136. 2-Primary Components of the Homotopy Groups of Spheres and Lie Groups

By Kunio ŌGUCHI

Department of Mathematics, International Christian University, Tokyo

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In our previous papers [1], [2], we have stated some results on 2-primary components of the homotopy groups of spheres and some Lie Groups. In the present note, we should like to report some additional results on the same subjects, and correct some errors in the preceeding papers due to misprints or otherwise. The same notations as in [1], [2] will be used.

In [1], to (1.5) on page 184, we should like to add

$$[\iota_6, \iota_6] \in \{\nu_6, \eta_9, 2\iota_{10}\} \quad \bmod 2\pi_{11}(S^6),$$

(I owe this remarks to professor Toda, to whom I express my sincere thanks for his interest in my papers and valuable discussion.) and to (2.6) on page 186,

$$[\iota_{13}, \iota_{13}] = \tau_{13}.$$

In [2], we have introduced the generators s^n_m, u^n_m, and r^n_m of the homotopy groups of Sp_n, U_n, and R_n, respectively. When I wrote [2], I did not know whether and how they could be represented in terms of other known elements, but I could show in the meantime that the following relations hold.

(1) $s^2_7 \in \{\sigma''^2_3, \alpha_3, 12\iota_6\}$ $\quad \bmod 12\pi_7(Sp_2)$,

$s^3_{11} \in \{i''^{3,2}, T''_2, 5!\iota_{10}\}$ $\quad \bmod 5!\pi_{11}(Sp_3)$.

(2) $u^3_5 \in \{\sigma'^3_3, \eta_3, 2\iota_4\}$ $\quad \bmod 2\pi_5(U_3)$,

$u^3_{10} \in \{\sigma'^3_3, \eta_3, \nu_4\circ\eta_7\circ\eta_8\}$ $\quad \bmod 0$,

$u^3_{11} \in \{\sigma'^3_3, \eta_3, \nu_4\circ\nu_7\}$ $\quad \bmod (\sigma'^3_3\circ\varepsilon_3)$,

$u'^3_{12} \in \{u^3_5, 4\nu_5, \nu_8\}$ $\quad \bmod (\sigma'^3_3\circ\delta_3)$,

$u^4_7 \in \{i'^{4,3}, \sigma'^3_3\circ\alpha_3, 6\iota_8\}$ $\quad \bmod (6\pi_7(U_4))$,

$u^5_9 \in \{i'^{5,4}, T'_4, 24\iota_8\}$ $\quad \bmod 24\pi_9(U_5)$,

$u^5_{12} \in \{i'^{5,4}, T'_4, 4\nu_8\}$ $\quad \bmod (u'^5_{12})$,

$u^7_{13} \in \{i'^{7,6}, T'_6, 6!\iota_{12}\}$ $\quad \bmod 6!\pi_{13}(U_7)$.

(3) $r^7_{11} \in \{i^{7,6}, 2r^6_8, \eta_8, 4\iota_9\}$ $\quad \bmod 4\pi_{11}(R_7)+(r^7_8\circ\nu_8)$,

$r^{11}_{12} \in \{i^{11,10}, T_{10}, \eta_9\circ\eta_{10}\}$ $\quad \bmod 0$.

The characteristic class T''_2 of Sp_3 and the characteristic class T'_4 of U_5 can be represented as follows:

(4) $T''_2 \in \{\sigma''^2_3, \alpha_3, \nu_6\}$ $\quad \bmod 4\pi_{10}(Sp_2)$,

$T'_4 \in \{i'^{4,3}, \sigma'^3_3\circ\alpha_3, \nu_6\}$ $\quad \bmod (u^4_5\circ\nu_5)$.

Using these relations (1)−(4), the following can be proved.

(5) $T''_2\circ\eta_{10} = \sigma''^2_3\circ\varepsilon_3$, $\qquad s^2_7\circ\nu_7 = 4T''_2$.

(6)

$$u_5^3 \circ \eta_5 = \sigma_3'^3 \circ \alpha_3, \qquad u_5^3 \circ \nu_5 \circ \nu_8 = 2u_{11}^3,$$
$$u_5^3 \circ \beta_5'' = 2u_{12}'^3, \qquad u_5^3 \circ \varepsilon_5 = \sigma_3'^3 \circ \varepsilon_3',$$
$$u_{10}^3 \circ \eta_{10} = 0, \qquad u_{10}^3 \circ \nu_{10} = 0,$$
$$u_{11}^3 \circ \eta_{11} = 0, \qquad u_{12}'^3 \circ \eta_{12} = 0,$$
$$u_7^4 \circ \eta_7 = 4T_4', \qquad u_7^4 \circ \nu_7 \nu_{10} = 2l^4 \circ T_2'' \circ \nu_{10},$$
$$u_9^5 \circ \eta_9 = 4i'^{5,4} \circ l^4 \circ T_2'', \qquad u_9^5 \circ \nu_9 = \pm 2u_{12}^5,$$
$$u_{12}^5 \circ \eta_{12} = 2i'^{5,4} \circ l^4 \circ T_2'' \circ \nu_{10}, \qquad T_4' \circ \nu_8 = \pm u_{11}^4.$$

(7)

$$r_7^5 \circ \nu_7 = 4r_{10}^5, \qquad r_7^6 \circ \eta_7 = 4r_8^6,$$
$$r_7^6 \circ \nu_7 = \pm 2r_{10}^5, \qquad \rho_7^7 \circ \nu_7 = \pm r_{10}^7 \text{ or } \pm 3r_{10}^7,$$
$$r_8^6 \circ \eta_8 \circ \eta_9 = k^6 \circ u_{10}^3 + 4r_{10}^6, \qquad r_{10}^5 \circ \eta_{10} = \sigma_3^5 \circ \varepsilon_3,$$
$$T_6 \circ \beta_5'' = 2k^6 \circ u_{12}^3.$$

Homotopy groups $\pi_n(G_2)$ $(n \leqq 13)$ can easily be computed by using these relations.

The following is the corrigenda for [1] and [2].

	Page	Line	Instead of	Read
[1]	185	14	$\beta_6' \circ \nu_{13} = -2(\nu_6 \circ \mu_9)$	$\beta_6' \circ \nu_{13} = \pm 2(\nu_6 \circ \mu_9)$
	185	15	$\beta_7 \circ \nu_{14} = -\nu_7 \circ \mu_{10}$	$\beta_7 \circ \nu_{14} = \pm \nu_7 \circ \mu_{10}$ or $\pm 3(\nu_7 \circ \mu_{10})$
	185	18	$\varepsilon_3 \in \{\alpha_3, \nu_6, \nu_9\}$	$\varepsilon_3 \in \{\alpha_3, \nu_6, \eta_9\}$
	185	29	$\varepsilon_3' \in \{\alpha_3, 2\nu_6, \nu_9\}$	$\pm \varepsilon_3' \in \{\alpha_3, 2\nu_6, \nu_9\}$
	187	8	$F = (\nu_6 \circ \nu_9)$	$F = (\nu_6 \circ \varepsilon_9)$
	186	Take out the last line.		
	187	1	$n \geqq 4$	$n \geqq 3$
[2]	236	19	$i''^{n,2} \circ T_2'' \in \pi_{10}(Sp_n)$ $(n=2, 3)$	$T_2'' \in \pi_{10}(Sp_2)$
	237	6	u_8^4	$u_5^4 \circ \nu_5$
	237	25	$T_4 = \rho_3^4 + 2\sigma_3^4$	$T_4 = -\rho_3^4 + 2\sigma_3^4$
	237	26	$T_8 = \rho_7^8 + 2\sigma_7^8$	$T_8 = -\rho_7^8 + 2\sigma_7^8$
	238	4	$i^{7,6} \circ k^6 \circ u_{10}^3 = 4r_{10}^7$	$i^{7,6} \circ k^6 \circ u_{10}^3 = 0$
	238	4	$r_{10}^9 = 2\sigma_7^9 \circ \nu_7$	$r_{10}^9 = \pm 2\sigma_7^9 \circ \nu_7$
	238	5	$k^8 \circ l^4 \circ T_2'' = \sigma_7^8 \circ \nu_7$	$k^8 \circ l^4 \circ T_2'' = r_{10}^8 + \sigma_7^8 \circ \nu_7$
	237	16	$\sigma_3^4 \circ \eta_3 \circ \nu_4$ should be added between $\rho_3^n \circ \eta_3 \circ \nu_4$ $(n=3, 4)$ and ρ_7^n $(n=7, 8)$.	

References

[1] K. Ōguchi: 2-primary components of the homotopy groups of spheres, Proc. Japan Acad., **38** (5), 183–187 (1962).

[2] K. Ōguchi: 2-primary components of the homotopy groups of some Lie groups, Proc. Japan Acad., **38** (6), 235–238 (1962).