95. *Axiom Systems of Distributive Lattice*

By Kiyoshi ISÉKI and Sakiko ÔHASHI

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In his paper [3], S. Tamura gave some axiom systems for semirings. In this Note, we shall give some axiom systems of distributive lattices.

In a letter of Dr. H. F. J. Lowig to Ôhashi, he noted that Theorem 2 in [1], is true under an additional condition: $r+1=1$. As easily seen, in a semiring R with 0 and 1 that the addition and multiplication operations are commutative and these are idempotent, if $r+1=1$ for every $r \in R$, then R is a distributive lattice. In such a semiring R, for every $a \in R$, we have $a+ar=a(1+r)=a$. Therefore we have the absorption law in R. Hence from Theorems 1–4, in [2] we have the following theorems.

Theorem 1. $\langle R, +, \cdot\cdot\cdot, 0, 1\rangle$ *is a distributive lattice, if and only if it satisfies the following conditions*:

1.1) $r+0=r$,

1.2) $r1=r$,

1.3) $0r=0$,

1.4) $r+1=1$,

1.5) $((a+br)+cz+d+d)r=br+(ar+z(cr)+dr)$ *for every* a, b, c, d, r, z *in* R.

Theorem 2. $\langle R, +, \cdot\cdot\cdot, 0, 1\rangle$ *is a distributive lattice, if and only if it satisfies the following conditions*:

2.1) $r+0=0+r=r$,

2.2) $0r=0$,

2.3) $r+1=1$,

2.4) $((a+br)+cz+d+d)r+s=br+(ar+z(cr)+dr)+s1$.

Theorem 3. $\langle R, +, \cdot\cdot\cdot, 0, 1\rangle$ *is a distributive lattice, if and only if the following conditions hold*:

3.1) $r+0=0+r=r$,

3.2) $r1=r$,

3.3) $r+1=1$,

3.4) $0e+((a+br)+cz+d+d)r=br+(ar+z(cr)+dr)$

for every a, b, c, d, e, r, z *in* R.

Theorem 4. $\langle R, +, \cdot\cdot\cdot, 0, 1\rangle$ *is a distributive lattice, if and only if it satisfies the following conditions*:

4.1) $r+0=0+r=r$,

4.2) $01=0$,

4.3) $r+1=1$,

4.4) $0e+((a+br)+cz+d+d)r+s=br+(ar+z(cr)+dr)+s1$.

References

[1] S. Ôhashi: On definitions of Boolean rings and distributive lattices. Proc. Japan Acad., **44**, 1015-1017 (1968).

[2] ——: On definitions for commutative idempotent semirings. Proc. Japan Acad., **46**, 113-115 (1970).

[3] S. Tamura: Axioms for commutative rings. Proc. Japan Acad., **46**, 116-120 (1970).