PAPERS COMMUNICATED

65. *Theory of Connections in the Generalized Finsler Manifold.*

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Recently the theory of connections in the Finsler manifold F_n has been developed somewhat by several authors,[1] in which the parameters of the connections are in general functions not only of positions x^ν but also of directions dx^ν. We shall consider some connections in the generalized Finsler manifold K_n similarly as in the Finsler manifold, the parameters of the connections depending on $x^\nu, dx^\nu, d^2x^\nu, \ldots\ldots, d^rx^\nu$.

1. *The base-connections.* Let X_n be a manifold of n dimensions, in which a system of coordinates x^ν is taken, and we express for brevity n differentials dx^ν of the coordinates by $\overset{(1)}{p}{}^\nu : dx^\nu = \overset{(1)}{p}{}^\nu$. Introduce an arbitrary linear connection $\overset{(1)}{U}$, whose parameters $\overset{(1)}{\Gamma}{}^\nu_{\lambda\mu}$ depend upon positions and directions, i.e. x^ν and dx^ν, then we get covariant differentials of a *line element* $\overset{(1)}{p}{}^\nu$, which can be considered as a contravariant vector and expressed by the parameters $\overset{(1)}{\Gamma}{}^\nu_{\lambda\mu}$ of the connection $\overset{(1)}{U}$ as follows:

$$(1.\ 1) \qquad \overset{(1)}{\delta}\overset{(1)}{p}{}^\nu = d\overset{(1)}{p}{}^\nu + \overset{(1)}{\Gamma}{}^\nu_{\lambda\mu}\overset{(1)}{p}{}^\lambda dx^\nu .$$

We shall write $\overset{(2)}{p}{}^\nu$ for $\overset{(1)}{\delta}\overset{(1)}{p}{}^\nu$, which is function of x^ν as well as $\overset{(1)}{p}{}^\nu$. Let us call this contravariant vector $\overset{(2)}{p}{}^\nu$ *the line element of the second kind.*

We introduce moreover a new linear connection $\overset{(2)}{U}$[2] for X_n, whose parameters $\overset{(2)}{\Gamma}{}^\nu_{\lambda\mu}$ are functions of x^ν as well as $\overset{(1)}{p}{}^\nu$ and $\overset{(2)}{p}{}^\nu$. This connection $\overset{(2)}{U}$ is applied to the affinors, which depend on x^ν and the foregoing line elements $\overset{(1)}{p}{}^\nu$ and $\overset{(2)}{p}{}^\nu$, and by which we get *the line element of the third kind*

$$(1.\ 2) \qquad \overset{(3)}{p}{}^\nu = \overset{(2)}{\delta}\overset{(2)}{p}{}^\nu = d\overset{(2)}{p}{}^\nu + \overset{(2)}{\Gamma}{}^\nu_{\lambda\mu}\overset{(2)}{p}{}^\lambda dx^\mu ,$$

1) E. Noether: Göttinger Nachrichten, math.-phys. Kl. (1918), S. 37–44. L. Berwald: Jahresberichte d. Deutschen Math.-Ver. **34** (1925), S. 213–220. J. L. Synge: Trans. of Amer. Math. Soc. **27** (1925), p. 61–67. J. H. Taylor: Trans. of Amer. Math. Soc. **27** (1925), p. 246–264.

2) $\overset{(2)}{U}$ is in general different from $\overset{(1)}{U}$, but we may also take same one with $\overset{(1)}{U}$ as $\overset{(2)}{U}$. It is completely analogous for following connections $\overset{(k)}{U}$.

$\overset{(3)}{p}{}^{\nu}$'s are functions of x^{ν}, $\overset{(1)}{p}{}^{\nu}$ and $\overset{(2)}{p}{}^{\nu}$, as we can see easily. Introducing another linear connections $\overset{(3)}{U}, \overset{(4)}{U}, \ldots\ldots, \overset{(r)}{U}$ and repeating this method we can get in general *a line element of the* $(k+1)$*-th kind*

$$\overset{(k+1)}{p}{}^{\nu}=\overset{(k)}{\delta}\overset{(k)}{p}{}^{\nu}=d\overset{(k)}{p}{}^{\nu}+\overset{(k)}{\Gamma}{}^{\nu}_{\lambda\mu}\overset{(k)}{p}{}^{\lambda}dx^{\mu}\,, \tag{1. 3}$$

where $\overset{(k)}{\Gamma}{}^{\nu}_{\lambda\mu}$'s are parameters of the connection $\overset{(k)}{U}$ and functions of x^{ν}, $\overset{(1)}{p}{}^{\nu}, \overset{(2)}{p}{}^{\nu}, \ldots\ldots, \overset{(k)}{p}{}^{\nu}$. These connections $\overset{(1)}{U}, \overset{(2)}{U}, \ldots\ldots, \overset{(r)}{U}$ are called *the base connections*. The manifold X_n, to whose every point are associated every system of the line elements $\overset{(1)}{p}{}^{\nu}, \overset{(2)}{p}{}^{\nu}, \ldots\ldots, \overset{(r)}{p}{}^{\nu}$, is defined as *the generalized Finsler manifold.*

2. *The connection U.* Consider a contravariant or covariant vector, which depends on $x^{\nu}, dx^{\nu}, \ldots\ldots, d^{r}x^{\nu}$, then components of the vector will be considered as functions of $x^{\nu}, \overset{(1)}{p}{}^{\nu}, \ldots\ldots, \overset{(r)}{p}{}^{\nu}$, for we can substitute $\overset{(1)}{p}{}^{\nu}, \ldots\ldots, \overset{(r)}{p}{}^{\nu}$ for $dx^{\nu}, \ldots\ldots, d^{r}x^{\nu}$ by (1. 1), (1. 2) and (1. 3). Now we define a connection U for such contravariant or covariant vectors v^{ν} or w_{λ}:

$$\begin{aligned}\delta v^{\nu}&=\nabla_{\mu}v^{\nu}dx^{\mu}=dv^{\nu}+\Gamma^{\nu}_{\lambda\mu}v^{\lambda}dx^{\mu}\,,\\ \delta w_{\lambda}&=\nabla_{\mu}w_{\lambda}dx^{\mu}=dw_{\lambda}-\Gamma'^{\nu}_{\lambda\mu}w_{\nu}dx^{\mu}\,,\end{aligned} \tag{2. 1}$$

where $\Gamma^{\nu}_{\lambda\mu}$'s as well as $\Gamma'^{\nu}_{\lambda\mu}$'s depend upon $x^{\nu}, \overset{(1)}{p}{}^{\nu}, \overset{(2)}{p}{}^{\nu}, \ldots\ldots, \overset{(r)}{p}{}^{\nu}$. Accordingly the covariant differential of any affinor can be defined from (2. 1) and we can calculate the covariant derivatives

$$\begin{aligned}\nabla_{\mu}v^{\nu}&=\frac{\partial v^{\nu}}{\partial x^{\mu}}+\Gamma^{\nu}_{\lambda\mu}v^{\lambda}+\sum_{i}^{1,r}\frac{\partial v^{\nu}}{\partial \overset{(i)}{p}{}^{\omega}}(\overset{(i)}{\nabla}_{\mu}\overset{(i)}{p}{}^{\omega}-\overset{(i)}{\Gamma}{}^{\omega}_{\lambda\mu}\overset{(i)}{p}{}^{\lambda})\,,\\ \nabla_{\mu}w_{\lambda}&=\frac{\partial w_{\lambda}}{\partial x^{\mu}}-\Gamma'^{\nu}_{\lambda\mu}w_{\nu}+\sum_{i}^{1,r}\frac{\partial w_{\lambda}}{\partial \overset{(i)}{p}{}^{\omega}}(\overset{(i)}{\nabla}_{\mu}\overset{(i)}{p}{}^{\omega}-\overset{(i)}{\Gamma}{}^{\omega}_{\nu\mu}\overset{(i)}{p}{}^{\nu})\,,\end{aligned} \tag{2. 2}$$

where $\overset{(i)}{\nabla}_{\mu}\overset{(i)}{p}{}^{\omega}$ means covariant derivative of $\overset{(i)}{p}{}^{\omega}$ referred to $\overset{(i)}{U}$.

3. *Construction of the parameters* $\Gamma^{\nu}_{\lambda\mu}$ *and* $\Gamma'^{\nu}_{\lambda\mu}$. The parameters $\Gamma^{\nu}_{\lambda\mu}$ and $\Gamma'^{\nu}_{\lambda\mu}$ are transformed in the same manner as that of the affine connection by any change of system of coordinates. Therefore

$$C_{\lambda\mu}^{\cdot\cdot\,\nu}=\Gamma^{\nu}_{\lambda\mu}-\Gamma'^{\nu}_{\lambda\mu} \tag{3. 1}$$

$$S_{\lambda\mu}^{\cdot\cdot\,\nu}=\tfrac{1}{2}(\Gamma^{\nu}_{\lambda\mu}-\Gamma^{\nu}_{\mu\lambda})\,,\qquad S'^{\cdot\cdot\,\nu}_{\lambda\mu}=\tfrac{1}{2}(\Gamma'^{\nu}_{\lambda\mu}-\Gamma'^{\nu}_{\mu\lambda}) \tag{3. 2}$$

are affinors. Let us consider a contravariant tensor $g^{\lambda\mu}$ and a corresponding covariant tensor $g_{\lambda\mu}$, which depend upon $x^{\nu}, \overset{(1)}{p}{}^{\nu}, \ldots\ldots, \overset{(r)}{p}{}^{\nu}$ also, and put their covariant derivatives

$$\nabla_{\nu}g^{\lambda\mu}=Q^{\lambda\mu}_{\cdot\cdot\,\nu}\,,\qquad \nabla_{\nu}g_{\lambda\mu}=Q'_{\lambda\mu\nu}\,. \tag{3. 3}$$

Then it follows from (2. 2)

$$(3.\ 4)\qquad Q^{\lambda\mu}_{\cdot\cdot\nu}=\frac{\partial g^{\lambda\mu}}{\partial x^{\nu}}+\Gamma'^{\lambda}_{\omega\nu}g^{\omega\mu}+\Gamma^{\mu}_{\omega\nu}g^{\lambda\omega}+\sum_{i}^{1,r}\frac{\partial g^{\lambda\mu}}{\partial \overset{(i)}{p^{\omega}}}(\nabla_{\nu}\overset{(i)}{p^{\omega}}-\overset{(i)}{\Gamma'^{\omega}_{\tau\nu}}\overset{(i)}{p^{\tau}}),$$

$$(3.\ 5)\qquad \Gamma'^{\omega}_{\lambda\nu}g_{\omega\mu}+\Gamma'^{\omega}_{\mu\nu}g_{\omega\lambda}=\frac{\partial g_{\lambda\mu}}{\partial x^{\nu}}+Q_{\lambda\mu\nu}+\sum_{i}^{1,r}\frac{\partial g_{\lambda\mu}}{\partial \overset{(i)}{p^{\omega}}}(\nabla_{\nu}\overset{(i)}{p^{\omega}}-\overset{(i)}{\Gamma'^{\omega}_{\tau\nu}}\overset{(i)}{p^{\tau}}),$$

where $Q_{\lambda\mu\nu}=g_{\lambda\omega}g_{\mu\tau}Q^{\omega\tau}_{\cdot\cdot\nu}$. We get in consequence of (3. 5)

$$(3.\ 6)\qquad \Gamma^{\nu}_{\lambda\mu}=\{{}^{\lambda\mu}_{\nu}\}+T^{\cdot\cdot\nu}_{\lambda\mu}+W^{\cdot\cdot\nu}_{\lambda\mu},\qquad \Gamma'^{\nu}_{\lambda\mu}=\{{}^{\lambda\mu}_{\nu}\}+T'^{\cdot\cdot\nu}_{\lambda\mu}+W^{\cdot\cdot\nu}_{\lambda\mu},$$

where we put

$$(3.\ 7)\qquad \{{}^{\lambda\mu}_{\nu}\}=\tfrac{1}{2}g^{\nu\omega}\Big(\frac{\partial g_{\lambda\omega}}{\partial x^{\mu}}+\frac{\partial g_{\omega\mu}}{\partial x^{\lambda}}-\frac{\partial g_{\lambda\mu}}{\partial x^{\omega}}\Big),$$

$$(3.\ 8)\qquad T^{\cdot\cdot\nu}_{\lambda\mu}=\tfrac{1}{2}(g_{\lambda\omega}Q^{\cdot\omega\nu}_{\mu}+g_{\mu\omega}Q^{\cdot\omega\nu}_{\lambda}-g^{\nu\sigma}g_{\mu\omega}g_{\lambda\tau}Q^{\cdot\omega\tau}_{\sigma}),$$
$$+S^{\cdot\cdot\nu}_{\lambda\mu}-g^{\nu\omega}(g_{\lambda\tau}S^{\cdot\cdot\tau}_{\omega\mu}+g_{\mu\tau}S^{\cdot\cdot\tau}_{\omega\lambda}),$$

$$(3.\ 9)\qquad T'^{\cdot\cdot\nu}_{\lambda\mu}=T^{\cdot\cdot\nu}_{\lambda\mu}-C^{\cdot\cdot\nu}_{\mu\lambda},$$

$$(3.\ 10)\qquad W^{\cdot\cdot\nu}_{\lambda\mu}=\tfrac{1}{2}g^{\nu\omega}\sum_{i}^{1,r}\Big(\frac{\partial g_{\lambda\omega}}{\partial \overset{(i)}{p^{\tau}}}\nabla_{\mu}\overset{(i)}{p^{\tau}}+\frac{\partial g_{\omega\mu}}{\partial \overset{(i)}{p^{\tau}}}\nabla_{\lambda}\overset{(i)}{p^{\tau}}-\frac{\partial g_{\lambda\mu}}{\partial \overset{(i)}{p^{\tau}}}\nabla_{\omega}\overset{(i)}{p^{\tau}}\Big)$$
$$-\tfrac{1}{2}g^{\nu\omega}\sum_{i}^{1,r}\Big(\frac{\partial g_{\lambda\omega}}{\partial \overset{(i)}{p^{\tau}}}\overset{(i)}{\Gamma'^{\tau}_{\sigma\mu}}+\frac{\partial g_{\omega\mu}}{\partial \overset{(i)}{p^{\tau}}}\overset{(i)}{\Gamma'^{\tau}_{\sigma\lambda}}-\frac{\partial g_{\lambda\mu}}{\partial \overset{(i)}{p^{\tau}}}\overset{(i)}{\Gamma'^{\tau}_{\sigma\omega}}\Big)\overset{(i)}{p^{\sigma}}.$$

4. *The curvature tensor.* We shall now find the curvature tensor. From (2. 2) it follows

$$\nabla_{\mu}\nabla_{\nu}v^{\lambda}=\frac{\partial}{\partial x^{\mu}}\nabla_{\nu}v^{\lambda}+\Gamma^{\lambda}_{\omega\mu}\nabla_{\nu}v^{\omega}-\Gamma'^{\omega}_{\nu\mu}\nabla_{\omega}v^{\lambda}+\sum_{i}\frac{\partial\nabla_{\nu}v^{\lambda}}{\partial \overset{(i)}{p^{\omega}}}(\nabla_{\nu}\overset{(i)}{p^{\omega}}-\overset{(i)}{\Gamma'^{\omega}_{\tau\mu}}\overset{(i)}{p^{\tau}})$$
$$=\frac{\partial^{2}v^{\lambda}}{\partial x^{\mu}\partial x^{\nu}}+\frac{\partial\Gamma^{\lambda}_{\rho\nu}}{\partial x^{\mu}}v^{\rho}+\Gamma^{\lambda}_{\rho\nu}\frac{\partial v^{\rho}}{\partial x^{\mu}}+\sum_{i}\frac{\partial^{2}v^{\lambda}}{\partial x^{\mu}\partial \overset{(i)}{p^{\omega}}}(\nabla_{\nu}\overset{(i)}{p^{\omega}}-\overset{(i)}{\Gamma'^{\omega}_{\rho\nu}}\overset{(i)}{p^{\rho}})$$
$$+\sum_{i}\frac{\partial v^{\lambda}}{\partial \overset{(i)}{p^{\omega}}}\Big(\frac{\partial\nabla_{\nu}\overset{(i)}{p^{\omega}}}{\partial x^{\mu}}-\frac{\partial\overset{(i)}{\Gamma'^{\omega}_{\rho\nu}}}{\partial x^{\mu}}\overset{(i)}{p^{\rho}}\Big)+\Gamma^{\lambda}_{\omega\mu}\Big\{\frac{\partial v^{\omega}}{\partial x^{\nu}}+\Gamma^{\omega}_{\rho\nu}v^{\rho}$$
$$+\sum_{i}\frac{\partial v^{\omega}}{\partial \overset{(i)}{p^{\sigma}}}(\nabla_{\nu}\overset{(i)}{p^{\sigma}}-\overset{(i)}{\Gamma'^{\sigma}_{\rho\nu}}\overset{(i)}{p^{\rho}})\Big\}-\Gamma'^{\omega}_{\lambda\mu}\nabla_{\omega}v^{\lambda}$$
$$+\sum_{j}\Big\{\frac{\partial^{2}v^{\lambda}}{\partial \overset{(j)}{p^{\omega}}\partial x^{\nu}}+\frac{\partial\Gamma^{\lambda}_{\rho\nu}}{\partial \overset{(j)}{p^{\omega}}}v^{\rho}+\Gamma^{\lambda}_{\rho\nu}\frac{\partial v^{\rho}}{\partial \overset{(j)}{p^{\omega}}}+\sum_{i}\frac{\partial^{2}v^{\lambda}}{\partial \overset{(j)}{p^{\omega}}\partial \overset{(i)}{p^{\tau}}}(\nabla_{\nu}\overset{(i)}{p^{\tau}}-\overset{(i)}{\Gamma'^{\tau}_{\rho\lambda}}\overset{(i)}{p^{\rho}})$$
$$+\sum_{i}\frac{\partial v^{\lambda}}{\partial \overset{(i)}{p^{\tau}}}\Big(\frac{\partial\nabla_{\nu}\overset{(i)}{p^{\tau}}}{\partial \overset{(j)}{p^{\omega}}}-\frac{\partial\overset{(i)}{\Gamma'^{\tau}_{\rho\nu}}}{\partial \overset{(j)}{p^{\omega}}}\overset{(i)}{p^{\rho}}\Big)-\frac{\partial v^{\lambda}}{\partial \overset{(j)}{p^{\tau}}}\overset{(i)}{\Gamma'^{\tau}_{\omega\nu}}\Big\}(\nabla_{\nu}\overset{(j)}{p^{\tau}}-\overset{(j)}{\Gamma'^{\tau}_{\rho\nu}}\overset{(j)}{p^{\rho}}),$$

hence

$$(4.\ 1)\qquad 2\nabla_{[\mu}\nabla_{\nu]}v^{\lambda}=R_{\lambda\mu\rho}^{\cdot\cdot\cdot\lambda}v^{\rho}-\sum_{i}\overset{(i)}{R}{}_{\lambda\mu\rho}^{\cdot\cdot\cdot\omega}\frac{\partial v^{\lambda}}{\partial \overset{(i)}{p}{}^{\omega}}\overset{(i)}{p}{}^{\rho}+\sum_{i}\frac{\partial v^{\lambda}}{\partial \overset{(i)}{p}{}^{\omega}}\overset{(i)}{V}{}_{\lambda\mu}^{\cdot\cdot\omega}+2S_{\mu\nu}^{\cdot\cdot\omega}\nabla_{\omega}v^{\lambda},$$

where

$$(4.\ 2)\qquad R_{\lambda\mu\rho}^{\cdot\cdot\cdot\lambda}=\frac{\partial\Gamma^{\lambda}_{\rho\nu}}{\partial x^{\mu}}-\frac{\partial\Gamma^{\lambda}_{\rho\mu}}{\partial x^{\nu}}+\Gamma^{\lambda}_{\omega\mu}\Gamma^{\omega}_{\rho\nu}-\Gamma^{\lambda}_{\omega\nu}\Gamma^{\omega}_{\rho\mu}+\sum_{j}\frac{\partial\Gamma^{\lambda}_{\rho\nu}}{\partial\overset{(j)}{p}{}^{\omega}}(\overset{(j)}{\nabla}_{\mu}\overset{(j)}{p}{}^{\omega}-\overset{(j)}{\Gamma}{}^{\omega}_{\tau\mu}\overset{(j)}{p}{}^{\tau})-\sum_{j}\frac{\partial\Gamma^{\lambda}_{\rho\mu}}{\partial\overset{(j)}{p}{}^{\omega}}(\overset{(j)}{\nabla}_{\nu}\overset{(j)}{p}{}^{\omega}-\overset{(j)}{\Gamma}{}^{\omega}_{\tau\nu}\overset{(j)}{p}{}^{\tau}),$$

$$(4.\ 3)\qquad \overset{(i)}{R}{}_{\nu\mu\rho}^{\cdot\cdot\cdot\lambda}=\frac{\partial\overset{(i)}{\Gamma}{}^{\lambda}_{\rho\nu}}{\partial x^{\mu}}-\frac{\partial\overset{(i)}{\Gamma}{}^{\lambda}_{\rho\mu}}{\partial x^{\nu}}+\overset{(i)}{\Gamma}{}^{\lambda}_{\omega\mu}\overset{(i)}{\Gamma}{}^{\omega}_{\rho\nu}-\overset{(i)}{\Gamma}{}^{\lambda}_{\omega\nu}\overset{(i)}{\Gamma}{}^{\omega}_{\rho\mu}-\sum_{j}\frac{\partial\overset{(i)}{\Gamma}{}^{\lambda}_{\rho\nu}}{\partial\overset{(j)}{p}{}^{\omega}}(\overset{(j)}{\nabla}_{\mu}\overset{(j)}{p}{}^{\omega}-\overset{(j)}{\Gamma}{}^{\omega}_{\tau\mu}\overset{(j)}{p}{}^{\tau})-\sum_{j}\frac{\partial\overset{(i)}{\Gamma}{}^{\lambda}_{\rho\mu}}{\partial\overset{(i)}{p}{}^{\omega}}(\overset{(j)}{\nabla}_{\nu}\overset{(j)}{p}{}^{\omega}-\overset{(j)}{\Gamma}{}^{\omega}_{\tau\mu}\overset{(j)}{p}{}^{\tau})$$

$$(4.\ 4)\qquad \overset{(i)}{\nabla}{}_{\lambda\mu}^{\cdot\cdot\omega}=2\overset{(i)}{\nabla}_{[\mu}\overset{(i)}{\nabla}_{\nu]}\overset{(i)}{p}{}^{\omega}+2\overset{(i)}{S}{}_{\nu\mu}^{\cdot\cdot\rho}\overset{(i)}{\nabla}_{\rho}\overset{(i)}{p}{}^{\omega},^{1)}\qquad \overset{(i)}{S}{}_{\nu\mu}^{\cdot\cdot\rho}=\overset{(i)}{\Gamma}{}^{\rho}_{\nu\mu}-\overset{(i)}{\Gamma}{}^{\rho}_{\mu\nu}.$$

We shall call $R_{\mu\nu\rho}^{\cdot\cdot\cdot\lambda}$ *the curvature tensor* of our connection U and $\overset{(i)}{R}{}_{\mu\nu\rho}^{\cdot\cdot\cdot\lambda}$ can be considered as the curvature tensors belonging to $\overset{(1)}{U}$, $\overset{(2)}{U}$, ..., $\overset{(r)}{U}$.

5. *Special cases.* When we put $r=1$, we get a connection in the general Finsler manifold. Let $\overset{(1)}{p}{}^{\nu}$ of x^{ν} be connected with that of $x^{\nu}+dx^{\nu}$, so that $\overset{(1)(1)}{\delta p}{}^{\nu}=0$, i.e. $dp^{\nu}=-\overset{(1)}{\Gamma}{}^{\nu}_{\lambda\mu}\overset{(1)}{p}{}^{\lambda}dx^{\mu}$, then we get the case, which T. Hosokawa[2] has studied. The connection of Berwald in the Finsler manifold[3] is a more special one of this case, i.e. $\Gamma^{\nu}_{\lambda\mu}=\overset{(1)}{\Gamma}{}^{\nu}_{\lambda\mu}$.

As the correspondence between line elements belonging to two consecutive points can be defined arbitrarily, we may put $\overset{(i)}{\nabla}_{\mu}\overset{(i)}{p}{}^{\nu}=0$, then $\overset{(i)}{V}{}_{\lambda\mu}^{\cdot\cdot\nu}=0$ and we get a direct generalisation of the connection of Berwald. Our connection contains that of Craig[4] as a special case too, in which case r is 2.

1) This expression consists only formally, because $\overset{(i)}{p}{}^{\nu}$ is not a vector field but a line element and $\overset{(i)}{\nabla}_{\mu}\overset{(i)}{p}{}^{\omega}$ define a correspondence between two line elements belonging to a point x^{ν} and its consecutive point $x^{\nu}+dx^{\nu}$ respectively. Accordingly this expression does not give us the curvature tensor.

2) T. Hosokawa: Science Reports, Tohoku Imp. University, series I, **19** (1930), p. 37–51.

3) L. Berwald: Math. Zeitschrift, **25** (1926), S. 40–73.

4) H. V. Craig: Trans. of Amer. Math. Soc., **33** (1931), p. 125–142.