58. *On Proof Retrieval: Problem-Solving Machines. I*

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1. Introduction. Various kinds of methods have been considered on a problem solving machine which can understand the sentences written in a natural language and solve the problems in mathematics described in sentences. Here are described one method to execute computer programs that retrieve the proof of each theorem in plane geometry, for example, which is done by transforming a sentence written in natural language into a unique code formula.

In order that we develop a computer into a general problem solving machine, it is required for the computer to recognize any given ploblem exactly and to make use of the knowledge obtained already for solving the problem. For this purpose, it is fundamental to retrieve the proof from the memory, encoding any given problem in plane geometry by a unique code expression of the problem, and using it as an index.

The code system of plane geometry, PG0, consists of terms such as 3000 (triangle), 3053 (exterior angle), etc., and predicate such as R(A, B, $\cdots$, C), where R is a predicate letter and A, B, and C are terms. The formula R(A, B, $\cdots$, C) is called atomic formula. These codes must be one to one correspondence with the semantic meaning of human language. In fact both of 'isosceles triangle' and 'triangle having two equal sides' have the same code '3100'. This encoding technique is possible using SEE ALSO technique.

The transformation procedure from a sentence into its code expression is as follows:

1. Smoothing routine, which supplements all omissions in given sentences.
2. Classification of each word into parts of speech, which are conjunction, verb, article and numeral, adjective, noun, preposition, and relative pronoun.
3. Coding routine, which finds the association of words and puts the code using the thesaurus in the memory.
4. Formation of predicate from codes according to formation grammar.
5. Boolean combination of atomic formulas.
6. Normalizing routine, which arranges the codes into lexi-

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cographic formulas. This expression is called 'normal form'.

For example, the sentence 'vertical angles are equal' is transformed into the code expression '221(20000, 20001)→710(20000, 20001)'. Thus, whatever sentences may be used to represent the same content of a proposition, the final code expression must be rigorously unique and logical.

2. Code system. In the code system of plane geometry, PG0, the primitive symbols are the following three kinds of symbols;

(1) 0, 1, 2, 3, 4, 5, 6, 7

(2) logical symbols: AND, OR, →, NOT

(3) auxiliary symbols: (,), ' .

All terms of plane geometry may be expressed by concatenations of numbers, but it is not essential to use octal number. Each number has following meanings:

0:	Point	4:	Quadrilateral	(2.1)
1:	Line	5:	Pentagon, Hexagon, Octagon, Polygon	
2:	Angle	6:	Circle	
3:	Triangle	7:	———	

In the predicate of the code system, a term is composed of the root of term and parameters. For example, 30100 means a vertex of a triangle, where 30 is the root of term, which means a triangle, 10 means a vertex and the last 0 is a parameter for a triangle. Similarly, a term 30110 means another vertex of the triangle. Therefore, the term in the code system is not a noun itself but a noun phrase. Adjective in plane geometry may be encoded in two ways. One is the case of being involved in a term, for example, '*middle* point of a side' is 3013. The other is the case of being translated into a predicate, for example, 'the *included* angle between two sides' is encoded into a predicate 713(30300, 30500, 30510).

Thus, the code expression is composed by abstracting all the relations involved in the given sentences, and so the contents of the sentences can be understood without ambiguity.

3. Thesaurus. In order to classify each word into parts of speech and derive all relations, a thesaurus is prepared. The thesaurus is a kind of dictionary which gives the same code to the same meaning expressions. In the code system, PG0, there are two kinds of dictionary, term dictionary and predicate dictionary.

The hierarchy of 'term' is the same as (2.1). The priority is set among the roots of term as follows:[1]

$$\text{Point} < \text{Line} < \text{Angle} < \text{Triangle} < \text{Quadrilateral} < \text{Pentagon} = \text{Hexagon} = \text{Octagon} = \text{Polygon} < \text{Circle} \qquad (3.1)^{1}$$

1) $A<B$ means that the priority of B is higher than that of A. $A=B$ means that both priorities of A and B are equal.

The code of the root of term is set according to the priority, for example, in the case of 'an angle of a triangle', 'triangle' has a higher priority than 'angle', then the root of this term is 30 (triangle).

The term dictionary is composed of adjective and noun as follows:

adjective	noun	code
	ANGLE	LZP50
EXTERIOR	ANGLE	LZP53
INCLUDED	ANGLE	−LZP50
EXTERIOR	ANGLE	LZP50
	⋮	

The '−' sign indicates SEE ALSO, refering the predicate dictionary. As a triangle has three angles, LZP (level zero parameter) shows which angle to be represented.

The predicate dictionary is as follows:

verb/adjective	preposition	code of predicate letter
⋮		
COMPLEMENT	OF	222
COMPLEMENTARY		222
COMPLEMENTARY	OF	222
CONCURRENT		070
CROSS		110
⋮		
EQUAL		710
EQUAL	TO	710
⋮		
INCLUDED		713
INCLUDED	BETWEEN	713
⋮		

'Complement' is a noun but 'complement of an angle' means 'complementary angle of an angle' and it represents a relation between two angles. Therefore, 'complement' is given a code of predicate letter.

4. Example 1. 'If two angles and the included side of one triangle are equal to the corresponding angles and the included side of the other, then the triangles are congruent.' This theorem is transformed according to the following procedure:

1. In the sentence of this theorem there is an omission, that is, 'the other' is an abbreviation of 'the other angle'. This is supplemented by finding the correspondence of 'one' with 'the other'.
2. Each word in this sentence is classified.

conj.	verb	article/numeral	adjective	noun	prep.
IF		TWO		ANGLES	
AND		THE	INCLUDED	SIDE	OF
		ONE		TRIANGLE	
	ARE		EQUAL		TO
		THE	CORRESPONDING		
				ANGLES	
AND		THE	INCLUDED	SIDE	OF
		THE	OTHER	TRIANGLE	
,					
THEN		THE	TRIANGLES		
	ARE		CONGRUENT		
.					

3. Scanning conjunctions, which include comma and period, it is found whether the statement form[2] is *subject-predicate* form or *if-then* form. If the sentence is subject-predicate form, the subject becomes the hypothesis and the predicate becomes the conclusion. (cf. Example 3) In this example, the highest priority word is 'triangle', therefore the root of terms is 30. Article and numeral are used to set the term parameters. Therefore, two angles are encoded 30500 and 30510.

conj.	verb	adj.	term 1	term 2	prep.
IF			30500	30510	
AND		713	30300		OF
			30000		
	710				TO
			30501	30511	
AND		713	30301		OF
			30001		
,					
THEN			30000	30001	
	742				
.					

4. Conjunction, verb and preposition are used to distinguish the block of the adjective form predicate such as 713(30300, 30500, 30510) and 713(30301, 30501, 30511).

2) The statement "A heated metal expands" and "If a metal is heated, then it expands" are two forms of the same idea.

5. The final code expression :

713(30300, 30500, 30510) AND 713(30301, 30501, 30511) AND 710(30500, 30501) AND 710(30510, 30511) AND 710(30300, 30301) →742(30000, 30001)

5. Example 2. The theorem 'If two triangles have two angles and the included side of one respectively equal to two angles and the included side of the other, the triangles are congruent.' is the same idea as example 1.

This code expression is the same as example 1 :

713(30300, 30500, 30510) AND 713(30301, 30501, 30511) AND 710(30500, 30501) AND 710(30510, 30511) AND 710(30300, 30301) →742(30000, 30001)

6. Example 3. The theorem 'Complements of the same or of equal angles are equal.' is subject-predicate form.

The final code expression :

222(20000, 20500) AND 222(20000, 20501) OR 222(20000, 20500) AND 222(20001, 20501) AND 710(20000, 20001)→710(20500, 20501)

Appendix

Table I. Predicate Letter

000	exist one point between two points	222	complement (ary)
001	divide internally	223	supplement (ary)
002	divide externally	660	inscribe
003	(four points make) harmonic range	661	circumscribe
		700	add
010	pass through (a point)	701	subtract
011	pass through (two points)	702	multiply
012	pass through (three points)	703	divide
013	pass through (four points)	704	twice
070	concurrent	705	half
110	cross	706	bisect
111	perpendicular (to) meet at right angle	710	equal (to)
		711	greater (than)
112	parallel (to)	712	less (than)
160	tangent (to)	720	move
165	common internal tangent	730	symmetry
166	common external tangent	740	similar
170	colinear, lie on a line	741	proportion(al)
220	adjacent	742	congruent
221	vertical	744	opposite

Table II. Term

root of term	parameter		root of term	parameter	
00	00	point	20	00	angle
01	02	distance		10	vertex of angle
	03	area		30-31	side of angle
10	00	straight line		33-34	exterior side of angle
14	00	segment			

continued

root of term	parameter		root of term	parameter	
	55	right angle		50-53	angle of quadrilateral
30	00	triangle			
	10-12	vertex of triangle	41	00	regular quadrilateral
	13-15	middle point of side			
	30-32	side of triangle	43	00	rectangle
	50-52	angle of triangle	44	00	parallelogram
31	00	regular triangle, equilateral triangle	45	00	trapezoid
			60	00	circle
				04	radius
32	00	isosceles triangle		05	diameter
33	00	right triangle		06	perimeter
40	00	quadrilateral		50	central angle
	10-13	vertex of quadrilateral		51	circum angle, inscribed angle
	30-33	side of quadrilateral	61	00	arc
	34-35	diagonal	62	00	chord

The codes of pentagon, hexagon, octagon, and polygon are not shown in above table but these codes are quite similar to those of quadrilateral.

References

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