ON $(1,2)^*$-SEMI-$T_{1/3}$ BITOPOLOGICAL SPACES

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Abstract. The aim of this paper is to introduce a separation axiom using $(1,2)^*$-ψ-closed sets.

1. Introduction

Levine [5], Mashhour et al. [7] and Njastad [8] have introduced the concepts of semi-open sets, preopen sets, and α-open sets, respectively. Levine [6] introduced generalized closed sets and studied their properties. Bhattacharya and Lahiri [2] introduced semi-generalized closed sets. Thivagar et al. [9] have introduced the concepts of $(1,2)^*$-semi-open sets, $(1,2)^*$-generalized closed sets, $(1,2)^*$-semi-generalized closed sets in bitopological spaces. In this paper we introduce the concept of $(1,2)^*$-ψ-closed sets in $(1,2)^*$-bitopological spaces and use them to define $(1,2)^*$-semi-$T_{1/3}$ bitopological spaces. We also study their basic properties and relative preservation properties of these spaces.

2. Preliminaries

Throughout this paper (X,τ_1,τ_2), (Y,σ_1,σ_2), and (Z,ν_1,ν_2) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1. [9] *A subset S of a bitopological space (X,τ_1,τ_2) is said to be $\tau_{1,2}$-open if $S = A \cup B$ where $A \in \tau_1$, and $B \in \tau_2$. A subset S of X is said to be $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open.*

Definition 2.2. [9] *Let S be a subset of X. Then*

(i) *The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined by $\cup\{G/G \subset S$ and G is $\tau_{1,2}$-open$\}$.*

(ii) *The $\tau_{1,2}$-closure of S denoted by $\tau_{1,2}$-cl(S), is defined by $\cap\{F/S \subset F$ and F is $\tau_{1,2}$-closed$\}$.*

Remark 2.3.

(i) *$\tau_{1,2}$-int(S) is $\tau_{1,2}$-open for each $S \subseteq X$ and $\tau_{1,2}$-cl(S) is $\tau_{1,2}$-closed for each $S \subseteq X$.*
(ii) *A set $S \subseteq X$ is $\tau_{1,2}$-open if and only if $S = \tau_{1,2}$-int(S) and is $\tau_{1,2}$-closed if and only if $S = \tau_{1,2}$-cl(S).*
(iii) *$\tau_{1,2}$-open sets need not form a topology.*

We recall the following definitions which are useful in the sequel.

Definition 2.4. [9] *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(i) *$(1,2)^*$-semi-open if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A))$*
(ii) *$(1,2)^*$-preopen if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$*
(iii) *$(1,2)^*$-α-open if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(\tau_{1,2}$-int$(A)))$*
(iv) *$(1,2)^*$-semi-preopen if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(\tau_{1,2}$-cl$(A)))$*
(v) *$(1,2)^*$-regular open if $A = \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$.*
(vi) *$(1,2)^*$semi-regular if A is both $(1,2)^*$-semi-open and $(1,2)^*$-semi-closed.*

The complements of the sets mentioned above from (i) to (v) are called their respective closed sets.

Definition 2.5. [4]

(i) *The $(1,2)^*$-semi-closure (resp. $(1,2)^*$-α-closure, $(1,2)^*$-semi-preclosure) of a subset A of X, denoted by $(1,2)^*$-scl(A) (resp.$(1,2)^*$-αcl(A), $(1,2)^*$-spcl(A)), is defined to be the intersection of all $(1,2)^*$-semi-closed (resp. $(1,2)^*$-α-closed, $(1,2)^*$-semi-preclosed) sets containing A.*
(ii) *The $(1,2)^*$-semi-interior (resp. $(1,2)^*$-α-interior, $(1,2)^*$-semi-preinterior) of a subset A of X, denoted by $(1,2)^*$-sint(A) (resp. $(1,2)^*$-αint(A), $(1,2)^*$-spint(A)), is defined to be the union of all $(1,2)^*$-semi-open (resp. $(1,2)^*$-α-open, $(1,2)^*$-semi-preopen) sets contained in A.*

Remark 2.6. [4]

(i) *Since arbitrary union (resp. intersection) of $(1,2)^*$-semi-open (resp.$(1,2)^*$-semi-closed) sets is $(1,2)^*$-semi-open (resp.$(1,2)^*$-semi-closed), $(1,2)^*$-sintA (resp.$(1,2)^*$-scl(A)) is $(1,2)^*$-semi-open (resp. $(1,2)^*$-semi-closed).*
(ii) *For a bitopological space (X, τ_1, τ_2), a subset A of X is $(1,2)^*$-semi-open (resp. $(1,2)^*$-semi-closed) if and only if $(1,2)^*$-sint(A) (resp. $(1,2)^*$-scl$(A)) = A$.*

Definition 2.7. [9] *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(i) *$(1,2)^*$-semi-generalized closed (briefly $(1,2)^*$-sg-closed) if $(1,2)^*$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$-semi-open in X.*

(ii) $(1,2)^*$-*g-closed (resp.* $(1,2)^*$-*gs-closed,*$(1,2)^*$-*gsp-closed,* $(1,2)^*$-αg-*closed) if* $\tau_{1,2}$-*cl*(A) *(resp.* $(1,2)^*$-*scl*(A),$(1,2)^*$-*spcl*(A), $(1,2)^*$-$\alpha cl(A)) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is* $\tau_{1,2}$-*open in* X.

(iii) $(1,2)^*$-*generalized* α-*closed (briefly* $(1,2)^*$-$g\alpha$-*closed) if* $(1,2)^*$-$\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is* $(1,2)^*$-α-*open in* X.

(iv) $(1,2)^*$-*Q-set if* $\tau_{1,2}$-*int*$(\tau_{1,2}$-*cl*$(A)) = \tau_{1,2}$-*cl*$(\tau_{1,2}$-*int*$(A))$.

The complements of the sets mentioned above from (i) to (iii) are called their respective open sets.

Definition 2.8. [10] *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called*

(i) $(1,2)^*$-*continuous (resp.*$(1,2)^*$-*semi-continuous,* $(1,2)^*$-*pre-continuous,* $(1,2)^*$-α-*continuous,*$(1,2)^*$-*g-continuous) if* $f^{-1}(V)$ *is* $\tau_{1,2}$-*open (resp.*$(1,2)^*$-*semi-open,* $(1,2)^*$-*preopen,* $(1,2)^*$-α-*open,* $(1,2)^*$-*g-open) in* (X, τ_1, τ_2) *for every* $\sigma_{1,2}$-*open set* V *in* (Y, σ_1, σ_2).

(ii) $(1,2)^*$-*sg-continuous (resp.*$(1,2)^*$-*gs-continuous,* $(1,2)^*$-$g\alpha$-*continuous,* $(1,2)^*$-αg-*continuous,* $(1,2)^*$-*gsp-continuous) if* $f^{-1}(V)$ *is* $(1,2)^*$-*sg-closed (resp.*$(1,2)^*$-*gs-closed,* $(1,2)^*$-$g\alpha$-*closed,* $(1,2)^*$-αg-*closed,* $(1,2)^*$-*gsp-closed) in* (X, τ_1, τ_2) *for every* $\sigma_{1,2}$-*closed set* V *in* (Y, σ_1, σ_2).

(iii) $(1,2)^*$-*irresolute if* $f^{-1}(V)$ *is* $(1,2)^*$-*semi-open in* (X, τ_1, τ_2) *for every* $(1,2)^*$-*semi-open set* V *in* (Y, σ_1, σ_2).

(iv) $(1,2)^*$-*sg-irresolute if* $f^{-1}(V)$ *is* $(1,2)^*$-*sg-closed in* (X, τ_1, τ_2) *for every* $(1,2)^*$-*sg-closed set* V *in* (Y, σ_1, σ_2).

(v) $(1,2)^*$-*pre-semi-closed (resp.* $(1,2)^*$-*pre-sg-closed) if* $f(U)$ *is* $(1,2)^*$-*semi-closed (resp.* $(1,2)^*$-*sg-closed) in* (Y, σ_1, σ_2) *for every* $(1,2)^*$-*semi-closed (resp.* $(1,2)^*$-*sg-closed) subset* U *of* (X, τ_1, τ_2)

Definition 2.9. [4] *A bitopological space* (X, τ_1, τ_2) *is called a*

(i) $(1,2)^*$-$T_{1/2}$-*space if every* $(1,2)^*$-*g-closed set is* $\tau_{1,2}$-*closed.*

(ii) $(1,2)^*$-*semi*-$T_{1/2}$-*space if every* $(1,2)^*$-*sg-closed set is* $(1,2)^*$-*semi-closed.*

(iii) $(1,2)^*$-*semi*-T_1 *space if to each pair of distinct points* x,y *of* X, *there exists a pair of* $(1,2)^*$-*semi-open sets, one containing* x *but not* y *and the other containing* y *but not* x.

Definition 2.10. [11] *A bitopological space* (X, τ_1, τ_2) *is called a*

(i) $(1,2)^*$-T_b-*space if every* $(1,2)^*$-*gs-closed set is* $\tau_{1,2}$-*closed.*

(ii) $(1,2)^*$-αT_b-*space if every* $(1,2)^*$-αg-*closed set is* $\tau_{1,2}$-*closed.*

3. $(1,2)^*$-ψ-Closed Sets

In this section, we define and study a new separation axiom by defining $(1,2)^*$-ψ-closed sets.

Definition 3.1. *A subset A of a bitopological space (X, τ_1, τ_2) is called $(1,2)^*$-ψ-closed if $(1,2)^*$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$-sg-open.*

A set A is called $(1,2)^*$-ψ-open if A^c is $(1,2)^*$-ψ-closed.

Remark 3.2. *If A is $(1,2)^*$-ψ-closed and U is $(1,2)^*$-sg-open with $A \subseteq U$, then $(1,2)^*$-scl$(A) \subseteq (1,2)^*$-sint(U).*

This follows from the definitions of $(1,2)^*$-ψ-closed set and $(1,2)^*$-sg-open set.

Lemma 3.3. *Let (X, τ_1, τ_2) be a bitopological space. If a subset A of X is $(1,2)^*$-sg-closed then it is $(1,2)^*$-semi-preclosed.*

Proof. Let $x \in (1,2)^*$-spcl(A). By Lemma 3.13 [4], $\{x\}$ is $(1,2)^*$-nowhere dense or $(1,2)^*$-preopen.
Case (i). $\{x\}$ is $(1,2)^*$-nowhere dense. Then $\{x\}$ is $(1,2)^*$-semi-closed. If $x \notin A$ then $A \subseteq (\{x\})^c$. Since A is $(1,2)^*$-sg-closed and $(\{x\})^c$ is $(1,2)^*$-semi-open, $(1,2)^*$-spcl$(A) \subseteq (1,2)^*$-scl$(A) \subseteq (\{x\})^c$, a contradiction.
Case (ii). $\{x\}$ is $(1,2)^*$-preopen.Then $\{x\}$ is $(1,2)^*$-semi-preopen.Also $x \in (1,2)^*$-spcl(A).Therefore $\{x\} \cap A \neq \phi$.This implies $x \in A$. Thus in both cases $x \in A$ and therefore $(1,2)^*$-spcl$(A) \subseteq A$. $(1,2)^*$-spcl$(A) \supseteq A$, always. □

Theorem 3.4.

(i) *Every $(1,2)^*$-semi-closed set, and thus every $\tau_{1,2}$-closed set and every $(1,2)^*$-α-closed set is $(1,2)^*$-ψ-closed.*
(ii) *Every $(1,2)^*$-ψ-closed set is $(1,2)^*$-sg-closed, and thus $(1,2)^*$-semi-preclosed and also $(1,2)^*$-gs-closed.*

Proof. (i) If A is a $(1,2)^*$-semi-closed set then $(1,2)^*$-scl$(A) = A$. Hence any $(1,2)^*$-sg-open set U containing A will also contain $(1,2)^*$-scl(A) and A is $(1,2)^*$-ψ-closed. If A is $\tau_{1,2}$-closed or $(1,2)^*$-α-closed then A is $(1,2)^*$-semi-closed and therefore $(1,2)^*$-ψ-closed.
(ii) Let A be $(1,2)^*$-ψ-closed and U, a $(1,2)^*$-semi-open set containing A. Then U is $(1,2)^*$-sg-open. Since A is $(1,2)^*$-ψ-closed, $(1,2)^*$-scl$(A) \subseteq U$. Thus, A is $(1,2)^*$-sg-closed and by Lemma 3.3, $(1,2)^*$-semi-preclosed. Since any $\tau_{1,2}$-open set is a $(1,2)^*$-semi-open set and A is $(1,2)^*$-sg-closed, from the definitions of $(1,2)^*$-sg-closed set and $(1,2)^*$-gs-closed set it follows that A is $(1,2)^*$-gs-closed. □

The following examples show that these implications are not reversible.

Example 3.5. *Let $X = \{a, b, c, d\}$; $\tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{a, c\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$; $A = \{b, c, d\}$ is $(1,2)^*$-ψ-closed but not $(1,2)^*$-semi-closed.*

Example 3.6. *Let* $X = \{a, b, c, d, e\}$*;* $\tau_1 = \{\phi, \{a, d, e\}, \{b, c\}, X\}$*;* $\tau_2 = \{\phi, \{b, c, d\}, X\}$*;* $\tau_{1,2}$*-open sets* $= \{\phi, \{a, d, e\}, \{b, c\}, \{b, c, d\}, X\}$*;* $A = \{b\}$ *is* $(1, 2)^*$*-sg-open and* $(1, 2)^*$*-sg-closed. Since* $(1, 2)^*$*-scl(A)* $= \{b, c\}$*, A is not* $(1, 2)^*$*-*ψ*-closed.*

Thus, the class of $(1, 2)^*$-ψ-closed sets properly contains the class of $(1, 2)^*$-semi-closed sets, and thus properly contains the class of $(1, 2)^*$-α-closed sets and also properly contains the class of $\tau_{1,2}$-closed sets. Also the class of $(1, 2)^*$-ψ-closed sets is properly contained in the class of $(1, 2)^*$-sg-closed sets and hence it is properly contained in the class of $(1, 2)^*$-semi-preclosed sets and contained in the class of $(1, 2)^*$-gs-closed sets.

Remark 3.7.

(i) *$(1, 2)^*$-ψ-closedness and $(1, 2)^*$-g-closedness are independent notions.*
(ii) *$(1, 2)^*$-ψ-closedness is independent from $(1, 2)^*$-gα-closedness, $(1, 2)^*$-αg-closedness and $(1, 2)^*$-preclosedness.*

This can be seen from the following examples.

Example 3.8. *Let* $X = \{a, b, c, d\}$*;* $\tau_1 = \{\phi, \{a\}, \{a, b\}, \{a, b, d\}, X\}$*;* $\tau_2 = \{\phi, \{b\}, \{a, c, d\}, X\}$*;* $\tau_{1,2}$*-open sets* $= \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, \{a, c, d\}, X\}$*;* $A = \{b, d\}$ *is* $(1, 2)^*$*-*ψ*-closed but not* $(1, 2)^*$*-g-closed.* $B = \{a, c\}$ *is* $(1, 2)^*$*-g-closed but not* $(1, 2)^*$*-*ψ*-closed.*

Example 3.9. *Let* $X = \{a, b, c, d\}$*;* $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$*;* $\tau_2 = \{\phi, \{b, c\}, X\}$*;* $\tau_{1,2}$*-open sets* $= \{\phi, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$*;* $A = \{b, c\}$ *is* $(1, 2)^*$*-*ψ*-closed but it is not* $(1, 2)^*$*-g*α*-closed, not* $(1, 2)^*$*-*α*g-closed and not* $(1, 2)^*$*-preclosed.*

Example 3.10. *Let* $X = \{a, b, c, d\}$*;* $\tau_1 = \{\phi, \{a, b\}, \{a, b, c\}, X\}$*;* $\tau_2 = \{\phi, \{a, c\}, X\}$*;* $\tau_{1,2}$*-open sets* $= \{\phi, \{a, b\}, \{a, b, c\}, \{a, c\}, X\}$*;* $A = \{a\}$ *is* $(1, 2)^*$*-preclosed but not* $(1, 2)^*$*-*ψ*-closed.* $B = \{a, b, d\}$ *is* $(1, 2)^*$*-*α*g-closed but not* $(1, 2)^*$*-*ψ*-closed.*

In Example 3.5, $B = \{b\}$ is $(1, 2)^*$-gα-closed but not $(1, 2)^*$-ψ-closed. The following theorem characterizes $(1, 2)^*$-ψ-closed sets.

Theorem 3.11. *Let A be a subset of a bitopological space* (X, τ_1, τ_2)*. Then*

(i) *A is $(1, 2)^*$-ψ-closed if and only if $(1, 2)^*$-scl(A)$-A$ does not contain any nonempty $(1, 2)^*$-sg-closed set.*
(ii) *If A is $(1, 2)^*$-ψ-closed and $A \subseteq B \subseteq (1, 2)^*$-scl(A), then B is $(1, 2)^*$-ψ-closed.*

Proof. (i) Necessity: Let A be $(1, 2)^*$-ψ-closed. Suppose F is a nonempty $(1, 2)^*$-sg-closed set contained in $(1, 2)^*$-scl$(A) - A$. Then $A \subseteq X - F$ and so $(1, 2)^*$-scl$(A) \subseteq X - F$. Hence, $F \subseteq X - (1, 2)^*$-scl(A), a contradiction.

Sufficiency: Suppose that $(1,2)^*$-scl$(A)-A$ does not contain any nonempty $(1,2)^*$-sg-closed set. Let U be a $(1,2)^*$-sg-open set such that $A \subseteq U$. If $(1,2)^*$-scl$(A) \nsubseteq U$ then $(1,2)^*$-scl$(A) \cap U^c \neq \phi$. This is a contradiction, since $(1,2)^*$-scl$(A) \cap U^c$, intersection of two $(1,2)^*$-sg-closed sets is also a $(1,2)^*$-sg-closed set [by Theorem 3.14, 4] contained in $(1,2)^*$-scl$(A) - A$.

(ii) $A \subseteq B \subseteq (1,2)^*$-scl$(A)$ imply that $(1,2)^*$-scl$(A) = (1,2)^*$-scl(B). Since A is $(1,2)^*$-ψ-closed B is also $(1,2)^*$-ψ-closed. □

Theorem 3.12. *Let A be a subset of a bitopological space (X, τ_1, τ_2). The following are equivalent:*

(i) *A is $(1,2)^*$-sg-open and $(1,2)^*$-ψ-closed.*
(ii) *A is $(1,2)^*$-semi-regular.*

Proof. (i) ⇒ (ii) By Remark 3.2, $(1,2)^*$-scl$(A) \subseteq (1,2)^*$-sint$(A) \subseteq A$. Therefore, $(1,2)^*$-scl$(A) = (1,2)^*$-sint$(A) = A$. That is A is $(1,2)^*$-semi-regular.
(ii) ⇒ (i) A is $(1,2)^*$-semi-open implies A is $(1,2)^*$-sg-open and by Theorem 3.4, A is $(1,2)^*$-semi-closed implies A is $(1,2)^*$-ψ-closed. □

Corollary 3.13. *Let A be a subset of a bitopological space (X, τ_1, τ_2). The following are equivalent:*

(i) *A is $(1,2)^*$-preopen, $(1,2)^*$-sg-open and $(1,2)^*$-ψ-closed.*
(ii) *A is $(1,2)^*$-regular open.*
(iii) *A is $(1,2)^*$-preopen, $(1,2)^*$-sg-open and $(1,2)^*$-semi-closed.*

Proof. (i) ⇒ (ii) By Theorem 3.12, A is $(1,2)^*$-semi-closed. Also since A is $(1,2)^*$-preopen, $A \supseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A)) \supseteq A$. Hence, $A = \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$ and so A is $(1,2)^*$-regular open.
(ii) ⇒ (iii) Since A is $(1,2)^*$-regular open, $A = \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$. Therefore A is $\tau_{1,2}$-open and also $(1,2)^*$-semi-closed. Hence (iii) follows.
(iii) ⇒ (i) A is $(1,2)^*$-semi-closed implies A is $(1,2)^*$-ψ-closed by Theorem 3.4. Hence (i) holds. □

Theorem 3.14. *Let A be a subset of a bitopological space (X, τ_1, τ_2). The following are equivalent:*

(i) *A is $\tau_{1,2}$-clopen.*
(ii) *A is $(1,2)^*$-preopen, $(1,2)^*$-sg-open, $(1,2)^*$-Q-set and $(1,2)^*$-ψ-closed.*

Proof. (i) ⇒ (ii) Since A is $\tau_{1,2}$-clopen, $\tau_{1,2}$-int$(\tau_{1,2}$-cl$(A)) = \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A)) = A$. Hence, A is $(1,2)^*$-preopen, $(1,2)^*$-sg-open, $(1,2)^*$-Q-set and $(1,2)^*$-semi-closed. Hence by Corollary 3.13, A is also $(1,2)^*$-ψ-closed.
(ii) ⇒ (i) By Theorem 3.12, A is $(1,2)^*$-semi-regular. Since A is $(1,2)^*$-preopen, $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$ and since A is $(1,2)^*$-semi-closed, $A \supseteq$

$\tau_{1,2}$-int($\tau_{1,2}$-cl(A)). Hence, $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) $= \tau_{1,2}$-cl($\tau_{1,2}$-int(A)) since A is a $(1,2)^*$-Q-set. It follows that A is $\tau_{1,2}$-clopen. □

Remark 3.15. *Union of two $(1,2)^*$-ψ-closed sets need not be $(1,2)^*$-ψ-closed as seen in the following example.*

Example 3.16. *Let $X = \{a,b,c,d\}$; $\tau_1 = \{\phi, \{b,c\}, X\}$; $\tau_2 = \{\phi, \{b,d\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{b,c\}, \{b,d\}, \{b,c,d\}, X\}$; $\{c\}$ and $\{d\}$ are $(1,2)^*$-ψ-closed but $\{c,d\}$ is not $(1,2)^*$-ψ-closed.*

4. $(1,2)^*$-Semi-$T_{1/3}$ Bitopological Space

Definition 4.1. *A bitopological space (X, τ_1, τ_2) is said to be a $(1,2)^*$-semi-$T_{1/3}$ space if every $(1,2)^*$-ψ-closed set is $(1,2)^*$-semi-closed.*

Theorem 4.2. *Every $(1,2)^*$-semi-$T_{1/2}$ space is a $(1,2)^*$-semi-$T_{1/3}$ space.*

Proof. Since every $(1,2)^*$-ψ-closed set is $(1,2)^*$-sg-closed, the theorem is valid. □

The converse of the above theorem is not true as it can be seen from the following example.

Example 4.3. *Let $X = \{a,b,c,d,e\}$; $\tau_1 = \{\phi, \{a,d,e\}, X\}$; $\tau_2 = \{\phi, \{b,c\}, \{b,c,d\}, \{a,b,c,e\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a,d,e\}, \{b,c\}, \{b,c,d\}, \{a,b,c,e\}, X\}$. (X, τ_1, τ_2) is not a $(1,2)^*$-semi-$T_{1/2}$ space, since $A = \{a,c,d,e\}$ is $(1,2)^*$-sg-closed but not $(1,2)^*$-semi-closed. However (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space.*

We characterize $(1,2)^*$-semi-$T_{1/3}$ space in the following theorem.

Theorem 4.4. *For a bitopological space (X, τ_1, τ_2), the following conditions are eqiuvalent:*

- (i) *(X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space.*
- (ii) *Every singleton of X is either $(1,2)^*$-sg-closed or $(1,2)^*$-semi-open.*
- (iii) *Every singleton of X is either $(1,2)^*$-sg-closed or $\tau_{1,2}$-open.*

Proof. (i) $\Rightarrow$ (ii) Let $x \in X$ and suppose that $\{x\}$ is not $(1,2)^*$-sg-closed. Then $X - \{x\}$ is not $(1,2)^*$-sg-open and so X is the only $(1,2)^*$-sg-open set containing $X - \{x\}$. Hence, $X - \{x\}$ is $(1,2)^*$-ψ-closed. Since (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space, $X - \{x\}$ is $(1,2)^*$-semi-closed or equivalently $\{x\}$ is $(1,2)^*$-semi-open.
(ii) $\Rightarrow$ (i) Let A be a $(1,2)^*$-ψ-closed set in (X, τ_1, τ_2). Let $x \in (1,2)^*$-scl(A).
Case 1. $\{x\}$ is $(1,2)^*$-sg-closed. Since $x \in (1,2)^*$-scl(A), by Theorem 3.11, $x \in A$.
Case 2. $\{x\}$ is $(1,2)^*$-semi-open. Since $x \in (1,2)^*$-scl(A), $\{x\} \cap A \neq \phi$. So

$x \in A$. Thus in any case, $(1,2)^*$-scl$(A) \subseteq A$. Therefore, $A = (1,2)^*$-scl(A) or equivalently A is a $(1,2)^*$-semi-closed set in (X, τ_1, τ_2). Hence, (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space.
(ii) $\Leftrightarrow$ (iii) follows from the fact that a singleton is $(1,2)^*$-semi-open if and only if it is $\tau_{1,2}$-open. □

Theorem 4.5. *Every $(1,2)^*$-semi-T_1 bitopological space is a $(1,2)^*$-semi-$T_{1/3}$ space but not conversely.*

Proof. Since every $(1,2)^*$-semi-T_1 bitopological space is a $(1,2)^*$-semi-$T_{1/2}$ space, the proof follows from Theorem 4.2. □

The bitopological space (X, τ_1, τ_2) in Example 4.3 is a $(1,2)^*$-semi-$T_{1/3}$ space but not even a $(1,2)^*$-semi-$T_{1/2}$ space.

Theorem 4.6. *Every $(1,2)^*$-T_b space is a $(1,2)^*$-semi-$T_{1/3}$ space but not conversely.*

Proof. Let (X, τ_1, τ_2) be a $(1,2)^*$-T_b bitopological space. First let us prove that (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/2}$ space. Let A be a $(1,2)^*$-sg-closed set in (X, τ_1, τ_2). Then A is $(1,2)^*$-gs-closed. Since (X, τ_1, τ_2) is a $(1,2)^*$-T_b space, A is $\tau_{1,2}$-closed and therefore $(1,2)^*$-semi-closed. Hence, (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/2}$ space and by Theorem 4.2, (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space. □

The bitopological space in Example 4.3 is a $(1,2)^*$-semi-$T_{1/3}$ space but not a $(1,2)^*$-T_b space since $\{e\}$ is $(1,2)^*$-gs-closed but not $\tau_{1,2}$-closed.

Theorem 4.7. *Every $(1,2)^*$-αT_b bitopological space is a $(1,2)^*$-semi-$T_{1/3}$ space but not conversely.*

Proof. Let (X, τ_1, τ_2) be a $(1,2)^*$-αT_b bitopological space. Let A be $(1,2)^*$-g-closed in (X, τ_1, τ_2). Then A is $(1,2)^*$-αg-closed in (X, τ_1, τ_2). Since (X, τ_1, τ_2) is $(1,2)^*$-αT_b, A is $\tau_{1,2}$-closed. Hence (X, τ_1, τ_2) is $(1,2)^*$-$T_{1/2}$ and therefore $(1,2)^*$-semi-$T_{1/2}$. By Theorem 4.2, (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space. □

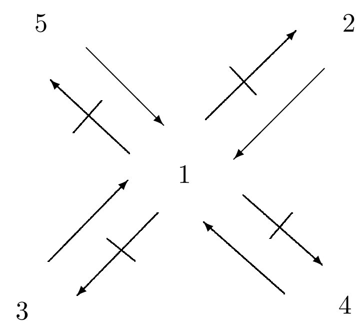
The bitopological space in Example 4.3 is a $(1,2)^*$-semi-$T_{1/3}$ space but not a $(1,2)^*$-αT_b space since $\{e\}$ is $(1,2)^*$-αg-closed but not $\tau_{1,2}$-closed.

Theorem 4.8. *If the domain of a bijective, $(1,2)^*$-pre-sg-closed and $(1,2)^*$-pre-semi-open map is a $(1,2)^*$-semi-$T_{1/3}$ space, then so is the codomain.*

Proof. Let $f\colon (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a bijective, $(1,2)^*$-pre-sg-closed and $(1,2)^*$-pre-semi-open map. Suppose (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space. Let $y \in Y$. Since f is a bijection, $y = f(x)$ for some $x \in X$. Since (X, τ_1, τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space, by Theorem 4.4, $\{x\}$ is $(1,2)^*$-sg-closed or $(1,2)^*$-semi-open. If $\{x\}$ is $(1,2)^*$-sg-closed, then $\{y\} = \{f(x)\}$ is

$(1,2)^*$-sg-closed since f is a $(1,2)^*$-pre-sg-closed map. If $\{x\}$ is $(1,2)^*$-semi-open, then $\{y\} = \{f(x)\}$ is $(1,2)^*$-semi-open since f is $(1,2)^*$-pre-semi-open map. Thus every singleton of Y is either $(1,2)^*$-sg-closed or $(1,2)^*$-semi-open in (Y, σ_1, σ_2). By Theorem 4.4, (Y, σ_1, σ_2) is also a $(1,2)^*$-semi-$T_{1/3}$ space. □

From the above results we have the following diagram, where $1 = (1,2)^*$-semi-$T_{1/3}$ space, $2 = (1,2)^*$-semi-$T_{1/2}$ space, $3 = (1,2)^*$-semi-T_1 space, $4 = (1,2)^*$-T_b space, $5 = (1,2)^*$-αT_b space.



5. $(1,2)^*$-ψ-Mappings

Definition 5.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$-ψ-continuous if $f^{-1}(V)$ is $(1,2)^*$-ψ-closed in (X, τ_1, τ_2) for every $\sigma_{1,2}$-closed set V in (Y, σ_1, σ_2).*

Theorem 5.2.

(i) *Every $(1,2)^*$-semi-continuous map and thus every $(1,2)^*$-continuous map and every $(1,2)^*$-α-continuous map is $(1,2)^*$-ψ-continuous.*

(ii) *Every $(1,2)^*$-ψ-continuous map is $(1,2)^*$-sg-continuous and thus $(1,2)^*$-sp-continuous, $(1,2)^*$-gs-continuous and $(1,2)^*$-gsp-continuous.*

Proof. (i) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$-semi-continuous map. Let V be a $\sigma_{1,2}$-closed set in (Y, σ_1, σ_2). Since f is $(1,2)^*$-semi-continuous, $f^{-1}(V)$ is $(1,2)^*$-semi-closed and so $(1,2)^*$-ψ-closed in (X, τ_1, τ_2). Therefore, f is a $(1,2)^*$-ψ-continuous map.
(ii) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$-ψ-continuous map. Let V be a $\sigma_{1,2}$-closed set in (Y, σ_1, σ_2). Since f is $(1,2)^*$-ψ-continuous, $f^{-1}(V)$ is $(1,2)^*$-ψ-closed and so by Theorem 3.4, $(1,2)^*$-sg-closed, $(1,2)^*$-gs-closed

and $(1,2)^*$-sp-closed in (X,τ_1,τ_2). Therefore f is $(1,2)^*$-sg-continuous and $(1,2)^*$-gs-continuous and $(1,2)^*$-sp-continuous. Any $(1,2)^*$-gs-closed set is $(1,2)^*$-gsp-closed since $(1,2)^*$-spcl$(A) \subseteq (1,2)^*$-scl(A). Therefore, f is $(1,2)^*$-gsp-continuous. □

The converses in Theorem 5.2 are not true as can be seen from the following examples.

Example 5.3. *Let $X = Y = \{a,b,c,d\}$; $\tau_1 = \{\phi,\{a,b\},X\}$; $\tau_2 = \{\phi,\{a,c\},X\}$; $\tau_{1,2}$-open sets $= \{\phi,\{a,b\},\{a,c\},\{a,b,c\},X\} = \sigma_{1,2}$-open sets. Define $f\colon (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $f(a) = a, f(b) = f(c) = f(d) = d$. $f^{-1}(\{d\}) = \{b,c,d\}$ is not $(1,2)^*$-semi-closed and therefore f is not $(1,2)^*$-semi-continuous. However f is $(1,2)^*$-ψ-continuous.*

Example 5.4. *Let $X = \{a,b,c,d,e\} = Y$; $\tau_1 = \{\phi,\{a,d,e\},\{b,c\},X\}$; $\tau_2 = \{\phi,\{b,c,d\},X\}$; $\tau_{1,2}$-open sets $= \{\phi,\{a,d,e\},\{b,c\},\{b,c,d\},X\} = \sigma_{1,2}$-open sets. Define $f\colon (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $f(a) = e, f(b) = b, f(c) = a, f(d) = d, f(e) = a$. $f^{-1}(\{b,c\}) = \{b\}$ is not $(1,2)^*$-ψ-closed and therefore, f is not $(1,2)^*$-ψ-continuous. However, f is $(1,2)^*$-sg-continuous.*

Thus the class of $(1,2)^*$-ψ-continuous maps properly contains the class of $(1,2)^*$-semi-continuous maps and thus it contains the class of $(1,2)^*$-continuous maps and the class of $(1,2)^*$-α-continuous maps. Also the class of $(1,2)^*$-ψ-continuous maps is properly contained in the class of $(1,2)^*$-sg-continuous maps and hence it is contained in the class of $(1,2)^*$-sp-continuous maps, $(1,2)^*$-gs-continuous maps and $(1,2)^*$-gsp-continuous maps.

Theorem 5.5.

(i) *$(1,2)^*$-ψ-continuity and $(1,2)^*$-g-continuity are independent of each other.*

(ii) *$(1,2)^*$-ψ-continuity is independent of $(1,2)^*$-αg-continuity, $(1,2)^*$-$g\alpha$-continuity and $(1,2)^*$-pre-continuity.*

Proof. 1. Let $X = Y = \{a,b,c,d\}$; $\tau_1 = \{\phi,\{a\},\{a,c,d\},X\}$; $\tau_2 = \{\phi,\{b\},\{a,b,d\},X\}$; $\tau_{1,2}$-open sets $= \{\phi,\{a\},\{b\},\{a,b\},\{a,c,d\},\{a,b,d\},X\} = \sigma_{1,2}$-open sets. Define $f\colon (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $f(a) = a, f(b) = c, f(c) = d, f(d) = c$. $f^{-1}(\{c\}) = \{b,d\}$ is not $(1,2)^*$-g-closed and therefore, f is not $(1,2)^*$-g-continuous. However, f is $(1,2)^*$-ψ-continuous. Let $Z = \{a,b,c,d\}$; $\nu_1 = \{\phi,\{a,c\},Z\}$; $\nu_2 = \{\phi,\{b,c\},Z\}$; $\nu_{1,2}$-open sets $= \{\phi,\{a,c\},\{b,c\},\{a,b,c\},Z\}$. Define $g\colon (X,\tau_1,\tau_2) \to (Z,\nu_1,\nu_2)$ by $g(a) = d, g(b) = a, g(c) = d, g(d) = b$. $g^{-1}(\{a,d\}) = \{a,b,c\}$ is not $(1,2)^*$-ψ-closed and therefore, g is not $(1,2)^*$-ψ-continuous. However, g is $(1,2)^*$-g-continuous.

2. $X = Y = \{a, b, c, d\}$; $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b, c\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ $\sigma_1 = \{\phi, \{a, b, c\}, X\}$; $\sigma_2 = \{\phi, \{b, c, d\}, X\}$; $\sigma_{1,2}$-open sets $= \{\phi, \{a, b, c\}, \{b, c, d\}, X\}$. Define $f\colon (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = c, f(b) = d, f(c) = d, f(d) = a$. $f^{-1}(\{d\}) = \{b, c\}$ is $(1,2)^*$-ψ-closed but not $(1,2)^*$-αg-closed, not $(1,2)^*$-$g\alpha$-closed and not $(1,2)^*$-preclosed and therefore, f is not $(1,2)^*$-αg-continuous, not $(1,2)^*$-$g\alpha$-continuous and not $(1,2)^*$-pre-continuous. However, f is $(1,2)^*$-ψ-continuous.

Let $X = \{a, b, c, d, e\} = Y$; $\tau_1 = \{\phi, \{a, d, e\}, X\}$; $\tau_2 = \{\phi, \{b, c\}, \{b, c, d\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a, d, e\}, \{b, c\}, \{b, c, d\}, X\} = \sigma_{1,2}$-open sets. Define $f\colon (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = e, f(b) = b, f(c) = a, f(d) = d, f(e) = a$. $f^{-1}(\{b, c\}) = \{b\}$ is not $(1,2)^*$-ψ-closed and therefore, f is not $(1,2)^*$-ψ-continuous. However, f is $(1,2)^*$-pre-continuous, $(1,2)^*$-αg-continuous and $(1,2)^*$-$g\alpha$-continuous. □

The composition of two $(1,2)^*$-ψ-continuous maps need not be $(1,2)^*$-ψ-continuous as can be seen from the following example.

Example 5.6. *$X = Y = \{a, b, c, d\}$; $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b, c\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$; $\sigma_1 = \{\phi, \{a, b, c\}, X\}$; $\sigma_2 = \{\phi, \{b, c, d\}, X\}$; $\sigma_{1,2}$-open sets $= \{\phi, \{a, b, c\}, \{b, c, d\}, X\}$. Let $f\colon (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map.*

Let $Z = \{a, b, c, d\}$; $\nu_1 = \{\phi, \{a, c, d\}, X\}$; $\nu_2 = \{\phi, \{a, b, d\}, X\}$; $\nu_{1,2}$-open sets $= \{\phi, \{a, c, d\}, \{a, b, d\}, X\}$. Define $g\colon (Y, \sigma_1, \sigma_2) \to (Z, \nu_1, \nu_2)$ by $g(a) = b, g(b) = b, g(c) = a, g(d) = b$. f and g are $(1,2)^*$-ψ-continuous but $g \circ f\colon (X, \tau_1, \tau_2) \to (Z, \nu_1, \nu_2)$ given by $g(a) = b, g(b) = b, g(c) = a, g(d) = b$ is not $(1,2)^*$-ψ-continuous since $(g \circ f)^{-1}(\{b\}) = \{a, b, d\}$ is not $(1,2)^*$-ψ-closed in (X, τ_1, τ_2).

We introduce the following definition.

Definition 5.7. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$-ψ-irresolute if $f^{-1}(V)$ is $(1,2)^*$-ψ-closed in (X, τ_1, τ_2) for every $(1,2)^*$-ψ-closed set V of (Y, σ_1, σ_2).*

Clearly every $(1,2)^*$-ψ-irresolute map is $(1,2)^*$-ψ-continuous. The converse is not true as can be seen from the following example.

Example 5.8. *$X = Y = \{a, b, c, d\}$; $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b, c\}, X\}$; $\tau_{1,2}$-open sets $= \{\phi, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$; $\sigma_1 = \{\phi, \{a, b, c\}, X\}$; $\sigma_2 = \{\phi, \{b, c, d\}, X\}$; $\sigma_{1,2}$-open sets $= \{\phi, \{a, b, c\}, \{b, c, d\}, X\}$. Define $f\colon (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. f is $(1,2)^*$-ψ-continuous but not $(1,2)^*$-ψ-irresolute.*

The following theorem can easily be proved.

Theorem 5.9. *Let $f\colon (X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g\colon (Y,\sigma_1,\sigma_2)\to(Z,\nu_1,\nu_2)$ be any two functions. Then*

(i) *$g\circ f\colon (X,\tau_1,\tau_2)\to(Z,\nu_1,\nu_2)$ is $(1,2)^*$-ψ-continuous if g is $(1,2)^*$-continuous and f is $(1,2)^*$-ψ-continuous.*

(ii) *$g\circ f\colon (X,\tau_1,\tau_2)\to(Z,\nu_1,\nu_2)$ is $(1,2)^*$-ψ-irresolute if g is $(1,2)^*$-ψ-irresolute and f is $(1,2)^*$-ψ-irresolute.*

(iii) *$g\circ f\colon (X,\tau_1,\tau_2)\to(Z,\nu_1,\nu_2)$ is $(1,2)^*$-ψ-continuous if g is $(1,2)^*$-ψ-continuous and f is $(1,2)^*$-ψ-irresolute.*

Theorem 5.10. *Let $f\colon (X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a bijective $(1,2)^*$-ψ-irresolute map. If (X,τ_1,τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space, then f is a $(1,2)^*$-irresolute map.*

Proof. Let V be a $(1,2)^*$-semi-open set in (Y,σ_1,σ_2). Then V^c is a $(1,2)^*$-semi-closed set and therefore, $(1,2)^*$-ψ-closed set in (Y,σ_1,σ_2). Since f is a $(1,2)^*$-ψ-irresolute map, $f^{-1}(V^c)$ is a $(1,2)^*$-ψ-closed set in (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space, $f^{-1}(V^c)$ is a $(1,2)^*$-semi-closed set in (X,τ_1,τ_2). Since f is a bijection, $f^{-1}(V)=(f^{-1}(V^c))^c$. Thus, $f^{-1}(V)$ is a $(1,2)^*$-semi-open set in (X,τ_1,τ_2). Therefore, f is a $(1,2)^*$-irresolute map. □

Theorem 5.11. *Let $f\colon (X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a surjective $(1,2)^*$-sg-irresolute and a $(1,2)^*$-pre-semi-closed map. Then for every $(1,2)^*$-ψ-closed subset A of (X,τ_1,τ_2), $f(A)$ is a $(1,2)^*$-ψ-closed subset of (Y,σ_1,σ_2).*

Proof. Let A be a $(1,2)^*$-ψ-closed set in (X,τ_1,τ_2) and U be a $(1,2)^*$-sg-open set in (Y,σ_1,σ_2) such that $f(A)\subseteq U$. Since f is a surjective $(1,2)^*$-sg-irresolute map, $f^{-1}(U)$ is a $(1,2)^*$-sg-open set in (X,τ_1,τ_2). Then $(1,2)^*$-scl$(A)\subseteq f^{-1}(U)$ since A is a $(1,2)^*$-ψ-closed set and $A\subseteq f^{-1}(U)$. This implies $f((1,2)^*\text{-scl}(A))\subseteq U$. Now $(1,2)^*$-scl$(f(A))\subseteq(1,2)^*$-scl$(f((1,2)^*$-scl$(A)))=f((1,2)^*$-scl$(A))$ since f is a $(1,2)^*$-pre-semi-closed map. Thus, $(1,2)^*$-scl$(f(A))\subseteq U$ and therefore, $f(A)$ is a $(1,2)^*$-ψ-closed subset of (Y,σ_1,σ_2). □

Theorem 5.12. *Let $f\colon (X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a surjective $(1,2)^*$-ψ-irresolute and $(1,2)^*$-pre-semi-closed map. If (X,τ_1,τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space then (Y,σ_1,σ_2) is also a $(1,2)^*$-semi-$T_{1/3}$ space.*

Proof. Let A be a $(1,2)^*$-ψ-closed subset of (Y,σ_1,σ_2). Since f is a $(1,2)^*$-ψ-irresolute map, $f^{-1}(A)$ is a $(1,2)^*$-ψ-closed subset of (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space, $f^{-1}(A)$ is a $(1,2)^*$-semi-closed set in (X,τ_1,τ_2). Then $f(f^{-1}(A))$ is $(1,2)^*$-semi-closed in (Y,σ_1,σ_2) since f is a $(1,2)^*$-pre-semi-closed map. Since f is a surjection, $A=f(f^{-1}(A))$. Thus, A is $(1,2)^*$-semi-closed in (Y,σ_1,σ_2) and therefore (Y,σ_1,σ_2) is a $(1,2)^*$-semi-$T_{1/3}$ space. □

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