129. Contributions to the Theory of Semi-groups. V

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S. Schwarz [3] considered a disjoint conjugate class decomposition of a semi-group. He proved any finite commutative semi-group has such a decomposition and any character on it takes the same value on each conjugate class. The present author [1, (IV)] proved that, in any commutative periodic semi-group S, there is a character $\chi(x)$ of S such that $\chi(a)\neq\chi(b)$ if a, b are two elements of distinct conjugate classes.

Let S be a strongly reversible (see G. Thierrin [4]) periodic semi-group, and let $G^{(\alpha)}$ be the maximal subgroup of $K^{(\alpha)}$ for an idempotent e_α (see K. Iséki [1, (I)]). Then, following S. Schwarz [3], for a of $G^{(\alpha)}$, the set T_a of all elements x of $K^{(\alpha)}$ satisfying $xe_\alpha=a$ is called a *conjugate class* of S. By Theorem 2 of K. Iséki [1, (I), p. 174], *the semi-group S is the set-sum of disjoint conjugate classes T_a*.

In any strongly reversible compact semi-group[1] S, if $G^{(\alpha)}$ is the maximal subgroup of $K^{(\alpha)}$ containing an idempotent e_α, S is the set-sum of $K^{(\alpha)}$ (see K. Iséki [2]) and $K^{(\alpha)}e_\alpha=e_\alpha K^{(\alpha)}=G^{(\alpha)}$. Therefore we can define conjugate classes of S, and S is the set-sum of disjoint conjugate classes T_a. Hence if a given semi-group is compact commutative, it is decomposed into disjoint conjugate classes T_a.

Now, let S be a strongly reversible periodic homogroup (see G. Thierrin [5]), then S has only one smallest idempotent e (see K. Iséki [1, (II)]).

Let χ be a character of S, i.e. a homomorphism into the multiplicative group of complex numbers of absolute value one. Let $G^{(e)}$ be a maximal group relative to the smallest idempotent e. From $T_ae_\alpha=a\in G^{(\alpha)}$ and $e_\alpha e=e=ee_\alpha$, we have

$$T_ae=T_ae_\alpha e=ae.$$

Hence

$$\chi(T_a)=\chi(T_ae)=\chi(ae).$$

Therefore we have the following

Theorem 1. In a strongly reversible periodic homogroup S, any character of S takes the same value on each conjugate class.

Further, we have

Theorem 2. Any character of a compact homogroup takes the same

1) For topological semi-groups, see A.D. Wallace [6] or [7].

value on each conjugate class.

Corollary 1. Any character of a commutative compact semi-group takes the same value on each conjugate class.

In any locally compact abelian group, there are sufficiently many characters.[2] Therefore the group ideal of any commutative compact semi-group has sufficiently many characters. Hence, from the extension theorem of characters (see K. Iséki [2]) and Corollary 1, we have the following

Theorem 3. Any distinct elements of the group ideal of a commutative compact semi-group are contained in distinct conjugate classes.

References

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2) See H. Cartan and R. Godement: Théorie de la dualité et analyse harmonique dans les groupes abéliens localement compact, Annales de l'Ecole Nor. Sup., **64** (1947).