57. On the 2-Components of the Unstable Homotopy Groups of Spheres. II

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This note is the continuation of the part I with the same title. We will state the results on the 2-components of the unstable homotopy groups of spheres for the following cases: π^n_{n+29} and π^n_{n+30} for all n*); π^n_{n+31} for n*) $\leqq 29$. Moreover, the following groups will be given: π^n_{n+32} and π^n_{n+33} for n*) $\leqq 8$. But the group π^9_{40} is not determined completely and the group extensions are not settled for π^{10}_{41} and π^n_{n+33} for $n=6, 7$ and 8.

5. On the 29-stem. *There are following new elements:* $\tilde{\varepsilon}'$, $\delta' \in \pi^6_{35}$ *and* $\delta'' \in \pi^7_{36}$ *with the Hopf invariants* $\pm\tilde{\varepsilon}_{11}$ *(mod other elements),* δ_{11} *(mod* $\bar{\mu}_{11} \circ \sigma_{28}$*), and* ϕ_{13} *(mod* $4\nu_{13} \circ \bar{\kappa}_{16}$*) respectively.*

$$\pi^3_{32} = Z_2\{\bar{\alpha} \circ \nu^2_{26}\} \oplus Z_2\{\nu' \circ \eta_6 \circ \mu_{3,7}\} \oplus Z_2\{\eta_3 \circ \varepsilon_4 \circ \bar{\kappa}_{12}\},$$

$$\pi^5_{34} = Z_2\{\phi_5 \circ \nu^2_{28}\} \oplus Z_2\{\nu_5 \circ \bar{\kappa}_8 \circ \nu^2_{28}\} \oplus Z_2\{\nu_5 \circ \bar{\sigma}_8 \circ \sigma_{27}\} \oplus Z_2\{\nu^3_5 \circ \bar{\kappa}_{14}\} \oplus Z_2\{\nu_5 \circ \eta_8 \circ \mu_{3,9}\} \oplus Z_2\{\eta_5 \circ \varepsilon'_6 \circ \bar{\kappa}_{14}\}.$$

In the above group, the following relation holds: $\phi_5 \circ \nu^2_{28} \equiv \nu_5 \circ \sigma_8 \circ \bar{\sigma}_{15}$ (*mod* $\nu_5 \circ \bar{\kappa}_8 \circ \nu^2_{28} - \nu^2_5 \circ \bar{\kappa}_{11} \circ \nu_{31}$).

Now we define elements by Toda brackets: $\delta' \in \{\sigma'' \circ \sigma_{13}, \sigma_{20}, 2\sigma_{27}\}_3$, $\delta'' \in \{\sigma' \circ \sigma_{14}, \sigma_{21}, 2\sigma_{28}\}_4$. Then we have $2\delta'' = -E\delta'$ *and* $E^2\delta'' = 2(\sigma_9 \circ \sigma^*_{16})$. Moreover there are following important results: $\Delta(\tilde{\varepsilon}_{13}) = 2\tilde{\varepsilon}'$ *for some* $\tilde{\varepsilon}' \in \pi^6_{35}$ *and* $2\delta' \equiv \nu^3_6 \circ \bar{\kappa}_{15} = \nu_6 \circ \bar{\kappa}_9 \circ \nu^2_{29}$ (*mod* $\nu_6 \circ \sigma_9 \circ \bar{\sigma}_{16}$). Using these results, we have

$$\pi^6_{35} = Z_4\{\delta'\} \oplus Z_4\{\tilde{\varepsilon}'\} \oplus Z_2\{\phi_6 \circ \nu^2_{29}\} \oplus Z_2\{\eta_6 \circ \varepsilon_7 \circ \bar{\kappa}_{15}\},$$

$$\pi^7_{36} = Z_8\{\delta''\} \oplus Z_2\{\sigma' \circ \varepsilon_{14} \circ \kappa_{22}\} \oplus Z_2\{\sigma' \circ \omega_{14} \circ \nu^2_{30}\} \oplus Z_2\{\phi_7 \circ \nu^2_{30}\} \oplus Z_2\{\eta_7 \circ \varepsilon_8 \circ \bar{\kappa}_{16}\}.$$

In the above group, we have $\sigma' \circ \omega_{14} \circ \nu^2_{30} \equiv E\tilde{\varepsilon}'$ (*mod* $E^2\pi^5_{34}$). This is obtained showing that $\sigma' \circ \omega_{14} \circ \nu^2_{30}$ *is not double suspended*: If $\sigma' \circ \omega_{14} \circ \nu^2_{30} \in E^2\pi^5_{34}$, we may construct the Toda bracket $\{\sigma' \circ \omega_{14} + x\phi_7 + y\nu_7 \circ \bar{\kappa}_{10}, \nu^2_{30}, 2\iota_{36}\}_1$ whose Hopf invariant is $\tilde{\varepsilon}_{13}$ (*mod other elements*). *Then we see* $\Delta\pi^{13}_{37} = 0$, *which contradicts the fact that* $H\Delta(\tilde{\varepsilon}_{13}) = 2\tilde{\varepsilon}_{11} \neq 0$.

$$\pi^9_{38} = Z_{16}\{\sigma_9 \circ \sigma^*_{16}\} \oplus Z_2\{\sigma_9 \circ \omega_{16} \circ \nu^2_{32}\} \oplus Z_2\{\sigma_9 \circ \varepsilon_{16} \circ \kappa_{24}\} \oplus Z_2\{\sigma_9 \circ \nu_{16} \circ \bar{\sigma}_{19}\} \oplus Z_2\{\eta_9 \circ \varepsilon_{10} \circ \bar{\kappa}_{18}\}.$$

$$\pi^{10}_{39} = Z_8\{\Delta(\bar{\kappa}_{21})\} \oplus Z_2\{\Delta(EA_2)\} \oplus Z_{16}\{\sigma_{10} \circ \sigma^*_{17}\} \oplus Z_2\{\sigma_{10} \circ \nu_{17} \circ \bar{\sigma}_{20}\}.$$

This results from the relation $4\Delta(\bar{\kappa}_{21}) = \sigma_{10} \circ \varepsilon_{17} \circ \kappa_{25}$.

We will use hereafter the metastable periodic elements: $\pi^{11}_{40} = Z_2\{C_1 \circ \bar{\mu}_{23}\} \oplus Z_{16}\{\sigma_{11} \circ \sigma^*_{18}\} \oplus Z_2\{\sigma_{11} \circ \nu_{18} \circ \bar{\sigma}_{21}\}$, $\pi^{12}_{41} = Z_4\{\Delta(\nu^*_{25}) + 2\sigma_{12} \circ \sigma^*_{19}\} \oplus Z_2\{A_1 \circ \bar{\mu}_{24}\} \oplus Z_2$

*) We omit the cases that $n=2, 4$ and 8 (c.f. Proposition 4.4 of [11]).

$\cdot\{EC_1 \circ \bar{\mu}_{24}\} \oplus Z_{16}\{\sigma_{12} \circ \sigma_{19}^*\}$, $\pi_{42}^{13} = Z_2\{EA_1 \circ \bar{\mu}_{25}\} \oplus Z_8\{\sigma_{13} \circ \sigma_{20}^*\}$, $\pi_{43}^{14} = Z_4\{\sigma_{14} \circ \sigma_{21}^*\}$, $\pi_{44}^{15} = Z_2\{L_1\} \oplus Z_2\{\sigma_{15} \circ \sigma_{22}^*\}$.

Let us choose an element $P_1 \in \{\sigma_{16}^2, 2\iota_{30}, \kappa_{30}\}_1$. This enables us to determine $\pi_{45}^{16} = Z_2\{\sigma_{16}^* \circ \sigma_{38}\} \oplus Z_2\{P_1\} \oplus \pi_{44}^{15}$, $\pi_{46}^{17} = Z_2\{\sigma_{17}^* \circ \sigma_{39}\} \oplus Z_2\{EP_1\}$, $\pi_{47}^{18} = \pi_{46}^{17}$, $\pi_{48}^{19} = Z_2\{C_2 \circ \mu_{39}\} \oplus \pi_{46}^{17}$, $\pi_{49}^{20} = Z_2\{A_2 \circ \mu_{40}\} \oplus \pi_{48}^{19}$.

Showing that $E^7P_1 = 0$ and that $\Delta(\varepsilon_{45}) = \Delta(\bar{\nu}_{45})$ *is divisible by* 2, we obtain the relations $E^5P_1 = 2M_2' \circ \nu_{47}$ *and* $\Delta(\varepsilon_{45}) = E^6P_1 = 2EM_2' \circ \nu_{48}$. It follows that $\pi_{50}^{21} = Z_4\{M_2' \circ \nu_{47}\} \oplus Z_2\{EA_2 \circ \mu_{41}\} \oplus Z_2\{\sigma_{21}^* \circ \sigma_{43}\}$, $\pi_{51}^{22} = Z_4\{EM_2' \circ \nu_{48}\} \oplus Z_2\{\sigma_{22}^* \circ \sigma_{44}\}$.

It is not difficult to show that $2(M_2 \circ \nu_{50}) = 0$ *and* $\Delta(\nu_{49}^2) = E^3M_2' \circ \nu_{50}$. Then we have $\pi_{52}^{23} = Z_2\{E^2M_2' \circ \nu_{49}\} \oplus Z_2\{\sigma_{23}^* \circ \sigma_{45}\}$, $\pi_{53}^{24} = Z_2\{M_2 \circ \nu_{50}\} \oplus Z_2\{E^3M_2' \circ \nu_{50}\}$, $\pi_{54}^{25} = Z_2\{EM_2 \circ \nu_{51}\}$, $\pi_{55}^{26} = \pi_{54}^{25}$, $\pi_{56}^{27} = Z_2\{C_3 \circ \eta_{55}\} \oplus \pi_{54}^{25}$, $\pi_{57}^{28} = Z_2\{A_3 \circ \eta_{56}\} \oplus Z_2\{EC_3 \circ \eta_{56}\}$, $\pi_{58}^{29} = Z_2\{EA_3 \circ \eta_{57}\}$, $\pi_{59}^{30} = Z\{\Delta(\iota_{61})\}$, $\pi_{n+29}^{n} = 0$ *for* $n \geqq 31$.

6. On the 30-stem. *There are following new elements:* $\theta^{\mathrm{VII}} \in \pi_{42}^{12}$, $\theta^{\mathrm{VI}} \in \pi_{44}^{14}$, $\theta^{\mathrm{V}} \in \pi_{45}^{15}$, $\theta^{\mathrm{IV}} \in \pi_{46}^{16}$, $\theta''' \in \pi_{50}^{20}$, $\theta'' \in \pi_{52}^{22}$, $\theta_{23}' \in \pi_{53}^{23}$ *with the Hopf invariants* $\bar{\zeta}_{23}$ *(mod* $2\bar{\zeta}_{23}$*)*, $\eta_{27} \circ \sigma_{28} \circ \mu_{35}$, $\sigma_{29} \circ \mu_{36}$, ρ_{31} *(mod* $2\rho_{31}$*)*, ζ_{39} *(mod* $2\zeta_{39}$*)*, $\eta_{43}^2 \circ \sigma_{45}$, $\eta_{45} \circ \sigma_{46}$ *respectively.*

For $n \leqq 8$, the group extensions are obtained making use of the known relations:

$$\pi_{33}^3 = Z_4\{\varepsilon' \circ \bar{\kappa}_{13}\} \oplus Z_2\{\varepsilon_3 \circ \nu_{11} \circ \bar{\sigma}_{14}\},$$
$$\pi_{35}^5 = Z_8\{\nu_5 \circ \sigma_8 \circ \bar{\kappa}_{15}\} \oplus Z_2\{\phi_5 \circ \sigma_{28}\} \oplus Z_2\{\nu_5 \circ \zeta_{3,8}\} \oplus Z_2\{\nu_5 \circ \bar{\nu}_8 \circ \bar{\sigma}_{16}\},$$
$$\pi_{36}^6 = Z_4\{\Delta(\xi_{13} \circ \sigma_{31})\} \oplus Z_4\{\sigma'' \circ \bar{\rho}_{13}\} \oplus Z_8\{\nu_6 \circ \sigma_9 \circ \bar{\kappa}_{16}\} \oplus Z_2\{\phi_6 \circ \sigma_{29}\},$$
$$\pi_{37}^7 = Z_8\{\sigma' \circ \bar{\rho}_{14}\} \oplus Z_2\{\sigma' \circ \phi_{14}\} \oplus Z_2\{\sigma' \circ \psi_{14}\} \oplus Z_8\{\nu_7 \circ \sigma_{10} \circ \bar{\kappa}_{17}\} \oplus Z_2\{\phi_7 \circ \sigma_{30}\}.$$

The relation $\Delta(\sigma_{21}^3) = 2\psi_{10} \circ \sigma_{33} = \sigma_{10} \circ \phi_{17}$ implies

$$\pi_{39}^9 = Z_{16}\{\sigma_9 \circ \bar{\rho}_{16}\} \oplus Z_8\{\sigma_9 \circ \nu_{16} \circ \bar{\kappa}_{19}\} \oplus Z_2\{\sigma_9 \circ \phi_{16}\} \oplus Z_2\{\sigma_9 \circ \psi_{16}\} \oplus Z_2\{\phi_9 \circ \sigma_{32}\},$$
$$\pi_{40}^{10} = Z_2\{\Delta(EA_2 \circ \eta_{41})\} \oplus Z_4\{\psi_{10} \circ \sigma_{33}\} \oplus Z_{16}\{\sigma_{10} \circ \bar{\rho}_{17}\} \oplus Z_4\{\sigma_{10} \circ \nu_{17} \circ \bar{\kappa}_{20}\} \oplus Z_2\{\sigma_{10} \circ \psi_{17}\},$$
$$\pi_{41}^{11} = Z_2\{\psi_{11} \circ \sigma_{34}\} \oplus Z_{16}\{\sigma_{11} \circ \bar{\rho}_{18}\} \oplus Z_2\{\sigma_{11} \circ \nu_{18} \circ \bar{\kappa}_{21}\} \oplus Z_2\{\sigma_{11} \circ \psi_{18}\}.$$

We have to define elements by Toda brackets: $\theta^{\mathrm{VII}} \in \{\sigma_{12}, \nu_{19}, \bar{\zeta}_{22}\}_1$, $\theta^{\mathrm{VI}} \in \{8\sigma_{14}, \sigma_{21}, \rho_{28}\}_1$, $\theta^{\mathrm{V}} \in \{4\sigma_{15}, \sigma_{22}, \rho_{29}\}_1$, $\theta^{\mathrm{IV}} \in \{2\sigma_{16}, \sigma_{23}, \rho_{30}\}_1$, $\theta''' \in \{A_2, \eta_{40}^2 \circ \sigma_{42}, 2\iota_{49}\}_1$, $\theta_{23}' \in \{2\sigma_{23}, \sigma_{30}, 2\sigma_{37}, \sigma_{44}\}_1$. This enables us to determine

$$\pi_{42}^{12} = Z_{32}\{\theta^{\mathrm{VII}}\} \oplus Z_4\{\sigma_{12} \circ \bar{\rho}_{19} \pm 2\theta^{\mathrm{VII}}\} \oplus Z_2\{\psi_{12} \circ \sigma_{35}\} \oplus Z_2\{\sigma_{12} \circ \psi_{19}\},$$
$$\pi_{43}^{13} = Z_{32}\{\rho_{13}^2\} \oplus Z_2\{\psi_{13} \circ \sigma_{36}\},$$
$$\pi_{44}^{14} = Z_{64}\{\theta^{\mathrm{VI}}\} \oplus Z_2\{\omega_{14} \circ \kappa_{30}\} \oplus Z_2\{\psi_{14} \circ \sigma_{37}\},$$
$$\pi_{45}^{15} = Z_{64}\{\theta^{\mathrm{V}}\} \oplus Z_2\{\omega_{15} \circ \kappa_{31}\} \oplus Z_2\{\psi_{15} \circ \sigma_{38}\},$$
$$\pi_{46}^{16} = Z_{128}\{\theta^{\mathrm{IV}}\} \oplus Z_{16}\{E\theta^{\mathrm{V}} \pm 2\theta^{\mathrm{IV}}\} \oplus Z_2\{B_1 \circ \kappa_{32}\} \oplus Z_2\{\omega_{16} \circ \kappa_{32}\} \oplus Z_2\{\psi_{16} \circ \sigma_{39}\}.$$

In the above groups, we have the following relations: $8\theta^{\mathrm{VII}} = \pm 4\sigma_{12} \circ \bar{\rho}_{19}$, $\rho_{13}^2 \equiv E\theta^{\mathrm{VII}}$ *(mod* $2E\theta^{\mathrm{VII}}$*)*, $2\theta^{\mathrm{VI}} \equiv \rho_{14}^2$ *(mod* $2\rho_{14}^2$*)*, $2\theta^{\mathrm{V}} = E\theta^{\mathrm{VI}}$ *(mod* $2E\theta^{\mathrm{VI}}$*)*, $32\theta^{\mathrm{IV}} = \pm 16E\theta^{\mathrm{V}}$.

Next, we have

$$\pi_{47}^{17} = Z_{64}\{E\theta^{\mathrm{IV}}\} \oplus Z_2\{EB_1 \circ \kappa_{33}\} \oplus Z_2\{\psi_{17} \circ \sigma_{40}\},\ \pi_{48}^{18} = Z_{64}\{E^2\theta^{\mathrm{IV}}\},\ \pi_{49}^{19} = \pi_{48}^{18}.$$

From the relation $4(E^4\theta^{\mathrm{IV}} - 2x\theta''') = 0$ $(x$*: odd*$)$, it follows that

$$\pi_{50}^{20} = Z_{128}\{\theta'''\} \oplus Z_4\{E^4\theta^{\mathrm{IV}} - 2x\theta'''\},\ \pi_{51}^{21} = Z_{64}\{E\theta'''\}.$$

We will use Proposition 11.13 of Toda [11], to show the existence of θ'' such that $E\theta''=2\theta'_{23}$ which appears in the next

$$\pi_{52}^{22}=Z_{64}\{\theta''\}\oplus Z_2\{V_2\circ\nu_{49}\},\ \pi_{53}^{23}=Z_{64}\{\theta'_{23}\}\oplus Z_2\{EV_2\circ\nu_{50}\}.$$

A result of J. F. Adams gives us an easy computation of the remaining part of the 31-stem: According to Corollary 1.3 of Adams [1], $\Delta(\iota_{63})=[\iota_{31},\iota_{31}]$ *is a 9-fold suspension but not a 10-fold suspension.*

Hence we see $\pi_{54}^{24}=Z_8\{\Delta(\sigma_{49})-4\theta'_{24}\}\oplus Z_{64}\{\theta'_{24}\}\oplus Z_2\{E^2V_2\circ\nu_{51}\}$, $\pi_{55}^{25}=Z_{32}\{\theta'_{25}\}\oplus Z_2\{E^3V_2\circ\nu_{52}\}$, $\pi_{56}^{26}=Z_{32}\{\theta'_{26}\}$, $\pi_{57}^{27}=\pi_{56}^{26}$, $\pi_{58}^{28}=Z_4\{\Delta(\nu_{57})-4\theta'_{28}\}\oplus Z_{32}\{\theta'_{28}\}$, $\pi_{59}^{29}=Z_{16}\{\theta'_{29}\}$, $\pi_{64}^{30}=Z_8\{\theta'_{30}\}$, $\pi_{61}^{31}=Z_4\{\theta'_{31}\}$, $\pi_{n+30}^{n}=Z_2\{\theta'_n\}$ *for* $n\geqq 32$.

7. On the 31-stem. *There are following new elements:* $\kappa_{10}^*\in\pi_{41}^{10}$, $\omega_{14}^*\in\pi_{45}^{14}$, $\kappa^{*\prime}\in\pi_{46}^{15}$ *with the Hopf invariants* $\nu_{19}\circ\bar{\sigma}_{22}$, $\pm\nu_{27}^*$, $\nu_{29}\circ\kappa_{32}$ *respectively.*

We have the relations: $2\alpha'_3=0$, $E\alpha'_3=2\alpha''_3$, $E\alpha''_3=2\alpha'''_3$, $E^2\alpha'''_3=2\alpha_3^{\mathrm{IV}}$ $(mod\ \pi_{33}^9\circ\sigma_{33})$.

$$\pi_{34}^3=Z_2\{\delta_3\circ\sigma_{27}\}\oplus Z_2\{\varepsilon_3\circ\nu_{11}\circ\bar{\kappa}_{14}\}\oplus Z_2\{\nu'\circ\varepsilon_6\circ\bar{\kappa}_{14}\}\oplus Z_2\{\phi'\circ\nu_{28}^2\},$$

$$\pi_{36}^5=Z_2\{\nu_5\circ E\phi'''\circ\nu_{33}\}\oplus Z_2\{\nu_5\circ\bar{\nu}_8\circ\bar{\kappa}_{16}\}\oplus Z_2\{\nu_5\circ\varepsilon_8\circ\bar{\kappa}_{16}\}\oplus Z_2\{\delta_5\circ\sigma_{29}\}\oplus Z_2\{\alpha'_3\}.$$

We note that the relation $2\bar{\nu}_6\circ\nu_{14}\circ\bar{\kappa}_{17}=\nu_6\circ\varepsilon_9\circ\bar{\kappa}_{17}$ holds in the next

$$\pi_{37}^6=Z_8\{\Delta(\tau^{\mathrm{IV}})\}\oplus Z_2\{\Delta(EA_1\circ\kappa_{25})\}\oplus Z_4\{\bar{\nu}_6\circ\nu_{14}\circ\bar{\kappa}_{17}\}\oplus Z_2\{\nu_6\circ E^2\phi'''\circ\nu_{34}\}\oplus Z_2\{\delta_6\circ\sigma_{30}\}\oplus Z_4\{\alpha''_3\}.$$

$$\pi_{38}^7=Z_2\{\sigma'\circ\bar{\sigma}'_{14}\}\oplus Z_2\{\sigma'\circ\bar{\mu}_{14}\circ\sigma_{31}\}\oplus Z_2\{\bar{\nu}_7\circ\nu_{15}\circ\bar{\kappa}_{18}\}\oplus Z_2\{\delta_7\circ\sigma_{31}\}\oplus Z_8\{\alpha'''_3\}.$$

Following two groups are not determined completely.

$$\pi_{40}^9=Z_2\{\sigma_9\circ\delta_{16}\}\oplus Z_2\{\sigma_9\circ\bar{\mu}_{16}\circ\sigma_{33}\}\oplus Z_2\{\sigma_9\circ\bar{\sigma}'_{16}\}\oplus Z_2\{\delta_9\circ\sigma_{33}\}\oplus(0\ or\ Z_2)\{\bar{\nu}_9\circ\nu_{17}\circ\bar{\kappa}_{20}\}\oplus Z_{16}\{\alpha_3^{\mathrm{IV}}\},$$

The last direct summand but one does not affect the next group, since $\bar{\nu}_{10}\circ\nu_{18}\circ\bar{\kappa}_{21}=0$.

$$\pi_{41}^{10}=Z_8\{\Delta(\sigma_{21}^*)\}\oplus(Z_4\ or\ Z_2\oplus Z_2)\{\kappa_{10}^*,\delta_{10}\circ\sigma_{34}\}\oplus Z_{16}\{E\alpha_3^{\mathrm{IV}}\}.$$

Although the above group extension is not a complete one, we have the relation $\delta_{11}\circ\sigma_{35}=0$. Hence we obtain the complete group structure in the next stage.

$$\pi_{42}^{11}=Z_2\{\kappa_{11}^*\}\oplus Z_{16}\{E^2\alpha_3^{\mathrm{IV}}\},\ \pi_{43}^{12}=Z_4\{\Delta(\bar{\kappa}_{25})\}\oplus\pi_{42}^{11}.$$

We define an element $\alpha_3^{\mathrm{V}}\in\{\rho_{13},32\iota_{28},\rho_{28}\}_1$. Then $2\alpha_3^{\mathrm{V}}=E^4\alpha_3^{\mathrm{IV}}$ *and* $H(\alpha_3^{\mathrm{V}})=4\bar{\zeta}_{25}$. Thus we have

$$\pi_{44}^{13}=Z_2\{\kappa_{13}^*\}\oplus Z_{32}\{\alpha_3^{\mathrm{V}}\}.$$

Let us choose an element $\kappa^{*\prime}\in\{\omega_{15},2\iota_{31},\kappa_{31}\}_1$, and make use of the periodic elements [9] to obtain the following isomorphisms: $\pi_{46}^{15}=Z_2\{D_1^{\mathrm{II}}\}\oplus Z_2\{D_1^{(1)}\circ\sigma_{39}\}\oplus Z_2\{\kappa^{*\prime}\}\oplus E\pi_{45}^{14}$, $\pi_{47}^{16}=Z_2\{B_1^{\mathrm{II}}\}\oplus Z_2\{B_1^{(1)}\circ\sigma_{40}\}\oplus\pi_{46}^{15}$, $\pi_{48}^{17}=Z_2\{EB_1^{\mathrm{II}}\}\oplus Z_2\{EB_1^{(1)}\circ\sigma_{41}\}\oplus Z_2\{E^2\kappa^{*\prime}\}\oplus E\pi_{45}^{14}$*), $\pi_{49}^{18}=Z_2\{E^2B_1^{\mathrm{II}}\}\oplus Z_2\{E^3\kappa^{*\prime}\}\oplus E\pi_{45}^{14}$, $\pi_{n+31}^{n}=E\pi_{45}^{14}$ *for* $n=19,20,21,22$.

Similarly we have $\pi_{54}^{23}=Z_2\{D_2^{\mathrm{I}}\}\oplus Z_2\{D_2\circ\sigma_{47}\}\oplus E\pi_{45}^{14}$, $\pi_{55}^{24}=Z_2\{B_2^{\mathrm{I}}\}\oplus Z_2\{B_2\circ\sigma_{48}\}\oplus\pi_{54}^{23}$, $\pi_{56}^{25}=Z_2\{EB_2^{\mathrm{I}}\}\oplus Z_2\{EB_2\circ\sigma_{49}\}\oplus E\pi_{45}^{14}$, $\pi_{57}^{26}=Z_2\{E^2B_2^{\mathrm{I}}\}\oplus E\pi_{45}^{14}$, $\pi_{n+31}^{n}=E\pi_{45}^{14}$ *for* $n=27,28,29$.

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*) The direct summand $E\pi_{45}^{14}$ must be understood as the image of the iterated suspension whose restriction to this group is incidentally monic.

We have to show the following results;

$$\pi_{45}^{14}=Z_8\{\omega_{14}^*\}\oplus Z_2\{\kappa_{14}^*\}\oplus Z_{64}\{\rho_{3,14}\},\ E\pi_{45}^{14}=Z_2\{\omega_{15}^*\}\oplus Z_2\{\kappa_{15}^*\}\oplus Z_{64}\{\rho_{3,15}\}.$$

We see $\Delta(\nu_{29}^*)=\pm 2\omega_{14}^*$ and $8\omega_{14}^*=0$. Moreover $E^{12}: E\pi_{45}^{14}\to\pi_{58}^{27}$ is *an isomorphism onto*. Hence by Proposition 3.6 [11], we conclude that *there exists an element* $\rho_{3,14}$ *of* π_{45}^{14} *such that* $\rho_{3,26}\in\{16\iota_{26}, 2\rho_{26}, \rho_{41}\}$ *and* $2\rho_{3,14}\equiv E\alpha_3^{\mathrm{V}}$ *(mod* $2E\alpha_3^{\mathrm{V}}, 2\omega_{14}^*$*), which also implies* $H(\rho_{3,14})\equiv\eta_{27}\circ\bar{\mu}_{28}$ *(mod* ν_{27}^**)*. This determines the group structures of π_{45}^{14} and $E\pi_{45}^{14}$.

8. Some results on the 32- and 33-stems. We have the following results.

$$\pi_{35}^3=Z_2\{\nu'\circ\eta_6\circ\varepsilon_7\circ\bar{\kappa}_{15}\}\oplus Z_4\{\phi'\circ\sigma_{28}\}\oplus Z_2,$$

$$\pi_{37}^5=Z_8\{\phi''\circ\sigma_{30}\}\oplus Z_2\{\nu_5\circ\eta_8\circ\varepsilon_9\circ\bar{\kappa}_{17}\}\oplus Z_2,$$

$$\pi_{38}^6=Z_8\{G_0^{(2)}\}\oplus Z_4\{\Delta(\sigma_{13}\circ\bar{\kappa}_{20})\}\oplus Z_2\{\Delta(EA_1^{\mathrm{II}})\}\oplus Z_2\{\Delta(EA_1^{(1)}\circ\sigma_{33})\}$$
$$\oplus Z_8\{E\phi''\circ\sigma_{31}\}\oplus Z_2,$$

$$\pi_{39}^7=Z_2\{\sigma'\circ\mu_{3,14}\}\oplus Z_2\{\sigma'\circ\eta_{14}\circ\sigma_{15}\circ\bar{\mu}_{22}\}\oplus Z_8\{E^2\phi''\circ\sigma_{32}\}\oplus Z_2.$$

In the above groups, *the last direct summand* Z_2 *must be read as* $Z_2\{\mu_{3,n}\circ\sigma_{n+25}\}$.

$$\pi_{36}^3=Z_2\{\nu'\circ\phi_6\circ\sigma_{29}\}\oplus Z_2\oplus Z_2,$$

$$\pi_{38}^5=Z_2\{\nu_5\circ\sigma_8\circ\nu_{15}\circ\bar{\kappa}_{18}\}\oplus Z_2\{\nu_5\circ\phi_8\circ\sigma_{31}\}\oplus Z_2\oplus Z_2.$$

Following groups are not determined completely. The element κ_6' has the Hopf invariant $\bar{\nu}_{11}\circ\bar{\kappa}_{19}$.

$$\pi_{39}^6=Z_2\{\Delta(EA_1^{(2)})\}\oplus Z_2\{\Delta(EA_1\circ\omega_{25})\}\oplus Z_2\{\Delta(EA_1\circ\sigma_{25}\circ\mu_{32})\}$$
$$\oplus(Z_2\oplus Z_2 \text{ or } Z_4)\{\kappa_6', \nu_6\circ\sigma_9\circ\nu_{16}\circ\bar{\kappa}_{19}\}\oplus Z_2\oplus Z_2,$$

$$\pi_{40}^7=Z_2\{\sigma'\circ\eta_{14}\circ\mu_{3,15}\}\oplus Z_2\{\bar{\sigma}_7\circ\sigma_{26}^2\}\oplus(Z_2\oplus Z_2 \text{ or } Z_4)\{\kappa_7', \nu_7\circ\sigma_{10}\circ\nu_{17}\circ\bar{\kappa}_{20}\}$$
$$\oplus Z_2\oplus Z_2.$$

In the above groups, *the direct summands* $Z_2\oplus Z_2$ *must be read as* $Z_2\{\mu_{4,n}\}\oplus Z_2\{\eta_n\circ\mu_{3,n+1}\circ\sigma_{n+26}\}$.

References are listed in the part I, which appeared in Proc. Japan Acad., 53, Ser. A, No. 6 (1977).