

## 22. On the Proposition $C\delta CpqC\delta p\delta q$ with a Variable Functor

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In this note, we shall prove that  $C\delta CpqC\delta p\delta q$  implies

- 1)  $CpCqp$ ,
- 2)  $CCpCqrCCpqCpr$ ,
- 3)  $CCCpqpp$ .

As the substitution rules for  $\delta$  are well-known, we omit them. (For example, see [1] Lukasiewicz, [2] Meredith or [3] Prior.) For the details on the propositional calculus with a variable functor  $\delta$ , see [1]-[3].

**Proof.**

- 1  $C\delta CpqC\delta p\delta q$ .  
1  $\delta / '-2$ ,
- 2  $CCpqCpq$ .  
1  $\delta / C'' * C2-3$ ,
- 3  $CCppCqq$ .  
3  $p / Cpq * C2-4$ ,
- 4  $Cqq$ .  
1  $\delta / Cp', p/q, q/r-5$ ,
- 5  $CCpCqrCCpqCpr$ .  
5  $p / Cpq, q/p, r/q * C2-6$ ,
- 6  $CCCpqpCCpq$ .  
1  $\delta / C' Cpq * C2-7$ ,
- 7  $CCpCpqCqCpq$ .  
1  $\delta / C'r-8$ ,
- 8  $CCCpqrCCprCqr$ .  
8  $q / Cpq, r / CqCpq, * C7-9$ ,
- 9  $CCpCqCpqCCpqCqCpq$ .  
1  $\delta / CCpCq' C' Cq' * C9-C4$   $q / CpCqp-10$ ,
- 10  $CCpCqqCqCqq$ .  
10  $p / Cpp, q/p * C4$   $q / Cpp-11$ ,
- 11  $CpCpp$ .  
8  $p / Cpq, q/p, r / CCpq * C6-12$ ,
- 12  $CCCpqCCpqCpCCpq$ .  
1  $\delta / CC' C' q Cp C' q * C12-C4$   $q / CpCpq-C11$   $p/q-13$ ,
- 13  $CpCqq$ .  
1  $\delta / CCpC' q C' Cpq * C13$   $p / CpCCpq, q / Cpq-C4$   $q / CpCpq$   
-C13-14,

- 14  $CqCpq,$   
     5  $p/CpCqr, q/Cpq, r/Cpr * C5-15,$   
 15  $CCCpCqrCpqCCpCqrCpr.$   
     15  $q/p, r/q * C13 p/CpCpq, q/p-16,$   
 16  $CCpCpqCpq.$   
     14  $q/CCpCpqCpq, p/CCCpqqp * C16-17,$   
 17  $CCCCpqqpCCpCpqCpq.$   
     1  $\delta/CCC'ppCCp'' * C17-C4 q/CCppp-18,$   
 18  $CCCqppCCpqq.$   
     18  $p/Cpq, q/p * C16-19,$   
 19  $CCCpqqp.$

Theses 5, 14, and 19 are axioms by Tarski-Bernays. Therefore the proof is complete.

### References

- [1] J. Lukasiewicz: Aristotle's Syllogistic. 2nd Edition, Oxford (1963).  
 [2] C. A. Meredith: On an extended system of the propositional calculus. Proc. Royal Irish Acad., Dublin, **54**, A 3 (1951).  
 [3] A. N. Prior: Formal Logic. 2nd Edition, Oxford (1962).