46. On Quasiideals in Regular Semigroup

A Remark on S. Lajos' Note

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In his paper [2], S. Lajos has given an interesting characterisations of quasiideals in regular rings. In this Note, we shall give a similar characterisation of quasiideals in regular semigroups. Lajos' method is also used for the case of semigroup. A subset A of a semigroup S such that $AS\cap SA\subseteq A$ is called a *quasiideal* in S. Such quasiideal has been previously studied by O. Steinfeld [3]. For details for semigroups and its related concepts, see E. C. Ляпин [4].

The present writer has proved that *a semigroup S is regular if and only if*

$$AB=A\cap B \tag{1}$$

for every right ideal A and every left ideal B of S. (See [1] or [4] p. 202–203.)

The main result of S. Lajos is formulated as follows:

Theorem 1. *A subset A of a semigroup S is a quasiideal if and if only*

$$ASA\subseteq A. \tag{2}$$

Proof. Suppose that A is a quasiideal in S, then we have $ASA\subseteq SA$ and $ASA\subseteq AS$. Therefore, by the definition of quasiideal,

$$ASA\subseteq SA\cap AS\subseteq A.$$

This shows that condition (2) holds.

Conversely, suppose that a subset A of a regular semigroup S satisfies the condition (2). The condition (2) shows that SA is a left ideal of S, and AS is a right ideal of S. Hence, by (1), we have

$$AS\cap SA=AS\cdot SA.$$

Therefore, by $AS\cdot SA\subseteq ASA$, and (2), we have

$$AS\cap AS\subseteq A.$$

This shows that the set A is a quasideal of S.

Corollary. *Let A, B be quasiideals of a regular semigroup S, then $A\cdot B$ is a quasiideal of S.*

For the proof, see S. Lajos (1).

References

[1] K. Iséki: A characterisation of regular semigroups, Proc. Japan Acad., **32**, 676–677 (1956).
[2] S. Lajos: On quasiideals of regular ring, Proc. Japan Acad., **38**, 210–211 (1962).
[3] O. Steinfeld: Über die Quasiideale von Ringen, Acta Sci. Math. (Szeged), **17**, 170–180 (1956).
[4] E. C. Ляпин: Полугруппы, Москва (1960).