

228. A Remark on Propositional Calculus with Variable Functors

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C. A. Meredith [1] has discussed a certain deductive equivalence in the δ -calculus. By the δ -calculus, we mean a propositional calculus with variable functors. In this paper, we shall concern with some deductive equivalences that C. A. Meredith omitted in his paper.

First of all, we shall prove that

$$g\delta 0\delta C00 \quad (1)$$

and

$$g00, g0C00, gC000, gC00C00 \quad (2)$$

are deductive equivalent under

$$T4 \quad C\delta 0C\delta C00\delta p,$$

$$T5 \quad C\delta CC000\delta 0,$$

$$T7 \quad C\delta CC00C00\delta C00,$$

where gpq denotes any function of p and q .

Proof of (1) \Rightarrow (2).

- 1 $g\delta 0\delta C00$.
 1 $\delta/' *2,$
- 2 $g0C00$.
 1 $\delta/C'0 *3,$
- 3 $gC00CC000$.
 T5 $\delta/gC00' *C3-4,$
- 4 $gC000$.
 1 $\delta/C'' *5,$
- 5 $gC00CC00C00$.
 T7 $\delta/gC00' *C5-6,$
- 6 $gC00C00$.
 1 $\delta/CC''0 *7,$
- 7 $gCC000CCC00C000$
 T7 $\delta/gCC000C'0 *C7-8,$
- 8 $gCC000CC000$.
 T5 $\delta/g'' *C8-9,$
- 9 $g00$.

Proof of (2) \Rightarrow (1).

- 1 $g00$.
- 2 $g0C00$.
- 3 $gC000$.

- 4 $gC00C00.$
 $T4 \delta/g0', p/\delta C00 *C1-C2-5,$
- 5 $g0\delta C00.$
 $T4 \delta/gC00', p/\delta C00 *C3-C4-6,$
- 6 $gC00\delta C00.$
 $T4 \delta/g'\delta C00, p/\delta 0 *C5-C6-7,$
- 7 $g\delta 0\delta C00.$
 We shall give another proof of $(2)\Rightarrow(1).$
 $T4 \delta/g'C00, p/\delta 0 *C2-C4-5,$
- 5 $g\delta 0C00.$
 $T4 \delta/g'0, p/\delta 0 *C1-C3-6,$
- 6 $g\delta 00.$
 $T4 \delta/g\delta 0', p/\delta C00 *C6-C5-7,$
- 7 $g\delta 0\delta C00.$

Reference

- [1] C. A. Meredith: On an extended system of the propositional calculus.
 Proc. Royal Irish Acad., 3, 37-47 (1951).