143. Non-Conjectural Theory of Relativity as a Non-Holonomic Laguerre Geometry Realized in the Three-Dimensional Torsioned Cartesian Space Fibered with Actions

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The purpose of the present paper consists in bringing the theory of general relativity, which has hitherto remained a conjecture of late Prof. A. Einstein, to its final, actual one,[1] namely a *three-dimensional non-holonomic Laguerre geometry* (i.e. *torsioned Laguerre space geometry*) *realized in the three-dimensional*

Laguerre space,	*torsioned Cartesian space,*

which is again realized in the ordinary three-dimensional Cartesian space (the receptacle of physical phenomena, so to speak) *fibered with actions, tracing physical necessities.*

Since the torsioned Laguerre space is, when it is abstractly grasped, a four-dimensional torsioned Cartesian space (a map of four-dimensional Cartesian space), *the geodesic curves* [which correspond to (the path of a free particle in four-dimension) = (the torsioned homocentric system in three-dimension)] in the four-dimensional torsioned Cartesian space ($\neq$ geodesic curves in the four-dimensional Cartesian space) *are free from singularities,* what is suitable for physics.

The author starts with the vector form

(1) $dS=\gamma_l\omega^l$, $(\omega^l=\omega^l_\mu dx^\mu)$, $(l, m, \cdots=1,2,3,4; \lambda, \mu, \nu\cdots=1,2,3,4)$,

where

(2) $\omega^l=$(superposed) action components, (3) $\omega^l_\mu=$(superposed) momentum components due to any kinds of energy,

(4) $\omega^i_4=$(superposed) statical potential components due to any kinds of energy emitted by a particle, $(i=1,2,3)$, (5) $x^4=t=$time,

(6) $|dS|=dS=$resultant (superposed) action,

(7) $dr=Edt=\omega^4=$radial (superposed) action emitted by the particle,

(8) $\gamma_h\gamma_k+\gamma_k\gamma_h=2\delta_{hk}$; $h, k, l=1, 2, 3, 5$; $\gamma_4=i\gamma_5$,

so that

(9) $dS^2=dSdS=\omega^l\omega^l=g_{\mu\nu}dx^\mu dx^\nu$,

where generally $A^lB^l\equiv A^4B^4-A^iB^i$, $(i=1, 2, 3)$ and

1) An insight, deeper than that in the introduction: T. Takasu, [3].[2]

2) The ciphers in the square brackets shall refer to the references attached to the end of this paper.

(10) $g_{\mu\nu}=g_{\underline{\mu\nu}}+g_{\underset{\vee}{\mu\nu}}$,

(11) $g_{\underline{\mu\nu}}=\omega^l_\mu\omega^l_\nu=\omega^4_\mu\omega^4_\nu-\omega^i_\mu\omega^i_\nu$,

(12) $g_{\underset{\vee}{\mu\nu}}=\gamma_4\gamma_1(\omega^4_\mu\omega^1_\nu-\omega^1_\mu\omega^4_\nu)+\cdots+\gamma_2\gamma_3(\omega^2_\mu\omega^3_\nu-\omega^3_\mu\omega^2_\nu)+\cdots$.

The general relativity of Einstein concerns only with gravitational $g_{\underline{\mu\nu}}$, while the present author's concerns with both $g_{\underline{\mu\nu}}$ (within the scheme of ordinary algebra) and $g_{\underset{\vee}{\mu\nu}}$ (within the scheme of exterior algebra), the $g_{\underline{\mu\nu}}$ and the $g_{\underset{\vee}{\mu\nu}}$ forming a duality in physics. Thereby the momentum, the statical potential, the action, the $g_{\underline{\mu\nu}}$ and the $g_{\underset{\vee}{\mu\nu}}$ are superposed ones in general and may be due to *any kind of energy, gravitational or electromagnetic etc.* emitted by the particle.

Takasu's theory is $4^2+4=20$-*dimensional* $(\omega^l_\mu, x^\lambda)$	*Einstein's theory is* 4-*dimensional* (x^λ)

in abstract sense. Thus *the author's general relativity is entirely different from Einstein's,* notwithstanding it is considerably concordant with Einstein's.

Most parts of theoretical physics are reducible to treatments of particles. Physicists have hitherto been accustomed to treat the particles themselves directly, what seems now to have made theoretical physics rather artificial. The present author has discovered that the author's non-holonomic Laguerre geometry, for which the vector form (1) is fundamental, describes physical phenomena extremely well, if the *energy front surfaces* (generalized spheres with geodesic curves of the second kind corresponding to ω^l_μ as generalized radii) *called geodesic spheres are taken as spatial elements.* It is shown that the second kind and the first kind coincide, provided that the ω^l_μ appears in (1).[3)]

Although the geometry itself is nothing other than Cartan-Weitzenböck's teleparallelism geometry[4)] (Einstein, [1]) corresponding to ω^l_μ in (1) keeping the Riemann metric system when it is grasped in 4-dimension, the point of view is different from the Einstein's (he aimed at unified field theory thereby) and has the following fortes: (i) it coheres to the physical phenomena in the very places concretely, where the phenomena take place, (ii) it is perceptional rather than to be conceptual (what was the case by Einstein); (iii) it implies necessities and needs no fundamental assumptions (which Einstein made); (iv) it is naive and has an extreme transparency and not only that it unifies substantially automatically gravitation

3) Cf. S. S. Chern: Lecture Note on Differential Geometry, 124 (1952).

4) It is remarkable that Einstein himself has already written (Einstein, [2]): "Es ist daher denkbar, dass diese Theorie die ursprüngliche Fassung der allgemeinen Relativitätstheorie verdrängen wird" and that, although my view is different from his, *this very conjecture of him is actually realizing.*

with electromagnetism in a similar manner as the light waves and the electromagnetic waves are unified, but also it admits of *superposed field of* (holonomic or non-holonomic) *actions of any kind of energy* emitted by each particle into unification; (v) it admits of the following fundamental principle: "The non-holonomic-Laguerre-geometrical quantum mechanics is obtainable from the Laguerre-geometrical (i.e. special-relativistic) quantum mechanics by replacing the special-relativistic action $dr=cdt$ in the latter by the corresponding non-holonomic-Laguerre-geometrical non-holonomic action $dr=Edt=\omega^4_\mu dx^\mu$ and the partial derivative operators $\frac{\partial}{\partial\xi^l}$ ($\xi^l=$ Cartesian) by their generalizations "$\frac{\partial}{\omega^l}$"$=\Omega^\lambda_l\frac{\partial}{\partial x^\lambda}$, where Ω^λ_l is defined by $\Omega^\lambda_l\omega^l_\mu=\delta^\lambda_\mu$, $\Omega^\lambda_m\omega^l_\lambda=\delta^l_m$"; (vi) treating of the vector-form $dS=\gamma_l\omega^l$, the fundamental form for the non-holonomic Laguerre geometry, is physically more powerful than treating the tensor form $g_{\mu\nu}dx^\mu dx^\nu$.

The author discovered that the equations of motion of a free particle are those of a geodesic curve of the second kind corresponding to ω^l_μ, which coincides with those of the first kind, provided that the ω^l_μ appears in (1), (in 4-dimensional abstract sense). He discovered also the well-known equation

$$\frac{d^2u}{d\varphi^2}+u=\frac{m}{h^2}+3mu^2$$

of planetary motion, which was supported by three famous observational data, from a quadratic form

$$dS^2=\gamma(\rho)dt^2-\bar{\gamma}(\rho)^{-1}d\rho^2-\rho^2d\theta^2-\rho^2\sin^2\theta\cdot d\varphi^2$$

accompanied by

$$-\bar{\gamma}=h^2\left(1-2mu-\frac{2m}{u}\right)+\frac{c}{u^2},\quad (u=\rho^{-1}),$$

where $\gamma(\rho)=$any function of ρ, other than that of Schwarzschild.

The author is entirely convinced with the following principles:

I. "*Physik ist Kugelgeometrie*".

II. *Physics is classified into branches corresponding to the branches of geometry based upon.*

III. *In physics a principle of duality of the following sort holds*:

Ordinary algebra	Exterior algebra
Velocity	Dual(transverse)velocity
Acceleration	Dual(transverse)acceleration
Force	Dual(transverse)force
Energy	Dual(transverse)energy
Potential	Dual(transverse)potential
Action	Dual(transverse)action
Work	Dual(transverse)work

Divergence	Curl
Heat quantity	Dual heat quantity
Temperature	Dual temperature
Dual entropy	Entropy
Capacity	Dual capacity
Specific heat	Dual specific heat
$pV=RT$	$\bar{p}V=\bar{R}\bar{T}$
$\frac{p}{p_0}=\left(\frac{V}{V_0}\right)^{\gamma}=\left(\frac{\rho}{\rho_0}\right)^{\gamma}$	$\frac{\bar{p}}{\bar{p}_0}=\left(\frac{V}{V_0}\right)^{\bar{\gamma}}=\left(\frac{\rho}{\rho_0}\right)^{\bar{\gamma}}$
$C^2=\frac{dp}{d\rho}=\frac{\gamma p}{\rho}$	$\bar{C}^2=\frac{d\bar{p}}{d\rho}=\frac{\bar{\gamma}\bar{p}}{\rho}$
$g_{\underline{\mu\nu}}=g_{\underline{\nu\mu}}$	$g_{\underset{\vee}{\mu\nu}}=-g_{\underset{\vee}{\nu\mu}}$
etc.	etc.

Thus the author is convinced with that *the whole theoretical physics ought once to be rewritten from the view-points* I, II, *and* III. *Geometrization of theoretical physics* has thus become successful to a considerable extent.

References

[1] Einstein, A.,: Riemann-Metrik mit Aufrechterhaltung des Begriffes der Fernparallelismus, Preuss. Akad. Wiss. Sitzungsber. Phys.-Math. Kl., **26**, 1–7 (1928).

[2] Einstein, A.,: Zur einheitlichen Feldtheorie, Preuss. Akad. Wiss. Sitzungsber. Phys.-Math. Kl., **27**, 2–7 (1929).

[3] Takasu, T.,: The general relativity as a three-dimensional non-holonomic Laguerre geometry of the second kind, its gravitation theory and its quantum mechanics, Yokohama Mathematical Journal, **1**, 89–104 (1953).