173. On Axiom Systems of Propositional Calculi. VIII

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In this note, we shall prove that the (R) axiom system 1∼6 of propositional calculus implies Sobosiński systems (S_1), (S_2) (for the notations and rules of inference, see [1]). In the fifth note [2] of this series, K. Iséki and S. Tanaka proved that Russell system implies Lukasiewicz (L_1)-system. And in the sixth note [3] it was proved that Lukasiewicz (L_2), (L_3)-systems, Hilbert system, and Mendelson system are deduced from the (R)-system.

1 $CpCqp$.

2 $CCpqCCqrCpr$.

3 $CCpCqrCqCpr$.

4 $CNNpp$.

5 $CCpNpNp$.

6 $CCpNqCqNp$.

3 r/p, $q/CpCqp$ *$C1$ $q/CpCqp$—$C1$—7,

7 Cpp.

6 p/Np, q/p *$C7$ p/Np—8,

8 $CpNNp$.

3 p/Cpq, q/Cqr, r/Cpr *$C2$—9,

9 $CCqrCCpqCpr$.

9 r/NNq *$C8$ p/q—10,

10 $CCpqCpNNq$.

2 p/Cpq, $q/CpNNq$, $r/CNqNp$ *$C10$—$C6$ q/Nq—11,

11 $CCpqCNqNp$.

9 q/NNq, r/q *$C4$ p/q—12,

12 $CCpNNqCpq$.

2 $p/CNpq$, $q/CNqNNp$, $r/CNqp$ *$C11$ p/Np—$C12$ p/Nq, q/p—13,

13 $CCNpqCNqp$.

9 $q/CNqp$, $r/CNpq$ *$C13$ p/q, q/p—$C1$ q/Nq—14,

14 $CpCNpq$.

3 q/Np, r/q *$C14$—15,

15 $CNpCpq$.

2 p/Cpq, $q/CCqrCpr$, r/s *$C2$—16,

16 $CCCCqrCprsCCpqs$.

16 q/Cqr, r/Csr, $s/CCsqCpCsr$ *$C16$ p/s, $s/CpCsr$—17,

17 $CCpCqrCCsqCpCsr$.

17 p/Nq, s/p *$C15$ p/q, q/r—18,

18 $CCpqCNqCpr$.

2 p/NNp, q/p, r/q *$C4$—19,

19 $CCpqCNNpq$.

2 p/Cpq, $q/CNNpq$ *$C19$—20,

20 $CCCNNpqrCCpqr$.

16 $s/CCCprsCCqrs$ *$C2$ p/Cqr, q/Cpr, r/s—21,

21 $CCpqCCCprsCCqrs$.

3 p/Cpq, $q/CCprs$, $r/CCqrs$ *$C21$—22.

22 $CCCprsCCpqCCqrs$.

22 $p/CNNpq$, $q/CNqNp$, $s/CCpqr$ *$C20$—$C13$ p/Np—23,

23 $CCCNqNprCCpqr$.

23 p/Np, q/p, r/NNp *$C5$ p/Np—24,

24 $CCNppNNp$.

12 $p/CNpp$, q/p *$C24$—25,

25 $CCNppp$.

22 p/Np, r/p, s/p *$C25$—26,

26 $CCNpqCCqpp$.

2 $p/CNpq$, $q/CNqp$, $r/CCpqq$ *$C13$—$C26$ p/q, q/p—27,

27 $CCNpqCCpqq$.

9 q/Crp, $r/CqCrp$ *$C1$ p/Crp—$C1$ q/r—28,

28 $CpCqCrp$.

2 $p/CpCNpq$, $q/CNpCpq$, $r/CCCpqpp$ *$C3$ q/Np, r/q—$C26$ q/Cpq—$C15$—29,

29 $CCCpqpp$.

22 p/Cpq, q/r, r/p, s/p *$C29$—30,

30 $CCCpqrCCrpp$.

2 $p/CCpqr$, $q/CCrpp$, $r/CCprr$ *$C30$—$C30$ p/r, q/p, r/p—31,

31 $CCCpqrCCprr$.

16 q/Cqr, r/Csr, $s/CCsqCpCsr$ *$C16$ p/s, $s/CpCsr$—32,

32 $CCpCqrCCsqCpCsr$.

32 p/Cpq, $q/CCprs$, $r/CCqrs$, s/t *$C21$—33,

33 $CCtCCprsCCpqCtCCqrs$.

33 $t/CCpqr$, q/s, s/r *$C31$—34,

34 $CCpsCCCpqrCCsrr$.

3 p/Cps, $q/CCpqr$, $r/CCsrr$ *$C34$—35,

35 $CCCpqrCCpsCCsrr$.

2 $p/CNpr$, $q/CCprr$, $r/CCpqCCqrr$ *$C27$ q/r—$C35$ q/r, s/q—36,

36 $CCNprCCpqCCqrr$.

2 $p/CNpr$, $q/CCpqCCqrr$, $r/CCqrCCpqr$ $*C36$—$C3$ p/Cpq, q/Cqr—37,

37 $CCNprCCqrCCpqr$.

Theses 15, 28, 37 are axioms of (S_1)-system, and theses 18, 27, 28 are axioms of (S_2)-system.

References

[1] Y. Imai and K. Iséki: On axiom systems of propositional calculi. I. Proc. Japan Acad., **41**, 436-439 (1965).

[2] K. Iséki and S. Tanaka: On axiom systems of propositional calculi. V. Proc. Japan Acad., **41**, 661-662 (1965).

[3] S. Tanaka: On axiom systems of propositional calculi. VI. Proc. Japan Acad., **41**, 663-666 (1965).