Altering distances and fixed point results for tangential mappings

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Abstract

The purpose of this paper is to prove a general fixed point theorem by altering distances for two owc pairs of mappings and to reduce the study of fixed points for pairs of mappings satisfying a contractive condition of integral type to the study of fixed points in a metric space by altering distances satisfying a new type of implicit relation generalizing a main result obtained by Pathak and Shahzad in a recent paper [17].

1 Introduction

Let (X, d) be a metric space and S, T two self-mappings of X. In [7], Jungck defined S and T to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$.

In 1994, Pant [15] introduced the notion of pairwise R-weakly commuting mappings. It is proved in [16] that the notion of pairwise R-weakly commuting is equivalent to commutativity at coincidence points.

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Jungck [8] defined S and T to be weakly compatible if $Sx = Tx$ implies $STx = TSx$. Thus, S and T are weakly compatible if and only if S and T are pairwise R-weakly commuting mappings.

Quite recently, Al-Thagafi and Naseer Shahzad [4] introduced the concept of occasionally weakly compatible mappings.

Definition 1.1. *Let X be a nonempty set and f, g self-mappings on X.*

A point $x \in X$ is called a coincidence point of f and g if $fx = gx$.

A point $w \in X$ is called a point of coincidence of f and g if there exists a coincidence point $x \in X$ of f and g such that $w = fx = gx$.

Definition 1.2. *Two self-maps f and g of a nonempty set X are are called occasionally weakly compatible maps (shortly owc) [4] if there exists a point x in X which is a coincidence point for f and g at which f and g commute.*

Remark 1.1 Two weakly compatible mappings having coincidence points are occasionally weakly compatible. The converse is not true, as the example of [4].

Weakly compatible does not imply occasionally weak compatibility as every $f : X \to X$ and *id*, the identity map of X, are weakly compatible, while f and *id* are occasionally weakly compatible if and only if f has a fixed point in X.

Lemma 1.1. *(Jungck and Rhoades [9]). Let X be a nonempty set and let f and g two occasionally weakly compatible self-mappings of X. If f and g have a unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.*

In 2000, Sastri and Krishna Murthy [24] introduced the following notion:

A point $z \in X$ is said to be *tangent* to the pair $\{A, B\}$ of self-mappings of a metric space (X, d) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$.

The pair pair $\{A, B\}$ of self-mappings of (X, d) is called *tangential* if there exists $z \in X$ which is tangent to the pair $\{A, B\}$.

Two years later, Aamri and Moutawakil [1] rediscovered this notion and called it as property (E.A).

Definition 1.3. *([1]) Let A and B be two self mappings of a metric space (X, d). We say that A and B satisfy property (E.A) if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$$

for some $z \in X$.

Recently, Liu et al. [13] defined a common property (E.A) as follows:

Definition 1.4. *([13]) Let A, B, S and T be four self mappings of a metric space (X, d). We say that the pair $\{A, S\}$ and $\{B, T\}$ satisfy a common property (E.A) if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$$

for some $z \in X$.

Quite recently, Pathak and Shahzad [17] introduced the concept of a weak tangent point for a pair of mappings and pairwise tangential property for two pairs of mappings.

Definition 1.5. *([13]) Let A, B, S and T be four self mappings of a metric space (X, d).*

A point $z \in X$ is said to be weak tangent to the pair $\{S, T\}$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = z.$$

We say also that z is a weak tangent point to the pair $\{S, T\}$.

We say that the pair $\{A, B\}$ is tangential w.r.t. the pair $\{S, T\}$ if

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = z,$$

whenever there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = z$$

for some $z \in X$.

Remark 1.2

1) Every pair of mappings (S, T) which satisfies property $(E.A)$ (or has a tangent point) also has a weak tangent point to (S, T), but the converse is not true. Hence, the notion of weak tangent point to the pair (S, T) is weaker than the notion of property $(E.A)$ of the pair (S, T) (and the notion of tangent point to (S, T)).

2) If $A = B$ and $S = T$, we say that the mapping A is tangential w.r.t. the mapping S.

3) If $S = A$ and $T = B$, we say that (A, B) is tangential with itself.

4) Obviously, every pair of mappings (S, T) satisfies property $(E.A)$ also has a point $z \in X$ which is tangent to (S, T). (To see this, just take $x_n = y_n$, but the converse need not be true. (See, Example 2.2 in [17]).

5) It may be noticed that if the pair (A, B) is tangential w.r.t. the pair (S, T), then the pair (S, T) need not be tangential w.r.t. the pair (A, B). (See, Example 2.3 in [17]).

2 Preliminaries

In [5], Branciari established the following result.

Theorem 2.1. *([5]) Let (X, d) be a complete metric space, $c \in (0, 1)$ and $f : X \to X$ be a mapping such that for all $x, y \in X$,*

$$\int_0^{d(fx,fy)} h(t)dt \le c \int_0^{d(x,y)} h(t)dt, \tag{2.1}$$

where $h : [0, \infty) \to [0, \infty)$ is a Lebesgue measurable mapping which is summable (i.e., with finite integral) on each compact subset of $[0, \infty)$, such that, for every $\epsilon > 0$ we have $\int_0^{\epsilon} h(t)dt > 0$. Then f has a unique fixed point $z \in X$, such that for each $x \in X$, $\lim_{n\to\infty} f^n(x) = z$.

Some fixed point theorems in metric and symmetric spaces for compatible and weakly compatible mappings satisfying a contractive condition of integral type are proved in [2], [11], [12], [14], [20] and other papers.

In [17], Gregus type fixed point results for tangential mappings satisfying contractive conditions of integral type are obtained. A main result by [17] is the following.

Theorem 2.2. *Let A, B, S and T be mappings from a metric space (X, d) into itself satisfying*

$$\begin{aligned}\Big[1+\alpha\int_0^{d(Sx,Ty)}\phi(t)dt\Big]\int_0^{d(Ax,By)}\phi(t)dt \;<\; &\\ \alpha\Big[\int_0^{d(Ax,Sx)}\phi(t)dt\cdot\int_0^{d(By,Ty)}\phi(t)\,dt+\int_0^{d(Ax,Ty)}\phi(t)dt\cdot\int_0^{d(Sx,By)}\phi(t)dt\Big]&\\ +a\int_0^{d(Sx,Ty)}\phi(t)\,dt+(1-a)\max\Big\{\int_0^{d(Ax,Sx)}\phi(t)dt,\int_0^{d(By,Ty)}\phi(t)dt,&\\ \Big(\int_0^{d(Ax,Sx)}\phi(t)dt\Big)^{\frac{1}{2}}\cdot\Big(\int_0^{d(Ax,Ty)}\phi(t)dt\Big)^{\frac{1}{2}},&\\ \Big(\int_0^{d(Sx,By)}\phi(t)dt\Big)^{\frac{1}{2}}\cdot\Big(\int_0^{d(Ax,Ty)}\phi(t)dt\Big)^{\frac{1}{2}}\Big\},& \quad (2.2)\end{aligned}$$

for all $x, y \in X$ for which the right-hand side of (2.2) is positive, where $0 < a < 1$, $\alpha > 0$ and ϕ is as in Theorem 2.1.

If there exists a weak tangent point $z \in S(X) \cap T(X)$ to (S, T) and (A, B) is tangential w.r.t. (S, T), and the pairs (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Definition 2.1. *An altering distance is a mapping $\psi : [0, \infty) \to [0, \infty)$ which satisfies:*
(ψ_1) : *ψ is increasing and continuous.*
(ψ_2) : *$\psi(t) = 0$ if and only if $t = 0$.*

Fixed point theorems involving altering distances have been studied in [10], [19], [22], [23] and other papers.

Lemma 2.1. *The function $\psi(t) := \int_0^t \phi(x)\,dx$, where ϕ is as in Theorem 2.1, is an altering distance.*

Proof. By definitions of ψ and ϕ it follows that ψ is increasing and $\psi(t) = 0$ if and only if $t = 0$. From the Lemma 2.5 of [14], ψ is continuous. ∎

In [18] a general fixed point theorem for compatible mappings satisfying implicit relations is proved. In [6] the results from [18] are improved relaxing compatibility to weak compatibility.

The purpose of this paper is to prove a general fixed point theorem by altering distances for two owc pairs of mappings and to reduce the study of fixed points for pairs of mappings satisfying a contractive condition of integral type at the study of fixed points in a metric space by altering distances satisfying a new type of implicit relations generalizing the result of Theorem 2.2.

After the two sections of Introduction and Preliminaries, this paper contains three other sections. In section three, we introduce a new class of implicit relations by which we define contractive conditions and give some examples. In the fourth and fifth sections we present our main results and explain relations with fixed point for mappings satisfying contractive conditions of integral type.

3 Implicit relations

Let $\mathcal{F}_t$ be the set of all real continuous functions $F(t_1, \dots, t_6) : \mathbb{R}^6 \to \mathbb{R}$ satisfying the following conditions:
(F_1): $F(t, 0, 0, t, t, 0) \leq 0$ implies $t = 0$.
(F_2): $F(t, 0, t, 0, 0, t) \leq 0$ implies $t = 0$.
(F_3): $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.1.
$F(t_1, \dots, t_6) = (1 + \alpha t_2)t_1 - \alpha(t_3t_4 + t_5t_6) - at_2 - (1 - a)\max\{t_3, t_4, \sqrt{t_3t_6}, \sqrt{t_5t_6}\}$, where $\alpha \geq 0$ and $0 < a < 1$.
(F_1): $F(t, 0, 0, t, t, 0) = at \leq 0$ implies $t = 0$.
(F_2): $F(t, 0, t, 0, 0, t) = at \leq 0$ implies $t = 0$.
(F_3): $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.2. $F(t_1, \dots, t_6) = t_1^p - at_2^p - (1 - a)\max\{t_2^p, t_3^p, t_4^p, (t_3t_6)^{\frac{p}{2}}, (t_5t_6)^{\frac{p}{2}}\}$, where $p > 0$ and $0 < a < 1$.
(F_1): $F(t, 0, 0, t, t, 0) = at^p \leq 0$ implies $t = 0$.
(F_2): $F(t, 0, t, 0, 0, t) = at^p \leq 0$ implies $t = 0$.
(F_3): $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.3. $F(t_1, \dots, t_6) = t_1 - \alpha\max\{t_2, t_3, t_4\} - (1 - \alpha)(at_5 + bt_6)$, where $a, b > 0, a + b = 1$ and $0 < \alpha < 1$.
(F_1): $F(t, 0, 0, t, t, 0) = t(1 - \alpha)(1 - a) \leq 0$ implies $t = 0$.
(F_2): $F(t, 0, t, 0, 0, t) = t(1 - \alpha)(1 - b) \leq 0$ implies $t = 0$.
(F_3): $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.4. $F(t_1, \dots, t_6) = t_1 - at_2 - b\max\{t_3, t_4\} - c\max\{t_2, t_5, t_6\}$, where $a, b, c \geq 0, b + c < 1$ and $a + c = 1$.
(F_1): $F(t, 0, 0, t, t, 0) = t(1 - (b + c)) \leq 0$ implies $t = 0$.
(F_2): $F(t, 0, t, 0, 0, t) = t(1 - (b + c)) \leq 0$ implies $t = 0$.
(F_3): $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.5. $F(t_1, \dots, t_6) = t_1 - at_2 - b\dfrac{t_5 + t_6}{1 + t_3 + t_4}$, where $a, b \geq 0$ and $a + 2b = 1$.
(F_1): $F(t, 0, 0, t, t, 0) = t(1 - \frac{b}{1+t}) \leq 0$ implies $t = 0$.

(F_2): $F(t,0,t,0,0,t) = t(1-\frac{b}{1+t}) \leq 0$ implies $t=0$.
(F_3): $F(t,t,0,0,t,t) = 0$, for every $t>0$.

Example 3.6. $F(t_1,\ldots,t_6) = t_1 - at_2 - b(t_3+t_4) - c(t_5+t_6)$, where $a,b,c \geq 0$, $b+c<1$ and $a+2c=1$.
(F_1): $F(t,0,0,t,t,0) = t(1-(b+c)) \leq 0$ implies $t=0$.
(F_2): $F(t,0,t,0,0,t) = t(1-(b+c)) \leq 0$ implies $t=0$.
(F_3): $F(t,t,0,0,t,t) = 0$, for every $t>0$.

Example 3.7. $F(t_1,\ldots,t_6) = t_1^2 - at_1(t_2+t_3+t_4) - bt_5t_6$, where $a,b>0$ and $a+b=1$.
(F_1): $F(t,0,0,t,t,0) = t^2(1-a) \leq 0$ implies $t=0$.
(F_2): $F(t,0,t,0,0,t) = t^2(1-a) \leq 0$ implies $t=0$.
(F_3): $F(t,t,0,0,t,t) = 0$, for every $t>0$.

Example 3.8. $F(t_1,\ldots,t_6) = t_1^3 - t_2^3 - \dfrac{t_3^2t_5 + t_4^2t_6}{1+t_3+t_4}$.
(F_1): $F(t,0,0,t,t,0) = t^3 \leq 0$ implies $t=0$.
(F_2): $F(t,0,t,0,0,t) = t^3 \leq 0$ implies $t=0$.
(F_3): $F(t,t,0,0,t,t) = 0$, for every $t>0$.

4 Altering distances and fixed points

Theorem 4.1. *Let (X,d) be a metric space and $A,B,S,T:(X,d)\to(X,d)$ be mappings satisfying the following inequality*

$$F\Big(\psi(d(Ax,By)),\psi(d(Sx,Ty)),\psi(d(Ax,Sx)),$$
$$\psi(d(Ty,By)),\psi(d(Sx,By)),\psi(d(Ty,Ax))\Big) < 0, \quad (4.1)$$

for all $x,y\in X$, where F satisfies property (F_3) and ψ is an altering distance. If there exist $x,y\in X$ such that $Ax=Sx$ and $By=Ty$, then A and S have a unique point of coincidence $u=Ax=Sx$ and B and T have a unique point of coincidence $v=By=Ty$.

Proof. First we prove that $Ax=By$. Suppose that $Ax\neq Ay$. By (4.1) we obtain

$$F\Big(\psi(d(Ax,By)),\psi(d(Ax,By)),0,0,\psi(d(Ax,By)),\psi(d(By,Ax))\Big) < 0,$$

a contradiction of (F_3). Hence $Ax=By=Sx=Ty$. Moreover, if there exists another point z such that $Az=Sz:=w$, with $Az\neq Ax$, then by (4.1) we obtain

$$F\Big(\psi(d(Az,By)),\psi(d(Az,By)),0,0,\psi(d(Az,By)),\psi(d(Az,By)))\Big) < 0,$$

a contradiction of (F_3). Therefore $u=Ax=Sx$ is the unique point of coincidence of A and S. Similarly, $v=By=Ty$ is the unique point of coincidence of B and T. ∎

Theorem 4.2. *Let (X, d) be a metric space and $A, B, S, T : (X, d) \to (X, d)$ be mappings satisfying the following inequality*

$$F\Big(\psi(d(Ax, By)), \psi(d(Sx, Ty)), \psi(d(Ax, Sx)), \\ \psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax))\Big) < 0, \quad (4.1)$$

for all $x, y \in X$, where $F \in \mathcal{F}_t$ and ψ is an altering distance. If there exists a weak tangent point $z \in S(X) \cap T(X)$ to the pair $\{S, T\}$ and $\{A, B\}$ is tangential w.r.t. $\{S, T\}$, then
a) A and S have a coincidence point,
b) B and T have a coincidence point.
Moreover, if the pairs $\{A, S\}$ and $\{B, T\}$ are owc, then A, B, S and T have a unique common fixed point.

Proof. Since the point $z \in S(X) \cap T(X)$ is a weak tangent point to (S, T), there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = z.$$

Because the pair of mappings (A, B) is tangential w.r.t. the pair (S, T), we have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = z.$$

Again, since $z \in S(X) \cap T(X)$, then $z = Su = Tv$ for some $u, v \in X$. Using (4.1) we have

$$F\Big(\psi(d(Ax_n, Bv)), \psi(d(Sx_n, Tv)), \psi(d(Sx_n, Ax_n)), \\ \psi(d(Tv, Bv)), \psi(d(Sx_n, Bv)), \psi(d(Tv, Ax_n))\Big) < 0.$$

Letting n tend to infinity, we obtain

$$F\Big(\psi(d(z, Bv)), 0, 0, \psi(d(z, Bv)), \psi(d(z, Bv)), 0)\Big) \leq 0.$$

By property (F_1), it follows that $\psi(d(z, Bv)) = 0$, which implies that $d(z, Bv) = 0$, i.e., $z = Bv$. Hence $z = Tv = Bv$ and v is a coincidence point of B and T.

Further, using (4.1) again, we get

$$F\Big(\psi(d(Au, By_n)), \psi(d(Su, Ty_n)), \psi(d(Su, Au)), \\ \psi(d(Ty_n, By_n)), \psi(d(Su, By_n)), \psi(d(Ty_n, Au))\Big) < 0.$$

Letting n tend to infinity, we obtain

$$F\Big(\psi(d(Au, z)), 0, \psi(d(z, Au)), 0, 0, \psi(d(z, Au))\Big) \leq 0.$$

By (F_2), it follows that $\psi(d(z, Au)) = 0$ which implies that $d(z, Au) = 0$, i.e., $z = Au$. Thus $z = Au = Su$ and u is a coincidence point of A and S.

Because F satisfies property (F_3), by Theorem 4.1, z is the unique point of coincidence of A and S and z is the unique point of coincidence of B and T.

If the pairs $\{A, S\}$ and $\{B, T\}$ are owc then by Lemma 1.1, z is the unique common fixed point of A, B, S and T. This ends the proof. ∎

For $\psi(t) = t$, we obtain

Corollary 4.1. *Let $A, B, S, T : (X, d) \to (X, d)$ be self-mappings of a metric space satisfying the inequality*

$$F(d(Ax, By), d(Sx, Ty), d(Ax, Sx), d(Ty, By)), d(Sx, By), d(Ty, Ax)) < 0, \quad (4.2)$$

for all $x, y \in X$, where $F \in \mathcal{F}_t$. If there exists a weak tangent point $z \in S(X) \cap T(X)$ to the pair $\{S, T\}$ and $\{A, B\}$ is tangential w.r.t. $\{S, T\}$, then
a) A and S have a coincidence point,
b) B and T have a coincidence point.
Moreover, if the pairs $\{A, S\}$ and $\{B, T\}$ are owc, then A, B, S and T have a unique common fixed point.

Remark 4.1.
1) By Corollary 4.1 and Example 3.1 we obtain a generalization of Corollary 2.8 of [17].
2) By Corollary 4.1 and Example 3.1 for $\alpha = 0$, we obtain a generalization of Corollary 2.9 of [17].

5 Altering distances and contractive conditions of integral type

Theorem 5.1. *Let (X, d) be a metric space and $A, B, S, T : (X, d) \to (X, d)$ be mappings satisfying the following inequality*

$$F\Big(\int_0^{d(Ax,By)} \phi(t)dt, \int_0^{d(Sx,Ty)} \phi(t)dt, \int_0^{d(Ax,Sx)} \phi(t)dt,$$
$$\int_0^{d(Ty,By)} \phi(t)dt, \int_0^{d(Sx,By)} \phi(t)dt, \int_0^{d(Ty,Ax)} \phi(t)dt\Big) < 0, \quad (5.1)$$

for all $x, y \in X$, where $F \in \mathcal{F}_t$ and ϕ is a function as in Theorem 2.1. If there exists a weak tangent point $z \in S(X) \cap T(X)$ to the pair $\{S, T\}$ and $\{A, B\}$ is tangential w.r.t. $\{S, T\}$, then
a) A and S have a coincidence point,
b) B and T have a coincidence point.
Moreover, if the pairs $\{A, S\}$ and $\{B, T\}$ are owc, then A, B, S and T have a unique common fixed point.

Proof. We set $\psi(t) := \int_0^t \phi(x)dx$ for all $t \in [0, \infty)$. By Lemma 2.1, ψ is an altering distance. The inequality (5.1) may be written in the following form:

$$F\Big(\psi(d(Ax, By)), \psi(d(Sx, Ty)), \psi(d(Sx, Ax)),$$
$$\psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax))\Big) < 0,$$

for all $x, y \in X$.

Hence all the conditions of Theorem 4.2 are satisfied and the result of Theorem 5.1 follows from Theorem 4.2. So our theorem is proved. ∎

Remark 5.1.

a) If $\phi(t) = 1$ (for all $t \in [0, \infty)$), then we obtain Corollary 4.1.

b) By Theorem 5.1 and Example 3.1, we obtain a generalization of Theorem 2.2.

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References

[1] M. Aamri and D. El Moutawakil, *Some new common fixed point theorems under strict contractive conditions,* Math. Anal. Appl. 270 (2002), 181-188.

[2] M. Aliouche, *A common fixed point theorem for weakly compatible mappings satisfying a contractive condition of integral type,* J. Math. Anal. Appl., 322 (2006), 796-802.

[3] M. Aliouche and V. Popa, *Common fixed point theorem for occasionally weakly compatible mappings via implicit relations,* Filomat, **22**(2) (2008), 99-107.

[4] M. A. Al-Thagafi and N. Shahzad, *Generalized I-nonexpansive self maps and invariant approximations,* Acta Math. Sinica, **24**(5) (2008), 867-876.

[5] A. Branciari, *A fixed point theorem for mappings satisfying a general contractive condition of integral type,* Internat. J. Math. Math. Sci., 29, 9 (2002), 531-536.

[6] M. Imdad, S.Kumar and M. S. Khan, *Remarks on some fixed point theorems satisfying implicit relations,* Radovi Math., **11** (2002), 135-143.

[7] G. Jungck, *Compatible mappings and common fixed points,* Internat. J. Math. Math. Sci., **9** (1986), 771-779.

[8] G. Jungck, *Common fixed points for noncontinuous nonself mappings on nonnumeric spaces,* Far East, J. Math.Sci., **4** (1996), 192-215.

[9] G. Jungck and B.E. Rhoades, *Fixed point theorems for occasionally weakly compatible mappings*, Fixed Point Theory, 7(2) (2006), 287-296.

[10] M. S. Khan, M. Swaleh and S. Sessa, *Fixed point theorems by altering distance between the points,* Bull. Austral. Math. Soc., 30 (1984), 1-9.

[11] J. K. Kohli and S. Vashistha, *Common fixed point theorems for compatible and weakly compatible mappings satisfying general contractive type conditions,* Stud. Cerc. St. Ser. Mat. Univ. Bacău, **16** (2006), 33-42.

[12] S. Kumar, R. Chung and R. Kumar, *Fixed point theorems for compatible mappings satisfying a contractive condition of integral type,* Soochow J. Math., 33, 2 (2007), 181-186.

[13] Y Liu, J. Wu and Z. Li, *Common fixed points for single valued and multi-valued maps,* Internat. J. Math. Math. Sci., **19** (2005), 3045-3055.

[14] M. Mocanu and V. Popa, *Some fixed point theorems for mappings satisfying implicit relations in symmetric spaces,* Libertas Math. (28) (2008), 1-13.

[15] R.P. Pant, *Common fixed points for noncommuting mappings,* J. Math. Anal. Appl., **188** (1994), 436-440.

[16] R.P. Pant, *Common fixed point for four mappings,* Bull. Calcutta Math. Soc., **9** (1998), 281-287.

[17] H. K. Pathak and Naseer Shahzad, *Gregus type fixed point results for tangential mappings satisfying contractive conditions of integral type,* Bull. Belg. Math. Soc. Simon Stevin, **16** (2009), 1-12.

[18] V. Popa, *Some fixed point theorems for compatible mappings satisfying an implicit relation,* Demonstratio Math. **32** (1999), 157-163.

[19] V.Popa, *A fixed point theorem for four compatible mappings in compact metric spaces,* U.P.B. Sci. Bull. Ser. A, 63, 4(2001), 43-46.

[20] V. Popa and M. Mocanu, *A new view point in the study of fixed points for mappings satisfying a contractive condition of integral type,* Bul. Inst. Politehn. Iasi, Ser. Mat. Mec. Teor., 53(57), 5(2007),269-272.

[21] B. E. Rhoades, *Two fixed point theorems for mapping satisfying a contractive condition of integral type,* Internat. J. Math. Math. Sci., 63 (2003), 4007-4013.

[22] K.P.R. Sastry and G.V.R. Babu, *Fixed point theorems in metric spaces by altering distances,* Bull. Cal. Math. Soc. 90 (1998), 175182.

[23] K.P.R. Sastry and G.V.R. Babu, *Some fixed point theorems by altering distances between the points,* Indian. J. Pure Appl. Math. 30 (1999), 641647.

[24] K.P.R. Sastry, I.S.R. Krishna Murthy, *Common fixed points of two partially commuting tangential selfmaps on a metric space,* J. Math. Anal. Appl., 250 (2000), 731-734.

[25] P. Viayaraju, B. E. Rhoades, R.Mohanraj, *A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type,* Internat. J. Math. Math. Sci. 15 (2005), 2359-2364.

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