54. On the 2-Components of the Unstable Homotopy Groups of Spheres. I

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The 2-components π_{n+i}^n of the homotopy groups of spheres $\pi_{n+i}(S^n)$ have been determined in the unstable range for all n when $i\leqq 24$ [4, 6, 7, 11]. The purpose of this note is to summarize briefly the results on the unstable homotopy groups of spheres which have been obtained as the application of the composition method of H. Toda [11]. Making use of the generators given in [2, 4–9, 11] and the new ones defined in this note, we will state, in this part I, the results on the 2-components π_{n+i}^n for all n*) when $25\leqq i\leqq 28$. Further results will be stated in the part II.

The results overlap in the metastable range with those of M. Mahowald [3] and S. Thomeier [10].

1. On the 25-stem. *There are following new elements*: $\phi'\in\pi_{28}^3$, $\phi''\in\pi_{30}^5$ *and* $\phi'''\in\pi_{32}^7$ *with the Hopf invariants* ϕ_5 *(mod* $4\nu_5\circ\bar{\kappa}_8, \alpha_2'$*)*, σ_9^3 *and* $\bar{\sigma}_{13}$ *respectively.*

Now, for an element $\alpha\in\{\eta_4, 2\iota_5, \phi_5\}$, we have $2\alpha\equiv\eta_4\circ\delta_5$ *(mod* $\eta_4\circ\bar{\mu}_5\circ\sigma_{22}, E\nu'\circ\varepsilon_7\circ\kappa_{15}$*)*. Hence we conclude that *there exists an element* ϕ' *such that* $2\phi'\equiv\eta_3\circ\delta_4$ *(mod* $\eta_3\circ\bar{\mu}_4\circ\sigma_{21}$*) and* $E\phi'\in\{\eta_4, 2\iota_5, \phi_5\}$ *(mod* $\nu_4\cdot\pi_{29}^7$*)*. This implies

$$\pi_{28}^3=Z_4\{\phi'\}\oplus Z_2\{\nu'\circ\varepsilon_6\circ\kappa_{14}\}\oplus Z_2\oplus Z_2.$$

In the above group and the following ones, *the last two direct summands* $Z_2\oplus Z_2$ *stand for* $Z_2\{\mu_{3,n}\}\oplus Z_2\{\eta_n\circ\bar{\mu}_{n+1}\circ\sigma_{n+18}\}$ $(n\geqq 3)$ *which survive in the stable range.* Next, the relation $2\phi''\equiv\pm E^2\phi'$ *(mod other elements)* holds for an element $\phi''\in\{\nu_5, E\sigma'\circ\sigma_{15}, \sigma_{22}\}_1$ and we see

$$\pi_{30}^5=Z_8\{\phi''\}\oplus Z_2\{\nu_5\circ\varepsilon_8\circ\kappa_{16}\}\oplus Z_2\{\nu_5^2\circ\bar{\sigma}_{11}\}\oplus Z_2\oplus Z_2,$$

$$\pi_{31}^6=Z_4\{\Delta(\bar{\kappa}_{13})\}\oplus Z_2\{\Delta(EA_1\circ\varepsilon_{25})\}\oplus Z_2\{\Delta(EA_1^{(1)})\}\oplus Z_8\{E\phi''\}\oplus Z_2\{\nu_6^2\circ\bar{\sigma}_{12}\}\oplus Z_2\oplus Z_2$$

Let us choose $\phi'''\in\{\sigma'\circ\sigma_{14}, \sigma_{21}, \nu_{28}\}_1$. Since $\nu_7^2\circ\bar{\sigma}_{13}\in 2\pi_{32}^7$, it follows that $2\phi'''\equiv\nu_7^2\circ\bar{\sigma}_{13}$ *(mod* $2E^2\phi''$*)*, which determines

$$\pi_{32}^7=(Z_4\oplus Z_8)\{\phi''', E^2\phi''\}\oplus Z_2\{\sigma'\circ\eta_{14}\circ\bar{\mu}_{15}\}\oplus Z_2\oplus Z_2.$$

Making use of the relations $E^2\phi'''\equiv\sigma_9\circ\nu_{16}^*$ *(mod* $\nu_9^2\circ\bar{\sigma}_{15}$*) and* $\sigma'\circ\xi_{14}\equiv E^2\phi''$ *(mod* $2E\phi''', 2E^2\phi'', \eta_7\circ\bar{\mu}_8\circ\sigma_{25}$*)*, we obtain

$$\pi_{34}^9=Z_8\{\sigma_9\circ\xi_{16}\}\oplus Z_2\{\sigma_9\circ\eta_{16}\circ\bar{\mu}_{17}\}\oplus Z_4\{\sigma_9\circ\nu_{16}^*\}\oplus Z_2\oplus Z_2,$$

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*) We omit the cases that $n=2$, 4 and 8; the results are immediate from Proposition 4.4 of [11].

$\pi^{10}_{35}=Z_4\{\sigma_{10}\circ\xi_{17}\}\oplus Z_4\{\sigma_{10}\circ\nu^*_{17}\}\oplus Z_2\oplus Z_2$.

We are in a position to use metastable periodic elements [9].

$\pi^{11}_{36}=Z_2\{K_1\}\oplus Z_4\{\sigma_{11}\circ\xi_{18}\}\oplus Z_2\{\sigma_{11}\circ\xi_{18}+\sigma_{11}\circ\nu^*_{18}\}\oplus Z_2\oplus Z_2$,

$\pi^{12}_{37}=Z_8\{\xi_{12}\cdot\sigma_{30}\}\oplus Z_2\{EK_1\}\oplus Z_2\{\sigma_{12}\circ\xi_{19}+\sigma_{12}\circ\nu^*_{19}\}\oplus Z_2\oplus Z_2$,

$\pi^{13}_{38}=Z_8\{\xi_{13}\circ\sigma_{31}\}\oplus Z_2\oplus Z_2$, where $2\xi_{12}\circ\sigma_{30}=\sigma_{12}\circ\xi_{19}$.

We have the following isomorphisms:*) $\pi^{14}_{39}=\pi^{13}_{38}$, $\pi^{15}_{40}=Z_2\{D_1\circ\mu_{31}\}\oplus\pi^{13}_{38}$, $\pi^{16}_{41}=Z_2\{B_1\circ\mu_{32}\}\oplus Z_2\{ED_1\circ\mu_{32}\}\oplus\pi^{13}_{38}$, $\pi^{17}_{42}=Z_2\{EB_1\circ\mu_{33}\}\oplus\pi^{13}_{38}$.

Since $\xi_{20}\circ\sigma_{38}=0$, we have $\pi^{18}_{43}=Z_4\{\xi_{18}\circ\sigma_{36}\}\oplus Z_2\oplus Z_2$, $\pi^{19}_{44}=Z_2\{\xi_{19}\circ\sigma_{37}\}\oplus Z_2\oplus Z_2$ *and* $\pi^n_{n+25}=Z_2\oplus Z_2$ *for* $n=20, 21, 22$.

Similarly, the periodic elements give the following results: $\pi^{23}_{48}=Z_2\{D_2\circ\eta_{47}\}\oplus Z_2\oplus Z_2$, $\pi^{24}_{49}=Z_2\{B_1\circ\eta_{48}\}\oplus\pi^{23}_{48}$, $\pi^{25}_{50}=Z_2\{EB_2\circ\eta_{49}\}\oplus Z_2\oplus Z_2$, $\pi^{26}_{51}=Z\{\Delta(\iota_{53})\}\oplus Z_2\oplus Z_2$, $\pi^n_{n+25}=Z_2\oplus Z_2$ *for* $n\geq 27$.

2. On the 26-stem. *There are following new elements:* $\tau''\in\pi^{10}_{36}$, $\tau'''\in\pi^{11}_{37}$ *and* $\tau^{\mathrm{IV}}\in\pi^{12}_{38}$ *with the Hopf invariants* $\eta_{19}\circ\sigma_{20}\circ\mu_{27}$, $\sigma_{21}\circ\mu_{28}$ *and* ρ_{23} *(mod* $2\rho_{23}$*) respectively.*

Now using the relations $2\bar{\mu}'=\eta^2_3\circ\bar{\mu}_5$ *and* $2\bar{\zeta}_5=E^2\bar{\mu}'$, we have

$\pi^3_{29}=Z_4\{\bar{\mu}'\circ\sigma_{22}\}\oplus Z_2\{\nu'\circ\phi_6\}\oplus Z_2\{\bar{\alpha}\circ\nu_{26}\}\oplus Z_2\{\eta_3\circ\mu_{3,4}\}$,

$\pi^5_{31}=Z_8\{\bar{\zeta}_5\circ\sigma_{24}\}\oplus Z_2\{\phi_5\circ\nu_{28}\}\oplus Z_2\{\nu_5\circ\bar{\kappa}_8\circ\nu_{28}\}\oplus Z_2\{\nu_5\circ\phi_8\}\oplus Z_2\oplus Z_2$.

The last two direct summands $Z_2\oplus Z_2$ *stand for* $Z_2\{\nu^2_n\circ\bar{\kappa}_{n+6}\}\oplus Z_2\{\eta_n\circ\mu_{3,n+1}\}$; *they survive in the stable range.*

Following relations hold: $2\bar{\sigma}_6\circ\sigma_{25}=\nu_6\circ\phi_9$ *and* $2\Delta(\lambda\circ\nu_{31})\equiv\nu^2_6\circ\bar{\kappa}_{12}-\nu_6\circ\bar{\kappa}_9\circ\nu_{29}$ *(mod* $\nu_6\circ\phi_9$*)*. They lead us to

$$\pi^6_{32}=Z_2\{\Delta(EA_1\circ\mu_{25})\}\oplus Z_4\{\Delta(\lambda\circ\nu_{31})\}\oplus Z_4\{\bar{\sigma}_6\circ\sigma_{25}\}\oplus Z_8\{\bar{\zeta}_6\circ\sigma_{25}\}\oplus Z_2\{\phi_6\circ\nu_{29}\}\oplus Z_2\oplus Z_2.$$

$$\pi^7_{33}=Z_2\{\sigma'\circ\omega_{14}\circ\nu_{30}\}\oplus Z_2\{\bar{\kappa}_7\circ\nu^2_{27}\}\oplus Z_2\{\bar{\sigma}_7\circ\sigma_{26}\}\oplus Z_8\{\bar{\zeta}_7\circ\sigma_{26}\}\oplus Z_2\{\phi_7\circ\nu_{30}\}\oplus Z_2\oplus Z_2.$$

Here we see a relation $\sigma'\circ\bar{\zeta}_{14}=x\bar{\zeta}_7\circ\sigma_{26}$ *for an odd integer* x.

$\pi^9_{35}=Z_2\{\sigma_9\circ\omega_{16}\circ\nu_{32}\}\oplus Z_2\{\sigma_9\circ\bar{\sigma}_{16}\}\oplus Z_8\{\sigma_9\circ\bar{\zeta}_{16}\}\oplus Z_2\{\bar{\kappa}_9\circ\nu^2_{29}\}\oplus Z_2\{\bar{\sigma}_9\circ\sigma_{28}\}\oplus Z_2\oplus Z_2$.

Now we have to define some elements by Toda brackets: $\tau''\in\{4\sigma_{10},\nu_{17},\rho_{20}\}_1$, $\tau'''\in\{2\sigma_{11},\nu_{18},\rho_{21}\}_1$ *and* $\tau^{\mathrm{IV}}\in\{\sigma_{12},\nu_{19},\rho_{22}\}_1$. This enable us to determine

$$\pi^{10}_{36}=Z_8\{\tau''\}\oplus Z_4\{R_1\}\oplus Z_2\{\sigma_{10}\circ\bar{\sigma}_{17}\}\oplus Z_2\oplus Z_2,$$

$$\pi^{11}_{37}=Z_8\{\tau'''\}\oplus Z_2\{C_1\circ\kappa_{23}\}\oplus Z_2\{\sigma_{11}\circ\bar{\sigma}_{18}\}\oplus Z_2\oplus Z_2.$$

In the above groups, we have the relations: $2R_1=\Delta(\nu_{21}\circ\kappa_{24})=\bar{\kappa}_{10}\circ\nu^2_{30}-\nu^2_{10}\circ\bar{\kappa}_{16}$, $2\tau''=\pm\sigma_{10}\circ\bar{\zeta}_{17}$, $2\tau'''=-E\tau''$ *and* $ER_1\equiv C_1\circ\kappa_{23}$ *(mod* $E^2\pi^9_{35}$*)*. Moreover we see $32\tau^{\mathrm{IV}}=\sigma_{12}\circ\bar{\zeta}_{19}=4E\tau'''$, which implies

$$\pi^{12}_{38}=Z_{64}\{\tau^{\mathrm{IV}}\}\oplus Z_4\{E\tau'''-8\tau^{\mathrm{IV}}\}\oplus Z_2\{A_1\circ\kappa_{24}\}\oplus Z_2\{EC_1\circ\kappa_{24}\}\oplus Z_2\{\sigma_{12}\circ\bar{\sigma}_{19}\}\oplus Z_2\oplus Z_2,$$

$$\pi^{13}_{39}=Z_8\{E\tau^{\mathrm{IV}}\}\oplus Z_2\{EA_1\circ\kappa_{25}\}\oplus Z_2\{\sigma_{13}\circ\bar{\sigma}_{20}\}\oplus Z_2\oplus Z_2,$$

$$\pi^{14}_{40}=Z_8\{E^2\tau^{\mathrm{IV}}\}\oplus Z_2\oplus Z_2,\qquad \pi^{15}_{41}=\pi^{14}_{40}.$$

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*) When we write $\pi^n_i=A\oplus\pi^{n-k}_{i-k}$, the latter direct summand must be understood as the image of the (iterated) suspension homomorphism, which is incidentally monic.

Next, we use the periodic elements [9] to obtain $\pi_{42}^{16}=Z_8\{M_1^{(1)}\}\oplus\pi_{40}^{14}$, $\pi_{43}^{17}=Z_8\{EM_1^{(1)}\}\oplus Z_2\oplus Z_2$, $\pi_{44}^{18}=Z_8\{M_2'''\}\oplus Z_2\oplus Z_2$, $\pi_{45}^{19}=Z_8\{M_2''\}\oplus Z_2\oplus Z_2$, $\pi_{46}^{20}=Z_{16}\{\Delta(\sigma_{41})\}\oplus Z_4\{EM_2''-2\Delta(\sigma_{41})\}\oplus Z_2\oplus Z_2$. $\pi_{n+26}^{n}=Z_8\{E^{n-21}M_2'\}\oplus Z_2\oplus Z_2$ *for* $n=21, 22, 23$.

In the above groups, we have the relations: $2M_2'''=\pm E^2M_1^{(1)}$, $2M_2''=EM_2'''$, $2M_2'=E^2M_2''$.

Finally we see: $\pi_{50}^{24}=Z_8\{M_2\}\oplus\pi_{47}^{21}$, $\pi_{51}^{25}=Z_8\{EM_2\}\oplus Z_2\oplus Z_2$, $\pi_{52}^{26}=Z_4\{E^2M_2\}\oplus Z_2\oplus Z_2$, $\pi_{53}^{27}=Z_2\{E^3M_2\}\oplus Z_2\oplus Z_2$, $\pi_{n+26}^{n}=Z_2\oplus Z_2$ *for* $n\geqq 28$.

3. On the 27-stem. *There appears no new elements.*

$$\pi_{30}^{3}=Z_2\{\nu'\circ\delta_6\}\oplus Z_2\{\nu'\circ\bar{\mu}_6\circ\sigma_{23}\}\oplus Z_2\{\varepsilon_3\circ\bar{\sigma}_{11}\}\oplus Z_4\{\mu_3'\},$$
$$\pi_{32}^{5}=Z_2\{\sigma'''\circ\bar{\kappa}_{12}\}\oplus Z_2\{\nu_5\circ\bar{\sigma}_8'\}\oplus Z_2\{\nu_5\circ\bar{\mu}_8\circ\sigma_{25}\}\oplus Z_2\{\nu_5\circ\delta_8\}\oplus Z_8.$$

The direct summand Z_8 *stands for* $Z_8\{\zeta_{3,n}\}$. We have the relations: $2\mu_3'=\eta_3^2\circ\mu_{3,5}$ *and* $2\zeta_{3,5}\equiv E^2\mu_3'$ (*mod* $\nu_5\circ\bar{\mu}_8\circ\sigma_{25}$).

$$\pi_{33}^{6}=Z_{16}\{\Delta(\rho_{13}\circ\sigma_{28})\}\oplus Z_4\{\sigma''\circ\bar{\kappa}_{13}\}\oplus Z_2\{\bar{\nu}_6\circ\bar{\sigma}_{14}\}\oplus Z_8,$$
$$\pi_{34}^{7}=Z_8\{\sigma'\circ\bar{\kappa}_{14}\}\oplus Z_2\{\bar{\nu}_7\circ\bar{\sigma}_{15}\}\oplus Z_8,$$
$$\pi_{36}^{9}=Z_8\{\sigma_9\circ\bar{\kappa}_{16}\}\oplus Z_2\{\bar{\nu}_9\circ\bar{\sigma}_{17}\}\oplus Z_8,$$
$$\pi_{37}^{10}=Z_4\{\Delta(\nu_{21}^*)\}\oplus Z_8\{\sigma_{10}\circ\bar{\kappa}_{17}\}\oplus Z_8.$$

With the help of the metastable periodic elements [9], we see $\pi_{38}^{11}=Z_2\{C_1^{\mathrm{II}}\}\oplus Z_2\{C_1^{(1)}\circ\sigma_{31}\}\oplus Z_8\{\sigma_{11}\circ\bar{\kappa}_{18}\}\oplus Z_8$, $\pi_{39}^{12}=Z_2\{A_1^{\mathrm{II}}\}\oplus Z_2\{A_1^{(1)}\circ\sigma_{32}\}\oplus Z_2\{EC_1^{\mathrm{II}}\}\oplus Z_2\{EC_1^{(1)}\circ\sigma_{32}\}\oplus Z_4\{\sigma_{12}\circ\bar{\kappa}_{19}\}\oplus Z_8$, $\pi_{40}^{13}=Z_2\{EA_1^{\mathrm{II}}\}\oplus Z_2\{EA_1^{(1)}\circ\sigma_{33}\}\oplus Z_4\{\sigma_{13}\circ\bar{\kappa}_{20}\}\oplus Z_8$, $\pi_{41}^{14}=Z_2\{E^2A_1^{\mathrm{II}}\}\oplus Z_2\{\sigma_{14}\circ\bar{\kappa}_{21}\}\oplus Z_8$, $\pi_{n+27}^{n}=Z_8$ *for* $n=15, 16, 17, 18$.

We make use of the periodic elements again to obtain $\pi_{46}^{19}=Z_2\{C_2^{\mathrm{I}}\}\oplus Z_2\{C_2\circ\sigma_{39}\}\oplus Z_8$, $\pi_{47}^{20}=Z_2\{A_2^{\mathrm{I}}\}\oplus Z_2\{A_2\circ\sigma_{40}\}\oplus\pi_{46}^{19}$, $\pi_{48}^{21}=Z_2\{EA_2^{\mathrm{I}}\}\oplus Z_2\{EA_2\circ\sigma_{41}\}\oplus Z_8$.

For the element V_2 of Y. Nomura [8], we have $\Delta(\nu_{45}^2)=2V_2=E^2A_2^{\mathrm{I}}$. Hence we obtain the following results: $\pi_{49}^{22}=Z_4\{V_2\}\oplus Z_8$, $\pi_{n+27}^{n}=Z_2\{E^{n-22}V_2\}\oplus Z_8$ *for* $n=23, 24, 25$, $\pi_{55}^{28}=Z\{\Delta(\iota_{57})\}\oplus Z_8$, $\pi_{n+27}^{n}=Z_8$ *for* $n=26$, 27 *and* $n\geqq 29$.

4. On the 28-stem. *There appears no new elements.*

$$\pi_{31}^{3}=Z_2\{\nu'\circ\eta_6\circ\bar{\mu}_7\circ\sigma_{24}\}\oplus Z_2\{\nu'\circ\mu_{3,6}\}\oplus Z_2\{\phi'\circ\nu_{28}\}\oplus Z_2.$$

The last direct summand Z_2 *stands for* $Z_2\{\varepsilon_n\circ\bar{\kappa}_{n+8}\}$ *which survives in the stable range.*

$$\pi_{33}^{5}=Z_4\{\nu_5\circ E\phi'''\}\oplus Z_2\{\nu_5\circ\mu_{3,8}\}\oplus Z_2\{\nu_5\circ\eta_8\circ\bar{\mu}_9\circ\sigma_{26}\}\oplus Z_2.$$

Now, $8\Delta(\bar{\rho}_{13})=\nu_6\circ\mu_{3,9}$ *and* $\nu_7\circ E^3\phi'''=0$, hence we have

$$\pi_{34}^{6}=Z_{16}\{\Delta(\bar{\rho}_{13})\}\oplus Z_8\{\bar{\nu}_6\circ\bar{\kappa}_{14}\}\oplus Z_2\{\nu_6\circ E^2\phi'''\}\oplus Z_2,$$
$$\pi_{35}^{7}=Z_2\{\phi'''\circ\nu_{32}\}\oplus Z_2\{\sigma'\circ\eta_{14}\circ\bar{\kappa}_{15}\}\oplus Z_2\{\bar{\nu}_7\circ\bar{\kappa}_{15}\}\oplus Z_2,$$
$$\pi_{37}^{9}=Z_2\{\sigma_9^4\}\oplus Z_2\{\sigma_9\circ\eta_{16}\circ\bar{\kappa}_{17}\}\oplus Z_2\{\sigma_9\circ\nu_{16}^*\circ\nu_{34}\}\oplus Z_2\{\bar{\nu}_9\circ\bar{\kappa}_{17}\}\oplus Z_2.$$

Here we remark that following two relations hold: $\sigma'\circ\eta_{14}\circ\bar{\kappa}_{15}=\varepsilon_7\circ\bar{\kappa}_{15}+\kappa_7^2$ *and* $\sigma_9\circ\nu_{16}^*\circ\nu_{34}=E^2\phi'''\circ\nu_{34}$.

Let us use the periodic elements. Then we obtain $\pi_{38}^{10}=Z_8\{F_1^{(1)}\}\oplus Z_2\{\sigma_{10}^4\}\oplus Z_2\{\sigma_{10}\circ\nu_{17}^*\circ\nu_{35}\}\oplus Z_2\{\bar{\nu}_{10}\circ\bar{\kappa}_{18}\}\oplus Z_2$, $\pi_{39}^{11}=Z_2\{C_1\circ\omega_{23}\}\oplus Z_2\{C_1^{(2)}\}\oplus Z_2\{EF_1^{(1)}\}$

$\oplus Z_2\{\sigma_{11}^4\} \oplus Z_2\{\bar{\nu}_{11} \circ \bar{\kappa}_{19}\} \oplus Z_2$, $\pi_{40}^{12} = Z_2\{A_1^{(2)}\} \oplus Z_2\{A_1 \circ \omega_{24}\} \oplus Z_2\{A_1 \circ \sigma_{24} \circ \mu_{31}\} \oplus Z_2\{EC_1^{(2)}\} \oplus Z_2\{EC_1 \circ \omega_{24}\} \oplus Z_2\{E^2 F_1^{(1)}\} \oplus Z_2\{\bar{\nu}_{12} \circ \bar{\kappa}_{20}\} \oplus Z_2$, $\pi_{41}^{13} = Z_2\{EA_1^{(2)}\} \oplus Z_2\{EA_1 \circ \omega_{25}\} \oplus Z_2\{EA_1 \circ \sigma_{25} \circ \mu_{32}\} \oplus Z_2$.

Since $16\varDelta(\rho_{29}) = E^2 A_1^{(2)}$, we have $\pi_{42}^{14} = Z_{32}\{\varDelta(\rho_{29})\} \oplus Z_2$, $\pi_{n+28}^n = Z_2$ *for* $n = 15, 16, 17$.

The last parts of the 28-stem are also determined by metastable periodic elements: $\pi_{46}^{18} = Z_8\{F_2\} \oplus Z_2$, $\pi_{47}^{19} = Z_2\{C_2^{(1)}\} \oplus Z_2\{EF_2\} \oplus Z_2$, $\pi_{48}^{20} = Z_2\{A_2^{(1)}\} \oplus Z_2\{A_2 \circ \varepsilon_{40}\} \oplus Z_2\{A_2 \circ \eta_{40} \circ \sigma_{41}\} \oplus \pi_{47}^{19}$, $\pi_{49}^{21} = Z_2\{EA_2^{(1)}\} \oplus Z_2\{EA_2 \circ \varepsilon_{41}\} \oplus Z_2\{EA_2 \circ \eta_{41} \circ \sigma_{42}\} \oplus Z_2$, $\pi_{50}^{22} = Z_{16}\{\varDelta(\sigma_{45})\} \oplus Z_2$ *where* $8\varDelta(\sigma_{45}) = E^2 A_2^{(1)}$, $\pi_{n+28}^n = Z_2$ *for* $n = 23, 24$ *and* 25, $\pi_{54}^{26} = Z_4\{\varDelta(\nu_{53})\} \oplus Z_2$, $\pi_{55}^{27} = Z_2\{C_3\} \oplus Z_2$, $\pi_{56}^{28} = Z_2\{A_3\} \oplus \pi_{55}^{27}$, $\pi_{57}^{29} = Z_2\{EA_3\} \oplus Z_2$, $\pi_{n+28}^n = Z_2$ *for* $n \geqq 30$.

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