182. *On Moduli of Open Holomorphic Maps of Compact Complex Manifolds*

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1. Let V and W be connected compact complex manifolds. According to Douady [1], the set $H(V, W)$ of all holomorphic maps of V into W admits an analytic space*) structure whose underlying topology is the compact-open topology. We denote by $O(V, W)$ the set of all open holomorphic maps of V onto W. Then $O(V, W)$ is an open subvariety of $H(V, W)$. Let $\mathrm{Aut}(V)$ and $\mathrm{Aut}(W)$ be the automorphism groups of V and W, respectively. It is well known that they are complex Lie groups. Now, $\mathrm{Aut}(W)$ and $\mathrm{Aut}(W)\times\mathrm{Aut}(V)$ act on $O(V, W)$ as follows:

$$(b, f) \in \mathrm{Aut}(W)\times O(V, W) \longrightarrow bf \in O(V, W),$$

$$(b, a, f) \in \mathrm{Aut}(W)\times\mathrm{Aut}(V)\times O(V, W) \longrightarrow bfa^{-1} \in O(V, W).$$

In this note, we state the following theorems. Details will be published elsewhere.

Theorem 1. *The orbit space $O(V, W)/\mathrm{Aut}(W)$ admits an analytic space structure such that the canonical projection map*

$$\pi: O(V, W) \longrightarrow O(V, W)/\mathrm{Aut}(W)$$

is holomorphic and is a principal fiber bundle with the structure group $\mathrm{Aut}(W)$.

Theorem 2. *Assume that* $\mathrm{Aut}(V)$ *is compact. Then the orbit space $O(V, W)/(\mathrm{Aut}(W)\times\mathrm{Aut}(V))$ with the quotient topology admits an analytic space structure such that* (1) *the canonical projection map*

$$\mu: O(V, W) \longrightarrow O(V, W)/(\mathrm{Aut}(W)\times\mathrm{Aut}(V))$$

is holomorphic and such that (2) *for any open subset U of $O(V, W)$ and for any holomorphic map F of U into an analytic space X which takes the same value at* $(\mathrm{Aut}(W)\times\mathrm{Aut}(V))$*-equivalent points, there is a holomorphic map $\hat{F}$ of $\mu(U)$ into X with $\hat{F}\mu=F$.*

Remark 1. The analytic space $O(V, W)/(\mathrm{Aut}(W)\times\mathrm{Aut}(V))$ in Theorem 2 is considered as *the moduli space of open holomorphic maps of V onto W.*

Remark 2. Theorems 1 and 2 are proved by applying Holmann's works [2] and [3].

2. $\mathrm{Aut}(V)$ acts on $O(V, W)/\mathrm{Aut}(W)$ as follows:

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*) By *an analytic space*, we mean a reduced, Hausdorff, complex analytic space.

$$(a, \pi(f)) \in \operatorname{Aut}(V) \times (O(V, W)/\operatorname{Aut}(W)) \longrightarrow \pi(fa^{-1}) \in O(V, W)/\operatorname{Aut}(W).$$

Assume that Aut(V) is compact. By Satz 20, [3], the orbit space $(O(V, W)/\operatorname{Aut}(W))/\operatorname{Aut}(V)$ is an analytic space. We have

Proposition. *Assume that* Aut(V) *is compact. Then there is a canonical holomorphic isomorphism:*

$$O(V, W)/(\operatorname{Aut}(W) \times \operatorname{Aut}(V)) \cong (O(V, W)/\operatorname{Aut}(W))/\operatorname{Aut}(V).$$

3. Let V be a compact Riemann surface of genus $g \geqq 1$. Let $\boldsymbol{P}^1$ be the complex projective line. Then $O(V, \boldsymbol{P}^1)$ is the set of all non-constant algebraic functions on V. The analytic space $O(V, \boldsymbol{P}^1)/(\operatorname{Aut}(\boldsymbol{P}^1) \times \operatorname{Aut}(V))$ is considered as *the moduli space of algebraic functions on* V.

In particular, for a complex 1-torus T, $O(T, \boldsymbol{P}^1)/(\operatorname{Aut}(\boldsymbol{P}^1) \times \operatorname{Aut}(T))$ is decomposed into the connected components as follows:

$$O(T, \boldsymbol{P}^1)/(\operatorname{Aut}(\boldsymbol{P}^1) \times \operatorname{Aut}(T)) = M_2 \cup M_3 \cup \cdots,$$

where each M_n, $n=2, 3, \cdots$, is *the moduli space of elliptic functions of order* n *on* T and is an irreducible normal analytic space of dimension $2n-4$. (M_2 is one point.)

On the other hand, we can easily show that the orbit space $O(\boldsymbol{P}^1, \boldsymbol{P}^1)/(\operatorname{Aut}(\boldsymbol{P}^1) \times \operatorname{Aut}(\boldsymbol{P}^1))$ is not Hausdorff.

References

[1] A. Douady: Le problème des modules pour les sous-espaces analytiques compacts d'un espace analytique donné. Ann. Inst. Fourier, Grenoble, **16**, 1–95 (1966).

[2] H. Holmann: Quotienten komplexer Räume. Math. Ann., **142**, 407–440 (1961).

[3] ——: Komplexe Räume mit komplexen Transformationsgruppen. Math. Ann., **150**, 327–360 (1963).