196. *On Axiom Systems of Propositional Calculi. XI*

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In this note we shall prove that the Sobociński (S_1) axiom system of propositional calculus implies (F), (H), (L_1), (L_2), (L_3), (M), (R), and (S_2) axiom systems. The notations and two rules of inference are mentioned in our previous notes published in this Proceedings. The (S_1)-system is read as follows:

1 $CNpCpq$,
2 $CpCqCrp$,
3 $CCNprCCqrCCpqr$.

Now we have the following theses from three axioms above.

3 p/q, q/p, r/Cqr *$C1$ p/q, q/r—4,
4 $CCpCqrCCqpCqr$.
4 r/Crp *$C2$—5,
5 $CCqpCqCrp$.
5p/Cpq, q/Np, r/s *$C1$—6,
6 $CNpCsCpq$.
3 q/Cqr, $r/CqCpr$ *$C6$ q/r, s/q—$C5$ p/r, r/p—7,
7 $CCpCqrCqCpr$.
7 p/Cqp, r/Crp *$C5$—8,
8 $CqCCqpCrp$.
3 $r/CCqrCpr$ *$C6$ q/r, s/Cqr—$C8$ p/r, r/p—9,
9 $CCpqCCqrCpr$.
7 p/Cpq, q/Cqr, r/Cpr *$C9$—10,
10 $CCqrCCpqCpr$.
3 q/Cqr, $r/CCpqCpr$ *$C6$ q/r, s/Cpq—$C10$—11,
11 $CCpCqrCCpqCpr$.
3 $r/CsCpq$ *$C6$—$C2$ p/q, q/s, r/p—12,
12 $CCpqCsCpq$.
4 $p/CqCpq$, q/Cpq, r/Cpq *$C4$ p/q, q/p, r/q—$C12$ s/q—13,
13 $CCpqCpq$.
4 p/Cqr *$C13$ p/q, q/r—14,
14 $CCqCqrCqr$.
14 q/p, r/Cqp *$C2$ q/p, r/q—15,
15 $CpCqp$.
14 q/p, r/p *$C15$ q/p—16,
16 Cpp.

3 q/Np, r/Np *$C16$ p/Np—$C16$ p/Np—17,

17 $CCpNpNp$.

7 p/Np, q/p, r/q *$C1$—18,

18 $CpCNpq$.

3 q/p, $r/CNNpp$ *$C18$ p/Np, q/p—$C15$ q/NNp—$C16$—19,

19 $CNNpp$.

3 p/Np, q/p, r/p *$C19$—$C16$—20,

20 $CCNppp$.

3 q/Nq, $r/CqNp$ *$C15$ p/Np—$C1$ p/q, q/Np—21,

21 $CCpNqCqNp$.

21 p/Np, q/p *$C16$ p/Np—22,

22 $CpNNp$.

7 $p/CNpr$, q/Cqr, $r/CCpqr$ *$C3$—23,

23 $CCqrCCNprCCpqr$.

23 r/q *$C16$ p/q—24,

24 $CCNpqCCpqq$.

9 p/Np, q/Cpq *$C1$—25,

25 $CCCpqrCNpr$.

9 q/Cqp *$C15$—26,

26 $CCCqprCpr$.

10 r/NNq *$C22$ p/q—27,

27 $CCpqCpNNq$.

9 p/Cpq, $q/CpNNq$, $r/CNqNp$ *$C27$—$C21$ q/Nq—28,

28 $CCpqCNqNp$.

7 $p/CNpq$, q/Cpq, r/q *$C24$—29,

29 $CCpqCCNpqq$.

10 p/s, q/NNp, r/p *$C19$—30,

30 $CCsNNpCsp$.

10 $p/CNpNq$, $q/CqNNp$, r/Cqp *$C30$ s/q—$C21$ p/Np—31,

31 $CCNpNqCqp$.

9 p/Cpq, $q/CCqrCpr$, r/s *$C9$—32,

32 $CCCCqrCprsCCpqs$.

32 q/Cqr, r/Csr, $s/CCsqCpCsr$ *$C32$ p/s, $s/CpCsr$—33,

33 $CCpCqrCCsqCpCsr$.

33 p/Nq, s/p *$C1$ p/q, q/r—34,

34 $CCpqCNqCpr$.

10 p/s, $q/CNpr$, $r/CCqrCCpqr$ *$C3$—35,

35 $CCsCNprCsCCqrCCpqr$.

9 p/NNp, q/p, r/q *$C18$—36,

36 $CCpqCNNpq$.

35 p/Np, s/Cpr $*C36$ q/r—37,

37 $CCprCCqrCCNpqr$.

37 p/q, q/p, r/q $*C16$ p/q—38,

38 $CCpqCCNqpq$.

10 $p/CNqNp$, q/Cpq, $r/CCNqpq$ $*C38$—$C31$ p/q, q/p—39,

39 $CCNqNpCCNqpq$.

Therefore, we have the following results:

1) Theses 11, 15, 19, 22, and 28 are axioms of (F),
2) theses 7, 10, 15, 18, and 29 are axioms of (H),
3) theses 9, 18, and 20 are axioms of (L_1),
4) axiom 3 of (S_1) and theses 25, 26 are axioms of (L_2),
5) theses 11, 15, and 31 are axioms of (L_3),
6) theses 11, 15, and 39 are axioms of (M),
7) theses 7, 9, 15, 17, 19, and 21 are axioms of (R),

and

8) axiom 2 of (S_1) and theses 24, 34 are axioms of (S_2).