85. *Classification of Homogeneous Siegel Domains of Type II of Dimensions 9 and 10*

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1. In the recent paper [2], Kaneyuki and Tsuji classified all homogeneous Siegel domains of type I (resp. of type II) up to dimension 10 (resp. 8). The purpose of this note is to state the results of classification of homogeneous Siegel domains of type II of dimensions 9 and 10. The detailed results with their complete proofs will appear elsewhere. A homogeneous Siegel domain is said to be *irreducible* if it is irreducible in the sense of Kähler geometry.

2. We will recall some of results about skeletons of type II in [2]. Put $m+1$ tiny circles $\circ$ in $\boldsymbol{R}^2$ such that they may form vertices of a regular $m+1$-polygon (by a 2-polygon we mean a line segment) and number these circles counterclockwise starting from the upper left corner and color the last $m+1$-th vertex $\bullet$ in black (the i-th vertex is called simply i). Some of these vertices may be joined by line segments. If i and j are joined (resp. not joined), we will write $i\sim j$ (resp. $i\not\sim j$). If $i\sim j(i<j)$, then a positive integer n_{ij} should be attached to the line segment $\overline{ij}$. The figure $(\mathfrak{S},(n_{ij}))$ thus obtained is called an *m-skeleton of type II* if the following conditions are satisfied:

(1) There exists at least one vertex $i(1\leq i\leq m)$ such that $i\sim m+1$. In this case $n_{i,m+1}$ is always an even number.

(2) If $i<j<k$, $i\sim j$ and $j\sim k$, then $i\sim k$ and $\max(n_{ij},n_{jk})\leq n_{ik}$.

(3) If $i<j<k<l$, $i\sim j$, $j\sim l$, $i\sim k$, $k\sim l$, $i\sim l$ and $j\not\sim k$, then $\max\cdot(n_{ij}+n_{ik},n_{ij}+n_{kl},n_{jl}+n_{ik},n_{jl}+n_{kl})\leq n_{il}$.

An m-skeleton $(\mathfrak{S},(n_{ij}))$ of type II is said to be *connected* if for any two vertices i and j $(i,j\neq m+1)$ there exists a series of vertices $i_0=i$, $i_1,\cdots,i_s=j$ such that $i_{k-1}\sim i_k$, $i_k\neq m+1(1\leq k\leq s)$. Let $(\mathfrak{S},(n_{ij}))$ and $(\mathfrak{S}',(n'_{ij}))$ be two m-skeletons of type II. Then $\mathfrak{S}$ is said to be *isomorphic* to $\mathfrak{S}'$ if there exists a permutation σ of the set $\{1,\cdots,m+1\}$ such that

(1) $\sigma(m+1)=m+1$,

(2) if $i<j$ and $\sigma(i)>\sigma(j)$, then $i\not\sim j$ in $\mathfrak{S}$,

(3) $\sigma(i)\sim\sigma(j)$ in $\mathfrak{S}'$ if and only if $i\sim j$ in $\mathfrak{S}$,

(4) $n'_{\sigma(i)\sigma(j)}=n_{ij}(1\leq i<j\leq m+1)$.

It can be seen that the above isomorphism is an equivalence relation. It is known in [2] that *to each holomorphic equivalence class of homo-*

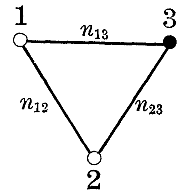
geneous Siegel domains of type II there corresponds an isomorphism class of certain skeletons of type II and that *a homogeneous Siegel domain of type II is irreducible if and only if the corresponding skeleton of type II is connected.*

3. In this paragraph we state the results obtained.

Lemma. *Let* $(\mathfrak{S}, (n_{ij}))$ *be an m-skeleton of type II which corresponds to an irreducible homogeneous Siegel domain of type II of dimensions* 9 *or* 10. *Then* $m \leq 5$ *and* $m + \sum_{1\leq i<j\leq m} n_{ij} + \frac{1}{2}\sum_{1\leq i\leq m} n_{i,m+1} = 9$ *or* 10. (*)

In view of the above facts and the results in [1], the classification of homogeneous Siegel domains of type II of dimensions 9 and 10 is, roughly speaking, reduced to the classification of connected m-skeletons of type II which satisfy the condition (*). Let $\mathfrak{S}_2$ be one of the following 2-skeletons of type II.

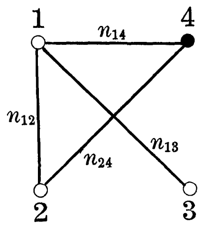
$\mathfrak{S}_2$



, $(n_{12}, n_{23}, n_{13}) = (2, 2, 8), (2, 2, 10), (2, 4, 6), (2, 4, 8), (2, 6, 6), (3, 2, 6), (3, 2, 8), (4, 2, 6).$

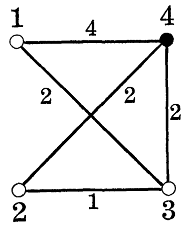
Let $\mathfrak{S}_3^k (1 \leq k \leq 3)$ be one of the following 3-skeletons of type II.

$\mathfrak{S}_3^1$

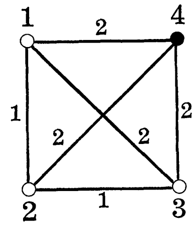


, $(n_{12}, n_{24}, n_{14}, n_{13}) = (2, 2, 4, 1), (2, 2, 6, 1), (2, 2, 4, 2), (2, 4, 4, 1).$

$\mathfrak{S}_3^2$



, $\mathfrak{S}_3^3$



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Then by using the analogous methods as in [2], we get

Theorem. (i) *To each of the connected m-skeletons of type II satisfying* (*) *which is not isomorphic to* $\mathfrak{S}_2$ *or* $\mathfrak{S}_3^k (1 \leq k \leq 3)$, *there corresponds a unique irreducible homogeneous Siegel domain of type II*;

(ii) *to each of the skeletons* $\mathfrak{S}_2$ *with* $(n_{12}, n_{23}, n_{13})=(2, 6, 6)$ *or* $\mathfrak{S}_3^1$ *with* $(n_{12}, n_{24}, n_{14}, n_{13})=(2, 4, 4, 1)$, *there correspond two non-equivalent irreducible homogeneous Siegel domains of type II*;

(iii) *to each of the skeletons* $\mathfrak{S}_2$ *with* $(n_{12}, n_{23}, n_{13})=(2, 2, 8), (2, 2, 10), (3, 2, 6), (3, 2, 8), (4, 2, 6), (2, 4, 6)$ *or* $\mathfrak{S}_3^1$ *with* $(n_{12}, n_{24}, n_{14}, n_{13})=(2, 2, 4, 1), (2, 2, 6, 1), (2, 2, 4, 2)$ *or* $\mathfrak{S}_3^2$, *there corresponds a one-parameter family of non-equivalent irreducible homogeneous Siegel domains of type II*;

(iv) *to the skeleton* $\mathfrak{S}_2$ *with* $(n_{12}, n_{23}, n_{13})=(2, 4, 8)$, *there corresponds a two-parameter family of non-equivalent irreducible homogeneous Siegel domains of type II*;

(v) *to the skeleton* $\mathfrak{S}_3^3$, *there corresponds no homogeneous Siegel domain of type II*;

the domains in (i)—(iv) *exhaust all irreducible homogeneous Siegel domains of type II of dimensions* 9 *and* 10.

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References

[1] S. Kaneyuki: On the automorphism groups of homogeneous bounded domains. J. Fac. Sci. Univ. Tokyo, **14**, 89–130 (1967).

[2] S. Kaneyuki and T. Tsuji: Classification of homogeneous bounded domains of lower dimension (to appear).