_\Gamma \delta\sigma(\xi)w(x,\xi^+)\{(n_\nu\xi^\mu-n_\mu\xi^\nu)\partial\!\!\!/-\varkappa(\xi^\mu\partial_\nu-\xi^\nu\partial_\mu)\}M(\xi)\cdot w(\xi^-,x')=0.$$

Here we have set $\gamma^{\mu\nu}=(1/2)[\gamma^\mu,\gamma^\nu]$. Then specializing x' to η^+ we obtain

$$(25)\qquad \{(x^\mu-\eta^\mu)\partial^x_\nu-(x^\nu-\eta^\nu)\partial^x_\mu\}w(x,\eta^+)+\frac{1}{2}[\gamma^{\mu\nu},w(x,\eta^+)]$$
$$+\int_\Gamma d\sigma(\xi)w(x,\xi^+)\cdot\{(n_\nu(\xi^\mu-\eta^\mu)-n_\mu(\xi^\nu-\eta^\nu))\partial\!\!\!/$$
$$-\varkappa((\xi^\mu-\eta^\mu)\partial_\nu-(\xi^\nu-\eta^\nu)\partial_\mu)\}M(\xi)\cdot w(\xi^-,\eta^+)=0.$$

We may regard (8) and (25) as linear equations for the quantity $w(x,\eta^+)$, having $w(\xi^-,\eta^+)$ as an unknown coefficient. On the other side the latter is governed by the non-linear functional differential equations (10). In this sense these are higher-dimensional analogues of the extended holonomic system II-(12), (15) and the deformation equations II-(17) [3].

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