82. A Table of Hecke Operators. II

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(Comm. by Kunihiko KODAIRA, M. J. A., June 12, 1973)

Let q be a prime number such that $q\equiv 1 \pmod 4$ and $\Gamma_0(q)$ be the congruence subgroup of level q i.e.,

$$\Gamma_0(q)=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\boldsymbol{Z})\,;\ c\equiv 0 \pmod q\right\}.$$

Let $S(q)$ be the set of cusp forms $f(z)$ such that

$$f\left(\frac{az+b}{cz+d}\right)=(cz+d)^2 f(z),\ \text{for all } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(q).$$

Then $S(q)$ forms a finite dimensional vector space over the complex number field. Let p be a prime number different from q. The Heck operator $T(p)$ is a linear transformation of $S(q)$ and it is known that if we choose suitable basis of $S(q)$ (independently of p), each $T(p)$ can be represented in the following form

$$T(p)=\begin{pmatrix} x_1 & & & 0 \\ & x_2 & & \\ & & \ddots & \\ 0 & & & x_n \end{pmatrix}$$

We can compute $x_1, x_2, \cdots, x_n$ using the Eichler-Selberg trace formula (cf. [1], [2]).

In general the characteristic polynomial of $T(p)$

$$F_{p,q}(x)=(x-x_1)(x-x_2)\cdots(x-x_n) \in \boldsymbol{Z}[x]$$

is not irreducible. But we have the following factorization algorithm: Suitable combinations of elementary symmetric functions in some of these roots $x_1, x_2, \cdots, x_n$ are tested in order to decide whether or not they are sufficiently close to rational integers to guarantee the existence of a corresponding proper factor of $F_{p,q}(x)$ in $\boldsymbol{Z}[x]$ (cf. [3]).

The author made a table of factorized $F_{p,q}(x)$ (with irreducible factors) for

(1) $0<q<250,\ q\neq 227, 239,$

(2) $0<p<1000,\ p\neq q.$

For computing this table, the author used the electronic computer TOSBAC-3000 installed in the Department of Mathematics, Tsuda College. This calculation required about three hundred hours computer time.

This table is very large. So in this paper there is only a list of

$F_{2,q}(x)$ and $F_{3,q}(x)$. There are copies of the whole table at Department of Mathematics, University of Tokyo and Department of Mathematics, Kyoto University.

q	$F_{2,q}(x)$
11	$x+2$
17	$x+1$
19	x
23	x^2+x-1
29	x^2+2x-1
31	x^2-x-1
37	$x(x+2)$
41	x^3+x^2-5x-1
43	$(x+2)(x^2-2)$
47	$x^4-x^3-5x^2+5x-1$
53	$(x+1)(x^3+x^2-3x-1)$
59	$x^5-9x^3+2x^2+16x-8$
61	$(x+1)(x^3-x^2-3x+1)$
67	$(x-2)(x^2+3x+1)(x^2+x-1)$
71	$(x^3-5x+3)(x^3+x^2-4x-3)$
73	$(x-1)(x^2+3x+1)(x^2-x-3)$
79	$(x+1)(x^5-6x^3+8x-1)$
83	$(x+1)(x^6-x^5-9x^4+7x^3+20x^2-12x-8)$
89	$(x+1)(x-1)(x^5+x^4-10x^3-10x^2+21x+17)$
97	$(x^3+4x^2+3x-1)(x^4-3x^3-x^2+6x-1)$
101	$x(x^7-13x^5+2x^4+47x^3-16x^2-43x+14)$
103	$(x^2+3x+1)(x^6-4x^5-x^4+17x^3-9x^2-16x+11)$
107	$(x^2+x-1)(x^7+x^6-10x^5-7x^4+29x^3+12x^2-20x-8)$
109	$(x-1)(x^3+2x^2-x-1)(x^4+x^3-5x^2-4x+3)$
113	$(x-1)^2(x+1)(x^3+2x^2-x-1)(x^3+2x^2-5x-9)$
127	$(x^3+3x^2-3)(x^7-2x^6-8x^5+15x^4+17x^3-28x^2-11x+15)$
131	$x(x^{10}-18x^8+2x^7+111x^6-18x^5-270x^4+28x^3+232x^2+16x-32)$
137	$(x^4+3x^3-4x-1)(x^7-10x^5+28x^3+3x^2-19x-7)$
139	$(x-1)(x^3+2x^2-x-1)(x^7-x^6-11x^5+8x^4+35x^3-10x^2-32x-8)$
149	$(x^3+x^2-2x-1)(x^9+x^8-15x^7-12x^6+75x^5+48x^4-137x^3-76x^2$ $+68x+39)$
151	$(x^3-5x+3)(x^3+2x^2-x-1)(x^6-x^5-7x^4+3x^3+13x^2+3x-1)$
157	$(x^5+5x^4+5x^3-6x^2-7x+1)(x^7-5x^6+2x^5+21x^4-22x^3-21x^2+27x$ $-1)$
163	$x(x^5+5x^4+3x^3-15x^2-16x+3)(x^7-3x^6-5x^5+19x^4-23x^2+4x+6)$
167	$(x^2+x-1)(x^{12}-2x^{11}-17x^{10}+33x^9+103x^8-189x^7-277x^6+447x^5$ $+363x^4-433x^3-205x^2+120x+9)$

q	$F_{2,q}(x)$
173	$(x^4+x^3-3x^2-x+1)(x^{10}-x^9-16x^8+16x^7+85x^6-80x^5-175x^4+136x^3+138x^2-71x-25)$
179	$(x-2)(x^3+x^2-2x-1)(x^{11}+3x^{10}-14x^9-45x^8+59x^7+225x^6-58x^5-427x^4-76x^3+240x^2+56x-16)$
181	$(x^5+3x^4-x^3-7x^2-2x+1)(x^9-3x^8-9x^7+29x^6+23x^5-84x^4-23x^3+89x^2+8x-27)$
191	$(x^2+x-1)(x^{14}-23x^{12}+x^{11}+205x^{10}-13x^9-895x^8+35x^7+1993x^6+103x^5-2135x^4-465x^3+853x^2+374x+41)$
193	$(x^2+3x+1)(x^5+2x^4-5x^3-7x^2+7x+1)(x^8-2x^7-9x^6+18x^5+21x^4-44x^3-11x^2+27x+1)$
197	$(x+2)(x^5-5x^3+x^2+3x-1)(x^{10}-15x^8+x^7+78x^6-7x^5-165x^4+15x^3+123x^2-9x-26)$
199	$(x^2+x-1)(x^4+3x^3-4x-1)(x^{10}-5x^9-4x^8+51x^7-32x^6-154x^5+151x^4+168x^3-168x^2-54x+27)$
211	$(x^2-x-1)(x^3+2x^2-x-1)(x^3-4x+1)(x^9+x^8-14x^7-11x^6+66x^5+36x^4-123x^3-38x^2+72x+8)$
223	$(x^2+2x-1)(x^4+4x^3+2x^2-5x-3)(x^{12}-7x^{11}+6x^{10}+57x^9-122x^8-105x^7+430x^6-73x^5-499x^4+242x^3+143x^2-52x-19)$
227	$(x-1)^2(x^2-5)(x^2-2)(x^3+2x^2-x-1)(x^{10}-17x^8-3x^7+98x^6+40x^5-218x^4-148x^3+136x^2+144x+32)$
229	$(x+1)(x^6+4x^5-12x^3-3x^2+9x-1)(x^{11}-5x^{10}-4x^9+50x^8-26x^7-165x^6+152x^5+193x^4-207x^3-50x^2+52x+1)$
233	$(x-1)(x^7+2x^6-6x^5-10x^4+10x^3+8x^2-7x+1)(x^{11}+2x^{10}-16x^9-30x^8+91x^7+158x^6-213x^5-349x^4+152x^3+290x^2+41x-19)$
239	$(x^3+x^2-2x-1)(x^{17}-28x^{15}+x^{14}+319x^{13}-17x^{12}-1903x^{11}+91x^{10}+6377x^9-125x^8-11967x^7-233x^6+11733x^5+503x^4-5015x^3-94x^2+609x+49)$
241	$(x^7+4x^6-14x^4-10x^3+6x^2+3x-1)(x^{12}-3x^{11}-14x^{10}+44x^9+65x^8-219x^7-123x^6+444x^5+105x^4-328x^3-45x^2+18x-1)$

q	$F_{3,q}(x)$
11	$x+1$
17	x
19	$x+2$
23	x^2-5
29	x^2-2x-1
31	x^2+2x-4
37	$(x-1)(x+3)$
41	x^3-4x+2
43	$(x+2)(x^2-2)$

q	$F_{3,q}(x)$
47	x^4-7x^2+4x+1
53	$(x+3)(x^3-3x^2-x+1)$
59	$x^5+2x^4-8x^3-11x^2+13x-1$
61	$(x+2)(x^3-2x^2-4x+4)$
67	$(x+2)(x^2+3x+1)(x^2-x-1)$
71	$(x^3+x^2-8x-3)(x^3-x^2-4x+3)$
73	$x(x^2+3x+1)(x^2-x-3)$
79	$(x+1)(x^5-x^4-12x^3+8x^2+24x-16)$
83	$(x+1)(x^6-x^5-10x^4+5x^3+30x^2-4x-25)$
89	$(x+1)(x-2)(x^5+3x^4-4x^3-16x^2-9x-1)$
97	$(x^3+4x^2+3x-1)(x^4-5x^2-x+4)$
101	$(x+2)(x^7-4x^6-7x^5+38x^4+4x^3-96x^2+13x+68)$
103	$(x+1)^2(x^6-13x^4+40x^2-8x-16)$
107	$(x^2+3x+1)(x^7-3x^6-9x^5+29x^4+14x^3-69x^2+12x+29)$
109	$x(x^3+4x^2+3x-1)(x^4-4x^3-x^2+15x-8)$
113	$(x-2)(x^2-2x-2)(x^3+5x^2+6x+1)(x^3+x^2-4x-1)$
127	$(x^3+3x^2-3)(x^7-3x^6-12x^5+39x^4+26x^3-128x^2+64x+16)$
131	$(x+1)(x^{10}-x^9-22x^8+24x^7+157x^6-184x^5-403x^4+533x^3+222x^2$ $-390x+67)$
137	$(x^4+5x^3+4x^2-10x-11)(x^7-3x^6-8x^5+26x^4+11x^3-58x^2+16x$ $+14)$
139	$(x-2)(x^3+2x^2-x-1)(x^7+2x^6-15x^5-25x^4+56x^3+52x^2-56x-16)$
149	$(x^3+4x^2+3x-1)(x^9-6x^8+55x^6-67x^5-125x^4+235x^3-6x^2-117x$ $+27)$
151	$(x-2)^3(x^3+x^2-2x-1)(x^6+5x^5-4x^4-51x^3-68x^2-12x+8)$
157	$(x^5+7x^4+15x^3+7x^2-8x-5)(x^7-5x^6-x^5+31x^4-20x^3-45x^2+44x$ $-4)$
163	$x(x^5+5x^4+x^3-23x^2-28x-9)(x^7-x^6-11x^5+13x^4+26x^3-39x^2$ $+16x-2)$
167	$(x^2+x-1)(x^{12}-3x^{11}-22x^{10}+71x^9+145x^8-552x^7-243x^6+1577x^5$ $-122x^4-1737x^3+384x^2+599x-91)$
173	$(x^4+6x^3+10x^2+3x-1)(x^{10}-8x^9+11x^8+59x^7-165x^6-55x^5+484x^4$ $-202x^3-390x^2+169x+113)$
179	$x(x^3+2x^2-x-1)(x^{11}-25x^9+5x^8+219x^7-98x^6-781x^5+589x^4$ $+901x^3-1000x^2+185x-9)$
181	$(x^5+5x^4+5x^3-6x^2-9x-1)(x^9-3x^8-15x^7+46x^6+63x^5-213x^4$ $-32x^3+272x^2-144x+16)$
191	$(x+1)^2(x^{14}-2x^{13}-30x^{12}+58x^{11}+334x^{10}-630x^9-1667x^8+3160x^7$ $+3418x^6-7088x^5-1483x^4+5142x^3-940x^2-122x+5)$
193	$(x+1)^2(x^5+5x^4-x^3-27x^2-10x+23)(x^8-5x^7-2x^6+40x^5-37x^4$ $-48x^3+36x^2+31x+4)$

q	$F_{3,q}(x)$
197	$x(x^5+8x^4+18x^3-x^2-38x-25)(x^{10}-10x^9+29x^8+17x^7-227x^6$ $+316x^5+184x^4-784x^3+646x^2-175x+2)$
199	$(x-2)^2(x^2+x-1)^2(x^{10}+4x^9-19x^8-88x^7+73x^6+552x^5+200x^4$ $-784x^3-480x^2+96x+64)$
211	$(x^2-3x+1)(x^3+x^2-2x-1)(x^3+3x^2-x-4)(x^9+x^8-20x^7-17x^6$ $+128x^5+80x^4-292x^3-72x^2+224x-32)$
223	$(x^2+2x-1)(x^4-4x^2+x+1)(x^{12}-27x^{10}+7x^9+263x^8-131x^7-1091x^6$ $+816x^5+1600x^4-1752x^3+128x^2+288x-64)$
227	?
229	$(x-1)(x^6+6x^5+7x^4-17x^3-36x^2-6x+13)(x^{11}-3x^{10}-19x^9+60x^8$ $+109x^7-402x^6-133x^5+987x^4-332x^3-572x^2+288x-16)$
233	$(x+2)(x^7+8x^6+18x^5-3x^4-44x^3-20x^2+12x+1)(x^{11}-10x^{10}+28x^9$ $+29x^8-277x^7+394x^6+162x^5-716x^4+250x^3+312x^2-138x-29)$
239	$(x^3+x^2-2x-1)(x^{17}-3x^{16}-35x^{15}+110x^{14}+468x^{13}-1573x^{12}-2977x^{11}$ $+11197x^{10}+8880x^9-42041x^8-8213x^7+80809x^6-11957x^5$ $-70374x^4+23710x^3+20383x^2-9684x+592)$
241	$(x^7+3x^6-5x^5-19x^4-4x^3+14x^2+8x+1)(x^{12}-x^{11}-25x^{10}+25x^9$ $+224x^8-210x^7-888x^6+725x^5+1540x^4-960x^3-992x^2+400x$ $+64)$

References

[1] H. Wada: A Table of Hecke Operators. I. United States-Japan Seminar on Modern Methods in Number Theory, pp. 1–10 (1971). Tokyo University.

[2] ——: On a Method of Calculations of Hecke Operators (in Japanese). Sūrikaiseki Kenkyūjo Kōkyūroku, No. 155, pp. 3–13.

[3] H. G. Zimmer: Computational Problems, Methods, and Results in Algebraic Number Theory. Lecture Notes in Math. Vol. 262, Springer (1972).