# Domains of $\mathbb{R}$-analytic existence in inductive limits of real separable Banach spaces

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**Abstract**

Every non void domain of a countable inductive limit of real separable Banach spaces is a domain of $\mathbb{R}$-analytic existence

## 1 Introduction

In [3], we proved that if $E = \text{ind}E_m$ is a countable inductive limit of real separable normed spaces, every non void, open and convex subset of $E$ is a domain of analytic existence. In this note, we prove that we can drop the hypothesis of convexity if we assume that the spaces $E_m$ are Banach spaces.

## 2 Result

**Theorem 2.1.** *If $E = \text{ind}E_m$ is an inductive limit of real separable Banach spaces, every non void domain $\Omega$ of $E$ is a domain of analytic existence.*

To prove this result in the case of an open and convex subset $\Omega$ of $E$, we used that $\omega = \Omega \cap (-\Omega)$ is an open and absolutely convex subset of $E$ such that $\Omega$ is open for its Minkowski functional $p_\omega$. Therefore, $\Omega$ is open for one semi-norm. But by the following result and examples of J.F. Colombeau and J. Mujica (cf. Lemma 3.5 in [1] and Example 3.1 in [2]), every arbitrary domain $\Omega$ in such a space $E$ is open for one semi-norm.

Received by the editors January 2000.
Communicated by Y. Félix.
1991 *Mathematics Subject Classification* : Primary: 46E50, 26E05; Secondary: 46A13.
*Key words and phrases* : real-analytic function, domain of existence, inductive limit.

**Theorem 2.2.** *Let $E$ be a locally convex space with the two following properties:*

*(i) For each sequence $(V_j)_{j\in\mathbb{N}}$ of neighbourhoods of zero in $E$, there exists a sequence $(\lambda_j)_{j\in\mathbb{N}}$ of strictly positive scalars such that $\cap_{j\in\mathbb{N}}\lambda_j V_j$ is also a neighbourhood of zero in $E$.*

*(ii) $E$ is hereditary Lindelöf i.e. every subset of $E$ is a Lindelöf topological space under the induced topology.*

*Then for each open set $\Omega \subset E$, there is a convex, balanced 0-neighbourhood $V$ in $E$ such that if $\alpha$ denotes the Minkowski functional of $V$, $\Omega$ is open in $(E, \alpha)$.*

**Examples 2.3.** *The strong dual of a Fréchet-Montel space and any countable inductive limit of separable Banach spaces are examples of spaces satisfying conditions (i) and (ii) in Theorem 2.2.*

Thanks to those results, the proof in [3] is valid for an arbitary non void domain of a countable inductive limit of real separable Banach spaces.

*Acknowledgements.* The author thanks a lot Professor D. Garçia of the University of Valencia for his help during a stay in Valencia (September 1999) and the Belgian National Science Foundation (FNRS) for its financial support.

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