34. *New Characterisations of Compact Spaces*

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In this Note, we shall correct some errors of results in my Note [2, 3] and [4],*) and give characterisations of compact space by fairly strong conditions.

Theorem 2 in [2] should be read as follows:

(1) *If the continuous convergence of $f_n(x)$ on a countably paracompact normal space S to $f(x)$ on S implies uniformly convergence, then S is countably compact.*

On the detail of countably paracompact spaces which was introduced by C. H. Dowker, see Yu. M. Smirnov [5].

Therefore, the hypothesis of Theorem 3 in [2] is not "completely regular", it should be read as "*countably paracompact*". Similar errors are contained in [3] and [4]. A part of Theorem 1 in [3] should be read as follows:

(2) "*If every sequence of continuous functions on a countably paracompact normal space is continuously convergent to a continuous function, then it is strictly continuous convergence to the continuous function*" *implies that the space is countably compact.*

The "if" part of Theorems 1 and 2 in [4] should be read as follows:

(3) *Let S be a countably paracompact normal space.*

(1) *Every sequence of continuous functions which is continuously convergent in the sense of Schaefer at each point of S is uniformly convergent.*

(2) *Under the same assumption (1), the convergence is strictly continuous.*

Then, each of them implies the countably compactness of S.

The proofs of these propositions are done by a construction of a sequence of continuous functions. Suppose that the space is not countably compact, then we can find a family of a countably point set $\{a_n\}$ without cluster points, i.e. $\{a_n\}$ is an isolated set. Therefore we can take open sets O_n containing a_n such that $O_m \frown O_n = \phi$ $(m \neq n)$. Further, by the normality we can find open sets U_n such that $\overline{U}_n \subset O_n$

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for every n. The family α of open sets $\{O_n\}$ $(n=1,2,\cdots)$ and $\{S-\overline{U}_n\}$ $(n=1,2,\cdots)$ is a countable open covering of S. By the countably paracompactness, we can find an open locally finite refinement β of α. Let V_n be the intersection of O_n and an element of β containing a_n. Then $\{V_n\}$ $(n=1,2,\cdots)$ is a family of locally finite open sets and pairwise disjoint. By the normality of S, there are continuous functions $f_n(x)$ on S such that

$$f_n(x)=\begin{cases} 0 & x \bar{\in} V_n \\ 1 & x=a_n \end{cases}$$

and $0\leq f_n(x)\leq 1$ on S. Since $\{V_n\}$ is locally finite, the sequence $\{f_n(x)\}$ is convergent to $f(x)\equiv 0$, and also its convergence is continuously.

However, $f_n(x)$ is not uniformly convergent to $f(x)$. This shows (1).

On the other hand, for the sequence $\{f_n(x)\}$, $\{f(a_n)\}$ and $f_n(a_n)$ are convergent, but $\lim f_n(a_n)=1\neq 0=\lim f(a_n)$. This shows that $\{f_n(x)\}$ is not strictly continuously convergent to $f(x)$. This completes the proofs of (2) and (3).

It is well known that, *if a paracompact (regular) space S is countably compact, S is compact.* Therefore we have the following

Theorem 1. *Let S be a paracompact space, then the following statements are equivalent:*

(a) *S is compact.*

(b) *Every continuously convergence implies uniformly convergence.*

(c) *Every continuously convergence implies strictly continuously convergence.*

(d) *Every continuously convergence in the sense of H. Schaefer implies uniformly convergence.*

(e) *Every continuously convergence in the sense of H. Schaefer implies strictly uniformly convergence.*

References

[1] K. Iséki: A remark on countably compact normal space, Proc. Japan Acad., **33**, 131-133 (1957).

[2] K. Iséki: A theorem on continuous convergence, Proc. Japan Acad., **33**, 355-356 (1957).

[3] K. Iséki: Pseudo-compactness and strictly continuous convergence, Proc. Japan Acad., **33**, 424-428 (1957).

[4] K. Iséki: On generalized continuous convergence, Proc. Japan Acad., **33**, 525-527 (1957).

[5] Yu. M. Smirnov (Ю. М. Смирнов): On strong-paracompact spaces, Izvestia Acad. Nauk (О сидьно-паракомлактных пространствах, Известия Акад. Наук СССР), **20**, 253-274 (1956).