A property of group laws

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Abstract

For a word in n letters, in [1] the author introduced a notion: *its standard exponent* and proved that the variety of residually finite groups defined by a word is almost nilpotent if and only if the standard exponent of this word is 1. In this paper we obtain the following result: let $\omega(x_1, \cdots, x_n)$ denote a word in $x_1, \cdots, x_n$. Then both $\omega(x_1, \cdots, x_n)$ and $\omega(x_1^{m_1}, \cdots, x_n^{m_n})$, where m_i are natural numbers, have the same standard exponents.

1 Introduction

Recall that an element of the free group $\mathcal{F}(x_1, \cdots, x_n)$ of rank n is called a word in n letters. Any word in $x_1, \cdots, x_n$ can be written as: $\prod_{i=1}^{s} x_{i^*}^{a_i}$, where $a_i \neq 0$ are integers and $*$ denotes a mapping from $\{1, \cdots, s\}$ to $\{1, \cdots, n\}$ for which $i^* \neq (i+1)^*$ for $i = 1, \cdots, s-1$ (see [1, Notation 1]). A word in $x_1, \cdots, x_n$ is called to be homogeneous if for each i, the exponent sum of x_i is zero. We say a group G satisfies a word $\omega(x_1, \cdots, x_n)$, if for any $g_1, \cdots, g_n \in G$ we have $\omega(g_1, \cdots, g_n) = 1$, *i.e.*, G satisfies the group law $\omega(x_1, \cdots, x_n) \equiv 1$. Let τ be a homomorphism from $\mathcal{F}(x_1, \cdots, x_n)$ to $\mathcal{F}(c, d)$ defined on the generators x_i. We can write it as: $\tau : x_i \mapsto f_i d^{k_i}, i = 1, \cdots, n$, where f_i are in $C^D (C =< c >, D =< d >)$ and k_i are integers. We call this the C^D form of τ. Similarly we can also write it as: $\tau : x_i \mapsto f_i' c^{k_i'}, i = 1, \cdots, n$, where f_i' are in D^C and k_i' are integers. Call this the D^C form of τ.

Suppose that $\omega(x_1, \cdots, x_n) = \prod_{i=1}^{s} x_{i^*}^{a_i}$, where $i^* \in \{1, \cdots, n\}$ and $i^* \neq (i+1)^*$ for $i = 1, \cdots, s-1$, is a homogeneous word. Applying the C^D form of $\tau : x_i \mapsto f_i d^{k_i}$

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to ω and using commutator collection, then *w.l.o.g.* we can write

$$\begin{aligned}\omega^\tau =&(f_1^{\delta_{11}})^{d^{\rho_{11}}}\cdots(f_1^{\delta_{1r_1}})^{d^{\rho_{1r_1}}}\\ &\cdots\cdots\\ &(f_l^{\delta_{l1}})^{d^{\rho_{l1}}}\cdots(f_l^{\delta_{lr_l}})^{d^{\rho_{lr_l}}}\cdot modulo(C^D)',\end{aligned} \tag{1}$$

where $l \leq n, \delta_{tj_t} \neq 0$ and $\rho_{tj_t} := v_{tj_t}^1 k_1 + \cdots + v_{tj_t}^n k_n$ for $j_t = 1, \cdots, r_t, t = 1, \cdots, l$ such that for each t the ordered n-tuples $(v_{t1}^1, \cdots, v_{t1}^n), \cdots, (v_{tr_t}^1, \cdots, v_{tr_t}^n)$ are distinct.

We call Form (1) a C^D standard form of ω^τ. If $r_1 + \cdots + r_l \neq 0$, then we call the number $\delta_\omega := gcd(\delta_{11}, \cdots, \delta_{1r_1}, \cdots, \delta_{l1}, \cdots, \delta_{lr_l})$ the standard exponent of ω and if $r_1 + \cdots + r_l = 0$, then we say $\delta_\omega = 0$. Finally, define the standard exponent of a non-homogeneous word as 1 (in [1, Section II] it has been pointed out that this *gcd* does not depend on the C^D standard form ω^τ).

By [1, Theorem 26], we note that there is a large class of groups, which includes all residually finite or soluble groups and all locally finite or soluble groups, such that every group in this class satisfying ω is nilpotent-by-finite-exponent if and only if $\delta_\omega = 1$. So it is of interest to investigate the properties of the standard exponent of a word. One notes that any group satisfying a non-homogeneous word is of finite exponent. Thus in the following we only consider homogeneous words.

2 A property of words

For convenience we write $\omega \sim \omega'$ to denote that the words ω and ω' have the same standard exponent.

Proposition 1. *Let $\omega(x_1, \cdots, x_n)$ be a homogeneous word in $x_1, \cdots, x_n$. Let m_i be natural numbers. Then $\omega(x_1, \cdots, x_n) \sim \omega(x_1^{m_1}, \cdots, x_n^{m_n})$.*

Proof. Let $\omega_1 = \omega(x_1, \cdots, x_n) = \prod_{i=1}^s x_{i^*}^{a_i}$, where $i^* \in \{1, \cdots, n\}$ and $i^* \neq (i+1)^*$ for $i = 1, \cdots, s-1$, be a homogeneous word and let $\omega_2 = \omega(x_1, \cdots, x_\lambda^m, \cdots, x_n)$, where $m \geq 1$ is a natural number. Thus $\omega_2 = \prod_{i=1}^s x_{i^*}^{b_i}$, where

$$b_i = \begin{cases} ma_i & \text{if } i^* = \lambda \\ a_i & \text{otherwise.} \end{cases}$$

In fact, it suffices to show that $\omega_1 \sim \omega_2$. For $m = 1$, it is trivial, so in the following we assume that $m > 1$.

Let τ be a homomorphism from $\mathcal{F}(x_1, \cdots, x_n)$ to $\mathcal{F}(c, d)$. Apply the C^D-form of $\tau : x_i \mapsto f_i d^{k_i}$ to ω_1 and ω_2 respectively. Then we have

$$\begin{aligned}\omega_1^\tau &= \prod_{i=1}^s (f_{i^*} d^{k_{i^*}})^{a_i} = \prod_{i=1}^s F_i d^{a_i k_{i^*}} = F_1 \prod_{i=2}^s F_i^{d^{-\sum_{j=1}^{i-1} a_j k_{j^*}}}\\ &= F_1 \prod_{i=2}^s F_i^{d^{\sum_{t=1}^n u_i^t k_t}}\end{aligned} \tag{2}$$

and

$$\omega_2^\tau = \prod_{i=1}^{s}(f_{i^*}d^{k_{i^*}})^{b_i} = \prod_{i=1}^{s} G_i d^{b_i k_{i^*}} = G_1 \prod_{i=2}^{s} G_i^{d^{-\sum_{j=1}^{i-1} b_j k_{j^*}}}$$
$$= G_1 \prod_{i=2}^{s} G_i^{d^{u_i^1 k_1 + \cdots + m u_i^\lambda k_\lambda + \cdots + u_i^n k_n}}, \tag{3}$$

where

$$u_1^t = 0$$
$$u_i^t = -\sum_{\{j|1\le j<i,\ j^*=t\}} a_j \qquad \text{for } i > 1$$
$$F_i = \begin{cases} f_{i^*}\ f_{i^*}^{d^{-k_{i^*}}}\ \cdots\ f_{i^*}^{d^{-(|a_i|-1)k_{i^*}}} & \text{if } a_i > 0 \\ (f_{i^*}^{-1})^{d^{k_{i^*}}}\ (f_{i^*}^{-1})^{d^{2k_{i^*}}}\ \cdots\ (f_{i^*}^{-1})^{d^{|a_i|k_{i^*}}} & \text{if } a_i < 0 \end{cases}$$
$$G_i = F_i \qquad \text{if } i^* \neq \lambda$$

and otherwise, if $i^* = \lambda$ then

$$G_i = \begin{cases} f_\lambda\ f_\lambda^{d^{-k_\lambda}}\ \cdots\ f_\lambda^{d^{-(m|a_i|-1)k_\lambda}} & \text{if } a_i > 0 \\ (f_\lambda^{-1})^{d^{k_\lambda}}\ (f_\lambda^{-1})^{d^{2k_\lambda}}\ \cdots\ (f_\lambda^{-1})^{d^{m|a_i|k_\lambda}} & \text{if } a_i < 0. \end{cases}$$

Then we can see that the general terms are respectively in Form (2):

$$\begin{cases} (f_\lambda^{sign(a_i)})^{d^{p_{il_i}k_\lambda + u_i^1 k_1 + \cdots + u_i^\lambda k_\lambda + \cdots + u_i^n k_n}} & \text{if } i^* = \lambda \\ (f_{i^*}^{sign(a_i)})^{d^{p_{il_i}k_{i^*} + u_i^1 k_1 + \cdots + u_i^\lambda k_\lambda + \cdots + u_i^n k_n}} & \text{if } i^* \neq \lambda \end{cases} \tag{4}$$

and in Form (3):

$$\begin{cases} (f_\lambda^{sign(a_i)})^{d^{q_{il_i'}k_r + u_i^1 k_1 + \cdots + m u_i^\lambda k_\lambda + \cdots + u_i^n k_n}} & \text{if } i^* = \lambda \\ (f_{i^*}^{sign(a_i)})^{d^{p_{il_i}k_{i^*} + u_i^1 k_1 + \cdots + m u_i^\lambda k_\lambda + \cdots + u_i^n k_n}} & \text{if } i^* \neq \lambda, \end{cases} \tag{5}$$

where

$$p_{il_i} = \begin{cases} l_i - a_i & \text{if } a_i > 0 \\ l_i & \text{if } a_i < 0 \qquad l_i = 1, \cdots, |a_i| \end{cases}$$
$$q_{il_i'} = \begin{cases} l_i' - m a_i & \text{if } a_i > 0 \\ l_i' & \text{if } a_i < 0 \qquad l_i' = 1, \cdots, m|a_i| \end{cases}$$

for $i = 1, 2, \cdots, s$.

Now by commutator collection we rewrite Form (2) as a C^D standard form of ω_1^τ. Without loss of generality, we can write

$$\begin{aligned} \omega_1^\tau =&(f_1^{\delta_{11}})^{d^{\rho_{11}}} \cdots (f_1^{\delta_{1r_1}})^{d^{\rho_{1r_1}}} \\ &\cdots\cdots \\ &(f_\lambda^{\delta_{\lambda 1}})^{d^{\rho_{\lambda 1}}} \cdots (f_\lambda^{\delta_{\lambda r_\lambda}})^{d^{\rho_{\lambda r_\lambda}}} \\ &\cdots\cdots \\ &(f_l^{\delta_{l1}})^{d^{\rho_{l1}}} \cdots (f_l^{\delta_{lr_l}})^{d^{\rho_{lr_l}}} \cdot modulo(C^D)', \end{aligned} \tag{6}$$

where $l \leq n, \delta_{tj_t} \neq 0$ and $\rho_{tj_t} := v^1_{tj_t}k_1 + \cdots + v^n_{tj_t}k_n$ for $j_t = 1, \cdots, r_t, t = 1, \cdots, l$ such that for each t the ordered n-tuples $(v^1_{t1}, \cdots, v^n_{t1}), \cdots, (v^1_{tr_t}, \cdots, v^n_{tr_t})$ are distinct.

We first consider such terms of ω^τ_1 for which the indexes of f's satisfy $i^* \neq \lambda$.

We note that between such terms in Form (4) and those in Form (5), there exists a one-to-one correspondence:

$$(f^{sign(a_i)}_{i^*})^{d^{p_{il_i}k_{i^*}+u^1_ik_1+\cdots+u^\lambda_ik_\lambda+\cdots+u^n_ik_n}} \mapsto (f^{sign(a_i)}_{i^*})^{d^{p_{il_i}k_{i^*}+u^1_ik_1+\cdots+mu^\lambda_ik_\lambda+\cdots+u^n_ik_n}},$$

where $l_i = 1, \cdots, |a_i|$ and all $i^* \neq \lambda$.

On the other hand, for any i, j satisfying $i^* = j^*$ we note that

$$(u^1_i, \cdots, p_{il_i} + u^{i^*}_i, \cdots, u^\lambda_i, \cdots, u^n_i) = (u^1_j, \cdots, p_{jl_j} + u^{j^*}_j, \cdots, u^\lambda_j, \cdots, u^n_j)$$
$$\Longleftrightarrow$$
$$(u^1_i, \cdots, p_{il_i} + u^{i^*}_i, \cdots, mu^\lambda_i, \cdots, u^n_i) = (u^1_j, \cdots, p_{jl_j} + u^{j^*}_j, \cdots, mu^\lambda_j, \cdots, u^n_j),$$

where $1 \leq l_i \leq |a_i|$ and $1 \leq l_j \leq |a_j|$.

Therefore, $(f^\delta_{i^*})^{d^{p_{il_i}k_{i^*}+u^1_ik_1+\cdots+u^\lambda_ik_\lambda+\cdots+u^n_ik_n}}$ appears in Form (6) if and only if $(f^\delta_{i^*})^{d^{p_{il_i}k_{i^*}+u^1_ik_1+\cdots+mu^\lambda_ik_\lambda+\cdots+u^n_ik_n}}$ appears in the C^D standard forms of ω^τ_2. So we can write a C^D standard form of ω^τ_2 as follows:

$$\begin{aligned}\omega^\tau_2 =& (f^{\delta_{11}}_1)^{d^{\rho_{11}}} \cdots (f^{\delta_{1r_1}}_1)^{d^{\rho_{1r_1}}}\\ &\cdots\cdots\\ &(f^{\bar\delta_{\lambda 1}}_\lambda)^{d^{\bar\rho_{\lambda 1}}} \cdots (f^{\bar\delta_{\lambda\bar r_\lambda}}_\lambda)^{d^{\bar\rho_{\lambda\bar r_\lambda}}}\\ &\cdots\cdots\\ &(f^{\delta_{l1}}_l)^{d^{\rho_{l1}}} \cdots (f^{\delta_{lr_l}}_l)^{d^{\rho_{lr_l}}} \cdot modulo(C^D)',\end{aligned} \tag{7}$$

where $\bar\delta_{\lambda\bar j_\lambda} \neq 0$ and $\bar\rho_{\lambda\bar j_\lambda} := \bar v^1_{\lambda\bar j_\lambda}k_1 + \cdots + \bar v^n_{\lambda\bar j_\lambda}k_n$ for $\bar j_\lambda = 1, \cdots, \bar r_\lambda$ such that the ordered n-tuples $(\bar v^1_{\lambda 1}, \cdots, \bar v^n_{\lambda 1}), \cdots, (\bar v^1_{\lambda\bar r_\lambda}, \cdots, \bar v^n_{\lambda\bar r_\lambda})$ are distinct.

Now we remain to consider the terms containing f_λ in ω^τ_1 and ω^τ_2. We start to consider Form (6) and assume that $r_\lambda \neq 0$. Suppose that $(f^\delta_\lambda)^{d^{v_1k_1+\cdots+v_\lambda k_\lambda+\cdots+v_nk_n}}$ is in Form (6) and suppose that $\delta = \sum^t_{i=1}\mathrm{sign}(a_{h_i})$. Just only for convenience to write we shall replace the indexes h_i of a by i below (note that it does not matter to do so though all $k^*_i = \lambda$). We further suppose that $(f^{\mathrm{sign}(a_i)}_\lambda)^{d^{p_{ij_i}k_\lambda+u^1_ik_1+\cdots+u^\lambda_ik_\lambda+\cdots+u^n_ik_n}}$, where $1 \leq j_i \leq |a_i|$, are all the terms containing f_λ in Form (4) for which

$$(u^1_i, \cdots, p_{ij_i} + u^\lambda_i, \cdots, u^n_i) = (v_1, \cdots, v_\lambda, \cdots, v_n).$$

It follows that

$$(u^1_i, \cdots, mp_{ij_i} + mu^\lambda_i, \cdots, u^n_i) = (v_1, \cdots, mv_\lambda, \cdots, v_n),$$

that is,

$$(u^1_i, \cdots, q_{i(mj_i)} + mu^\lambda_i, \cdots, u^n_i) = (v_1, \cdots, mv_\lambda, \cdots, v_n).$$

Now we claim that $(f^{\mathrm{sign}(a_i)}_\lambda)^{d^{q_{i(mj_i)}k_\lambda+u^1_ik_1+\cdots+mu^\lambda_ik_\lambda+\cdots+u^n_ik_n}}$, $i = 1, \cdots, t$ exhaust the terms containing f_λ in Form (5) which are conjugated by powers $d^{v_1k_1+\cdots+mv_\lambda k_\lambda+\cdots+v_nk_n}$ of d.

Indeed, suppose that there is one more such term in Form (5). Similarly for convenience to write, let us suppose that

$$(f_\lambda^{\mathrm{sign}(a_{t+1})})^{d^{q_{(t+1)j}k_\lambda+u_{t+1}^1k_1+\cdots+mu_{t+1}^\lambda k_\lambda+\cdots+u_{t+1}^nk_n}},$$

where $1 \le j \le m|a_{t+1}|$, satisfies

$$(u_{t+1}^1, \cdots, q_{(t+1)j} + mu_{t+1}^\lambda, \cdots, u_{t+1}^n) = (v_1, \cdots, mv_\lambda, \cdots, v_n).$$

Notice that $m|q_{(t+1)j}$, so we have $m|j$ and thus $q_{(t+1)j} \ge q_{(t+1)m} = mp_{(t+1)1}$. Setting $j' = j/m$, then $1 \le j' \le |a_{t+1}|$ and $p_{(t+1)j'} = q_{(t+1)j}/m$. It follows that there is a term $(f_\lambda^{\mathrm{sign}(a_{t+1})})^{d^{p_{(t+1)j'}k_\lambda+u_{t+1}^1k_1+\cdots+u_{t+1}^\lambda k_\lambda+\cdots+u_{t+1}^nk_n}}$ in Form (4) such that $(u_{t+1}^1, \cdots, p_{(t+1)j'} + u_{t+1}^\lambda, \cdots, u_{t+1}^n) = (v_1, \cdots, v_\lambda, \cdots, v_n)$, a contradiction. We get the claim. So $(f_\lambda^\delta)^{d^{v_1k_1+\cdots+mv_\lambda k_\lambda+\cdots+v_nk_n}}$ occurs in Form (7) and thus $\bar{r}_\lambda \ne 0$. It follows that

$$\gcd(\bar{\delta}_{\lambda 1}, \cdots, \bar{\delta}_{\lambda \bar{r}_\lambda}) \mid \gcd(\delta_{\lambda 1}, \cdots, \delta_{\lambda r_\lambda}). \tag{8}$$

Conversely, suppose that $\bar{r}_\lambda \ne 0$ and that of the exponents of f_λ in Form (7), all the distinct ones are $\theta_1, \theta_2, \cdots, \theta_s$. Thus

$$\gcd(\theta_1, \theta_2, \cdots, \theta_s) = \gcd(\bar{\delta}_{\lambda 1}, \cdots, \bar{\delta}_{\lambda \bar{r}_\lambda}).$$

Let us suppose that$(f_\lambda^{\theta_j})^{d^{v_1k_1+\cdots+mv_\lambda k_\lambda+\cdots+v_nk_n}}$ is a term in Form (7). Similar to that above we can suppose that $\theta_j = \sum_{i=1}^e \mathrm{sign}(a_i)$ and that the terms

$$(f_\lambda^{\mathrm{sign}(a_i)})^{d^{q_{i\gamma_i}k_\lambda+u_i^1k_1+\cdots+mu_i^\lambda k_\lambda+\cdots+u_i^nk_n}},$$

where $1 \le \gamma_i \le m|a_i|$, run through the terms containing f_λ in Form (7) which have the property:

$$(u_i^1, \cdots, q_{i\gamma_i} + mu_i^\lambda, \cdots, u_i^n) = (v_1, \cdots, mv_\lambda, \cdots, v_n).$$

Thus we have

$$q_{1\gamma_1} + mu_1^\lambda = \cdots = q_{e\gamma_e} + mu_e^\lambda = mv_\lambda.$$

Note that $q_{i\gamma_i} + mu_i^\lambda = \gamma_i + \mu_i$, where

$$\mu_i = \begin{cases} mu_{i+1}^\lambda & \text{if } a_i > 0 \\ mu_i^\lambda & \text{if } a_i < 0 \end{cases}$$

for $i = 1, \cdots, e$. Let us write $\gamma_i + \mu_i = \zeta m - \eta$, where ζ is an integer and $0 \le \eta < m$. Then we consider the following equations:

$$\eta + q_{1\gamma_1} + mu_1^\lambda = \cdots = \eta + q_{e\gamma_e} + mu_e^\lambda = \eta + mv_\lambda.$$

Note that $m|(\eta + \gamma_i)$ and $\gamma_i \le m|a_i|$, so $\eta + \gamma_i \le m|a_i|$ and thus $\eta + q_{i\gamma_i} = q_{i(\gamma_i+\eta)}$. Setting $\gamma_i' = (\gamma_i + \eta)/m$, then $1 \le \gamma_i' \le |a_i|$. Note that $q_{i(m\gamma_i')} = mp_{i\gamma_i'}$, so we have

$$p_{1\gamma_1'} + u_1^\lambda = \cdots = p_{e\gamma_e'} + u_e^\lambda = (\eta + mv_\lambda)/m := v_\lambda'.$$

Thus $(u_i^1, \cdots, p_{i\gamma_i'} + u_i^\lambda, \cdots, u_i^n) = (v_1, \cdots, v_\lambda', \cdots, v_n)$ for $i = 1, \cdots, e$. It follows that all $(f_\lambda^{\mathrm{sign}(a_i)})^{d^{p_{i\gamma_i'} k_\lambda + u_i^1 k_1 + \cdots + u_i^\lambda k_\lambda + \cdots + u_i^n k_n}}$ for $i = 1, \cdots, e$ are in Form (6).

If $(f_\lambda^{\mathrm{sign}(a_r)})^{d^{p_{rh} k_\lambda + u_r^1 k_1 + \cdots + u_r^\lambda k_\lambda + \cdots + u_r^n k_n}}$, where $1 \le h \le |a_r|$ and $r > e$, is one more term in Form (6) for which $(u_r^1, \cdots, p_{ih} + u_r^\lambda, \cdots, u_r^n) = (v_1, \cdots, v_\lambda', \cdots, v_n)$, then we have

$$(u_r^1, \cdots, mp_{ih} + mu_r^\lambda, \cdots, u_r^n) = (v_1, \cdots, \eta + mv_\lambda, \cdots, v_n).$$

This is equivalent to

$$(u_r^1, \cdots, mp_{ih} + mu_r^\lambda - \eta, \cdots, u_r^n) = (v_1, \cdots, mv_\lambda, \cdots, v_n).$$

Note that $0 < mh - \eta \le m|a_r|$, so $mp_{rh} - \eta = q_{r(mh-\eta)}$. Thus we get one more term $(f_\lambda^{sign(a_r)})^{d^{q_{r(mh-\eta)} k_\lambda + u_r^1 k_1 + \cdots + mu_r^\lambda k_\lambda + \cdots + u_r^n k_n}}$ in Form (7) for which $(u_r^1, \cdots, q_{r(mh-\eta)} + mu_r^\lambda, \cdots, u_r^n) = (v_1, \cdots, mv_\lambda, \cdots, v_n)$, a contradiction. So $(f_\lambda^{\theta_j})^{d^{v_1 k_1 + \cdots + v_\lambda' k_\lambda + \cdots + v_n k_n}}$ occurs in Form (6) and thus $r_\lambda \neq 0$. It follows that

$$\gcd(\delta_{\lambda 1}, \cdots, \delta_{\lambda r_\lambda}) \mid \gcd(\theta_1, \cdots, \theta_s).$$

So by Form (8) we have $\gcd(\delta_{\lambda 1}, \cdots, \delta_{\lambda r_\lambda}) = \gcd(\bar{\delta}_{\lambda 1}, \cdots, \bar{\delta}_{\lambda \bar{r}_\lambda})$ and thus $\delta_{\omega_1} = \delta_{\omega_2}$. This completes the proof. ■

Now we apply these techniques to determine the standard exponents of some words which appear in [2].

Examples

1. Let n_0 be a natural number and

$$\omega_{3,n_0} = [x^{n_0},\ y^{n_0}] \quad (or \quad [x^{n_0},\ y])$$
$$\omega_{n_0} = [x^{n_0},\ y^{n_0},\ x^{n_0}].$$

By Proposition 1

$$\omega_{3,n_0} \sim [x,\ y] = x^{-1} x^y$$
$$\omega_{n_0} \sim [x,\ y,\ x] = y^{-1} (y^2)^x (y^{-1})^{x^2} \cdot modulo(Y^X)'.$$

Thus $\delta_{\omega_{3,n_0}} = \delta_{[x,\ y]} = 1$ and $\delta_{\omega_{n_0}} = \delta_{[x,\ y,\ x]} = 1$.

2. (see [2, Introduction]) Let $\omega = x^{-2}y^{-2}x^2y^2x^{-10}y^{-2}x^{10}y^2x^{-2}y^{-4}x^{-6}y^6x^{-2}y^{-2}x^{10}y^{-4}x^{-8}y^6x^8y^{-2}$.

By Proposition 1

$$\omega \sim x^{-1}y^{-1}xyx^{-5}y^{-1}x^5yx^{-1}y^{-2}x^{-3}y^3x^{-1}y^{-1}x^5y^{-2}x^{-4}y^3x^4y^{-1}$$
$$= y^{-1}(y^{-3})^x(y^6)^{x^4}(y^{-2})^{x^5} \cdot modulo(Y^X)'.$$

So $\delta_\omega = 1$.

Note that compared with direct calculation of the standard exponent of ω (made in [3, Example (3) of Section 4]), now the result follows very easily using Proposition 1.

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