WEAKLY CONNECTED SUBSETS

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Abstract. A generalization of the notion of connectedness of subsets in a topological space is given. Several properties of this type of subsets (also those being similar to fundamental results of the theory of connected subsets) are obtained.

Introduction. Over the last thirty years several concepts of connectedness have been considered. In 1975, Pipitone and Russo [13] introduced the so-called *semi-connected spaces*. Many characterizations of such spaces have been given in [4, 5, 10, 12, 17, 20]. *Preconnectedness* of topological spaces has been defined by Popa in 1987 [14]. Furthermore, in 1994, Popa with Noiri and Aho with Nieminen introduced, respectively, *β-preconnectedness* [15] and *semi-preconnectedness* [2] of spaces (both notions are equivalent). The author has investigated some other forms of connectedness of spaces in [5]. In all the above listed definitions, 'connectedness' was described in terms of some classes of generalized open subsets of a space (α-open, semi-open, preopen, semi-preopen sets). For a more detailed study of various forms of connectedness, the reader is referred to [18, 19]. In [6, 10, 12] one can find some properties of so-called *β-connected* and *hyperconnected* subsets.

The present paper offers another extension for the classical meaning of connectedness (of subsets in topological spaces) that is defined without making use of any other class of subsets of the space. We note in advance that this form of connectedness is most applicable in spaces which allow a non-trivial clopen set.

2. Characterizations. Let (X, τ) denote a topological space.

Definition 1. A subset $S \subset X$ is said to be *weakly connected* in (X, τ) if there are no two clopen subsets X_1, X_2 in (X, τ) such that $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$, and $X_1 \cap S \neq \emptyset \neq S \cap X_2$.

In a standard manner, a subset $S \subset X$ will be called *weakly disconnected* if it is not weakly connected. If S is weakly disconnected in (X, τ), then S is disconnected in (X, τ). The reverse implication can fail in general.

Example 1. Let $X = \{a, b, c\}$ and $\tau = \big\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\big\}$. Consider $S = \{a, b\}$.

Every singleton set is weakly connected in any (X, τ). In the antidiscrete space each subset is weakly connected. In the discrete space each subset S with cardinality greater than 1 is weakly disconnected. In the space of reals (with the usual topology) each subset is weakly connected.

More generally, if (X,τ) is connected, then each subset of X is weakly connected in (X,τ).

Remark 1. For disconnected spaces (X,τ_1), (X,τ_2) with $\tau_1 \subset \tau_2$, and for any S weakly connected in (X,τ_2), S is weakly connected in (X,τ_1).

The following theorem does not require a proof.

Theorem 1. A subset S of a disconnected space (X,τ) is weakly connected if and only if for each pair of subsets X_1, X_2 of X with $\mathrm{int}(X_i) = X_i = \mathrm{cl}(X_i)$, $i = 1,2$; $X_1 \cap X_2 = \emptyset$; and $X = X_1 \cup X_2$; we have $S \subset X_1$ or $S \subset X_2$.

Theorem 2. In an arbitrary disconnected space (X,τ), the following statements are equivalent.

(a) $S \subset X$ is weakly connected.
(b) for each non-empty clopen subset A of X we have $S \subset A$ or $S \subset X \setminus A$.
(c) for each non-empty subset A of X with $\mathrm{Fr}(A) = \emptyset$, $S \subset A$ or $S \subset X \setminus A$.

Proof. (a)$\Rightarrow$(b). Suppose there exists a non-empty clopen subset A of (X,τ) such that $S \not\subset A$ and $S \not\subset X \setminus A = B$. This implies that $S = (S \cap A) \cup (S \cap B)$, where $S \cap A \neq \emptyset \neq S \cap B$, $X = A \cup B$, $A \cap B = \emptyset$, and A, B are both clopen in (X,τ). Thus, S is weakly disconnected in (X,τ).
(b)$\Rightarrow$(c). Obvious.
(c)$\Rightarrow$(a). Let A, B be two clopen subsets of (X,τ) such that $X = A \cup B$, $A \cap B = \emptyset$, and $A \cap S \neq \emptyset \neq B \cap S$. Thus, there exists a non-void subset A with $\mathrm{Fr}(A) = \emptyset$, $S \not\subset A$, and $S \not\subset X \setminus A$. This completes the proof.

Recall that subsets A, B of X are called *separated* in (X,τ) if $B \cap \mathrm{cl}(A) = \emptyset = A \cap \mathrm{cl}(B)$.

Theorem 3. A subset S of a disconnected space (X,τ) is weakly connected in this space if and only if the following condition holds.

$$\text{For every pair } A, B \text{ of non-empty separated subsets of } X \text{ such that } X = A \cup B \text{ we have } S \subset A \text{ or } S \subset B. \tag{1}$$

Proof. ($\Rightarrow$). Suppose S is weakly connected and let non-empty $A, B \subset X$ be such that $X = A \cup B$ and $A \cap S \neq \emptyset \neq B \cap S$. The sets A and B may not be closed simultaneously (by weak connectedness of S). So, one of them is not closed, say A. Thus, there exists an $x_0 \in \mathrm{cl}(A) \setminus A$ and hence, $x_0 \in B$. This shows that $x_0 \in B \cap \mathrm{cl}(A) = \emptyset$, a contradiction.
($\Leftarrow$). Assume that condition (1) holds, but S is not weakly connected in (X,τ). Hence, there exist clopen subsets X_1, X_2 of (X,τ) with $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$, $S \not\subset X_1$, and $S \not\subset X_2$. Since X_1, X_2 are closed, they are separated. So, by (1), we get $S \subset X_1$ or $S \subset X_2$, a contradiction.

3. Properties. The example below shows that a subset $S \subset X_0 \subset X$ can be weakly connected in X, but weakly disconnected in X_0.

Example 2. Let $X = \{a, b, c, d\}$ and $\tau = \big\{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\big\}$. Consider $S = \{a, b\}$ and $X_0 = \{a, b, c\}$.

Observe that if we put above $S = \{c, d\}$ and $X_0 = \{b, c, d\}$, S would be weakly connected both in X and X_0.

Theorem 4. Let a set $S \subset X_0$ be weakly connected in a subspace X_0 of a disconnected (X, τ). Then, S is weakly connected in (X, τ).

Proof. Clear.

Recall that a topological space is called *totally* (or *globally*) *disconnected* if each open subset is closed. It is worthwhile to observe that there exist finite totally disconnected spaces which are neither discrete nor antidiscrete.

Example 3.

(1) Let $X = \{a, b, c, d\}$ and $\tau_1 = \big\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\big\}$.
(2) Let X be as above and $\tau_2 = \big\{\emptyset, X, \{a, b\}, \{c, d\}\big\}$.

Theorem 5. Let an $S \subset X_0 \subset X$ be weakly connected in (X, τ). If (X, τ) is totally disconnected, then S is weakly connected in X_0.

Proof. Suppose S is weakly disconnected in $\big(X_0, \tau_{X_0}\big)$. Hence, there exist $X_1^0, X_2^0 \in \tau_{X_0}$ such that $X_0 = X_1^0 \cup X_2^0$, $X_1^0 \cap X_2^0 = \emptyset$, and $X_1^0 \cap S \neq \emptyset \neq X_2^0 \cap S$. We have $X_i^0 = X_0 \cap G_i$ for certain $G_i \in \tau$, $i = 1, 2$. Put $G = G_1 \cup G_2$. By hypothesis, $G_3 = X \setminus G \in \tau$. So, $\big\{G_1 \setminus (G_1 \cap G_2), G_2 \cup G_3\big\}$ is a partition of (X, τ) and $\big(G_1 \setminus (G_1 \cap G_2)\big) \cap S \neq \emptyset \neq (G_2 \cup G_3) \cap S$. Therefore, S is weakly disconnected in (X, τ).

Corollary 1. Let (X, τ) be totally disconnected and $S \subset X_0 \subset X$. Then, S is weakly connected in X if and only if it is weakly connected in X_0.

The following is easily observed. Let (X, τ) be disconnected and $S \subset X_0 \subset X$; if S is connected in X, then S is weakly connected in X_0 (since S is connected as a subspace).

Theorem 6. Let (X, τ) be disconnected and $S \subset X_0 \subset X$, and X_0 be clopen and disconected in X. If S is weakly connected in X, then it is weakly connected in X_0.

Proof. Suppose S is weakly disconnected in X_0. Then, there exist subsets $A_0, B_0 \in \tau_{X_0}$ with $X_0 = A_0 \cup B_0$, $A_0 \cap B_0 = \emptyset$, and $A_0 \cap S \neq \emptyset \neq S \cap B_0$, where $A_0 = X_0 \cap A$, $B_0 = X_0 \cap B$ for some $A, B \in \tau$. By hypothesis, $A_0, B_0 \in \tau$, $X = A_0 \cup \big(B_0 \cup (X \setminus X_0)\big)$, where $X \setminus X_0 \in \tau$. So, S is weakly disconnected in X.

Theorem 7. Let subsets $M, N \subset X$ be non-empty and separated in a disconnected space (X, τ), and let a set $S \subset X_0 = M \cup N$ be weakly connected in X_0. Then, $S \subset M$ or $S \subset N$.

Proof. Since M and N are separated in X_0, by hypothesis, and with Theorem 3, we obtain that $S \subset M$ or $S \subset N$.

Theorem 8. A topological space (X, τ) is connected if and only if for each pair of distinct points from X, there exists a weakly connected subset containing these points.

Proof. $(\Rightarrow)$ Obvious. $(\Leftarrow)$ Suppose the sufficiency condition holds but the space (X, τ) is disconnected. For if, for certain nonempty $X_1, X_2 \in \tau$ we would have $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$. Let $x_1 \in X_1$, $x_2 \in X_2$, and let $x_1, x_2 \in S$ be weakly connected in X. Our supposition leads to a contradiction.

Theorem 9. Let $A \subset X$ be dense and weakly connected in (X, τ). Then the space (X, τ) is connected.

Proof. Suppose (X, τ) is disconnected. Thus, $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$ for certain non-empty $X_1, X_2 \in \tau$. Clearly, we have $X_1 \cap A \neq \emptyset \neq X_2 \cap A$. This proves that A is not weakly connected in (X, τ), a contradiction.

Corollary 2. Let an $S \subset X$ be weakly connected in a totally disconnected space (X, τ) and let $S \subset X_0 \subset \mathrm{cl}_X(S)$. Then, X_0 is connected in X.

Proof. Observe that $\mathrm{cl}_{X_0}(S) = X_0 \cap \mathrm{cl}_X(S) = X_0$, i.e., S is dense in $\left(X_0, \tau_{X_0}\right)$. By Theorem 5, S is weakly connected in X_0. Therefore, by Theorem 9, X_0 is a connected subspace of X.

The following property is obvious.

Theorem 10. Let (X, τ) be a disconnected space and let $S_1 \subset S_2 \subset X$. If S_2 is weakly connected in X, then S_1 is weakly connected in X.

Corollary 3. Let $\{S_i\}_{i \in \mathcal{I}}$ be a family of subsets of disconnected (X, τ) and let, for some $i_0 \in \mathcal{I}$, S_{i_0} be weakly connected in X. Then, $\bigcap_{i \in \mathcal{I}} S_i$ is weakly connected in X.

Corollary 4. Let Y_1, Y_2, U be such subsets of a disconnected (X, τ) that $Y_1 \subset Y_2$ and $U \subset Y_2 \setminus Y_1$. If Y_2 is weakly connected in X, then the union $Y_1 \cup U$ is weakly connected in X.

Theorem 11. Let S_1, S_2 be weakly connected in a disconnected space (X, τ) and let $S_1 \cap S_2 \neq \emptyset$. Then $S_1 \cup S_2$ is weakly connected.

Proof. Suppose $S = S_1 \cup S_2$ is weakly disconnected, i.e., for certain clopen X_1, X_2 such that $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$, we have $X_1 \cap S \neq \emptyset \neq X_2 \cap S$. By Theorem 1 and our hypothesis, $S_i \subset X_1$ or $S_i \subset X_2$, $i = 1, 2$.

Since $S_1 \cap S_2 \neq \emptyset$, then $S_1 \cup S_2 \subset X_1$ or $S_1 \cup S_2 \subset X_2$. Thus, $S \cap X_1 = \emptyset$ or $S \cap X_2 = \emptyset$, a contradiction.

Corollary 5. Let $\{S_i\}_{i\in\mathbb{N}}$ be a family of weakly connected subsets of (X,τ). If $\bigcap_{i\in\mathbb{N}} S_i \neq \emptyset$, then $S_{i_1} \cup \cdots \cup S_{i_N}$ is weakly connected for any $i_1,\ldots,i_N \in \mathbb{N}$.

Proof. By induction (with respect to $i \in \mathbb{N}$) it is clear that every union $S_1 \cup \cdots \cup S_i$, $i \in \mathbb{N}$ is weakly connected in (X,τ) (apply Theorem 11). Now, put $i = \max\{i_1,\ldots,i_N\} + 1$ and use Theorem 10.

It is well-known that the family $\{S_i : i = 1,\ldots,n\}$ of sets is called a *chain joining* S_1 and S_n, if $S_i \cap S_{i+1} \neq \emptyset$ for each $i = 1,2,\ldots,n-1$. A family $\mathcal{F}$ is said to be a *chaining family* if for any two $S_1, S_2 \in \mathcal{F}$ there exists a chain of subsets from $\mathcal{F}$ joining S_1 and S_2.

Theorem 12. If $\{S_1, S_2, \ldots, S_n\}$ is a chain of weakly connected subsets in a disconnected (X,τ) joining S_1 and S_n, then the union $\bigcup_{i=1}^n S_i$ is weakly connected in X.

Proof. Induction with respect to n and Theorem 11.

Theorem 13. Let $\mathcal{F}$ be a chaining family of weakly connected subsets of (X,τ). If $\mathcal{F}$ is a cover of X, then X is connected.

Proof. Let $x_1, x_2 \in X$, $x_1 \neq x_2$, be arbitrarily chosen and let $S_1, S_2 \in \mathcal{F}$ be such that $x_1 \in S_1$, $x_2 \in S_2$. There exists a chain $\mathcal{F}' \subset \mathcal{F}$ joining S_1 and S_2. So, $\bigcup \mathcal{F}'$ is weakly connected (Theorem 12) and contains x_1, x_2. Thus, by Theorem 8, the space (X,τ) is connected.

Theorem 14. Let S be a subset of (X,τ). If S is weakly connected, then $\mathrm{cl}(S)$ is weakly connected.

Proof. Let $\mathrm{cl}(S)$ be weakly disconnected in X. Then, for certain clopen X_1, X_2 with $X = X_1 \cup X_2$, $X_1 \cap X_2 = \emptyset$, we have $X_1 \cap \mathrm{cl}(S) \neq \emptyset \neq X_2 \cap \mathrm{cl}(S)$. But, $\emptyset \neq X_i \cap \mathrm{cl}(S) \subset \mathrm{cl}(X_i \cap S)$, $i = 1,2$. Hence, $X_1 \cap S \neq \emptyset \neq X_2 \cap S$ and S is weakly disconnected.

Thus, by Theorem 10, we have the following corollary.

Corollary 6. A subset S of a disconnected space (X,τ) is weakly connected if and only if $\mathrm{cl}(S)$ is weakly connected.

By Theorems 10 and 14, one obtains the following corollary.

Corollary 7. A subset S of a disconnected space (X,τ) is weakly connected in X if and only if each $S_1 \subset X$ such that $S \subset S_1 \subset \mathrm{cl}(S)$ is weakly connected.

Obviously, if S is weakly connected in disconnected (X,τ), then every $S_1 \subset \mathrm{cl}(S)$ is also weakly connected (compare Corollary 4).

Theorem 15. Let $\{S_i\}_{i\in\mathcal{I}}$ be a family of subsets of disconnected (X,τ). If $\bigcup_{i\in\mathcal{I}} S_i \subset S$ and S is weakly connected, then $\bigcup_{i\in\mathcal{I}} \mathrm{cl}(S_i)$ is weakly connected.

Proof. From Theorem 12 we have that $\mathrm{cl}(S)$ is weakly connected in X. So, the inclusions $\bigcup_{i\in\mathcal{I}} \mathrm{cl}(S_i) \subset \mathrm{cl}\big(\bigcup_{i\in\mathcal{I}} S_i\big) \subset \mathrm{cl}(S)$ and Theorem 10 imply that $\bigcup_{i\in\mathcal{I}} \mathrm{cl}(S_i)$ is weakly connected in X.

Directly from Theorems 10 and 14, we get the following corollary.

Corollary 8. If an S is weakly connected in a disconnected (X,τ), then $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(S)))$ is also weakly connected.

Theorem 16. Let S_1, S_2 be two weakly connected sets in a disconnected (X,τ) and let $S_1\cap S_2 \neq \emptyset$. Then, $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(S_1)))\cup \mathrm{cl}(\mathrm{int}(\mathrm{cl}(S_2)))$ is weakly connected.

Proof. By hypothesis, $\mathrm{cl}(S_1)\cap\mathrm{cl}(S_2)\neq\emptyset$. From Theorem 14 we obtain that $\mathrm{cl}(S_1), \mathrm{cl}(S_2)$ are weakly connected. Hence, by Theorem 11, we get that $\mathrm{cl}(S_1)\cup\mathrm{cl}(S_2)$ is weakly connected. So, by Theorems 10 and 14, we obtain that

$$\mathrm{cl}\big(\mathrm{int}\big(\mathrm{cl}(S_1)\cup\mathrm{cl}(S_2)\big)\big) = \mathrm{cl}(\mathrm{int}(\mathrm{cl}(S_1)))\cup\mathrm{cl}(\mathrm{int}(\mathrm{cl}(S_2)))$$

is weakly connected.

The following corollaries are immediate consequences of Theorems 10 and 14.

Corollary 9. If an S is weakly connected in a disconnected (X,τ), then the sets $\mathrm{cl}(\mathrm{int}(S))$, $\mathrm{int}(\mathrm{cl}(S))$, and $\mathrm{int}(\mathrm{cl}(\mathrm{int}(S)))$ are also weakly connected in X.

Recall that a subset S of (X,τ) is said to be semi-open [7] (resp. α-open [9]; preopen [8]; semi-preopen [1, 3]) if $S\subset\mathrm{cl}(\mathrm{int}(S))$ (resp. $S\subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(S)))$; $S\subset\mathrm{int}(\mathrm{cl}(S))$; $S\subset\mathrm{cl}(\mathrm{int}(\mathrm{cl}(S)))$).

Corollary 10. Let (X,τ) be a disconnected space and let $S\subset X$. If $\mathrm{int}(S)$ is weakly connected and S is semi-open, then S is weakly connected.

From Corollaries 8 and 9 and Theorem 10 we get the following.

Corollary 11. Let S be a semi-open (resp. α-open; preopen; semi-preopen) subset of a disconnected (X,τ). Then S is weakly connected in X if and only if $\mathrm{cl}(\mathrm{int}(S))$ (resp. $\mathrm{int}(\mathrm{cl}(\mathrm{int}(S)))$; $\mathrm{int}(\mathrm{cl}(S))$; $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(S)))$) is weakly connected in X.

Definition 2. Let an (X,τ) be disconnected and let a $p\in X$. A subset S_p of X is called a *w-component* of p, if $p\in S_p$, S_p is weakly connected in X, and it is a proper subset of no weakly connected set in X.

Theorem 17. Let a space (X,τ) be disconnected. Then,

(1) there exists a subset $X' \subset X$ such that

$$X = \bigcup_{p \in X'} S_p,$$

where sets S_p are w-components of respective p's and $S_{p_1} \cap S_{p_2} = \emptyset$ if $p_1 \neq p_2$;

(2) for each point $p \in X$ its w-component S_p is closed in (X, τ).

Proof. (1) and (2) hold for any discrete space.

(1) Let (X, τ) be a disconnected and not discrete space such that there is no subset $X' \subset X$ with $X = \bigcup_{p \in X'} S_p$ and $S_{p_1} \cap S_{p_2} = \emptyset$ for $p_1 \neq p_2$ ($p_1, p_2 \in X'$). Thus, for any subset $X'' \subset X$ such that $X = \bigcup_{p \in X''} S_p$, there exists a pair of distinct points $p_1, p_2 \in X''$ such that $S_{p_1} \cap S_{p_2} \neq \emptyset$ and $S_{p_1} \neq S_{p_2}$. Obviously, $S_{p_1} \not\subset S_{p_2}$ and $S_{p_2} \not\subset S_{p_1}$. Let $q \in S_{p_2} \setminus S_{p_1}$. The set $S_{p_1} \cup \{q\}$ is weakly disconnected. It follows from the inclusion $S_{p_1} \cup \{q\} \subset S_{p_1} \cup S_{p_2}$ that $S_{p_1} \cup S_{p_2}$ is weakly disconnected (Theorem 10). On the other hand, with Theorem 11, we obtain that $S_{p_1} \cup S_{p_2}$ is weakly connected, a contradiction.

(2) The proof is obvious by maximality of each S_p (Definition 2) and by Theorem 14.

Problem 1. Is it true that in any disconnected space, w-components are connected?

Applying Theorem 3 we obtain the following.

Corollary 12. Suppose two points p_1, p_2 belong to the same w-component S_q in a disconnected (X, τ). If $X = A \cup B$, where A, B are non-empty and separated, both p_1 and p_2 must belong either to A or to B.

The family of all α-open subsets of (X, τ) one usually denotes with τ^α. This family forms a topology on X such that $\tau \subset \tau^\alpha$ [9]. The reverse inclusion is false, in general.

Theorem 18. Let S be a subset of a disconnected (X, τ). Then, S is weakly disconnected in (X, τ) if and only if it is weakly disconnected in (X, τ^α).

Proof. Clear, because the family of all clopen subsets of X with respect to τ coincides with the family of all clopen subsets with respect to τ^α.

A function $f \colon (X, \tau) \to (Y, \sigma)$ is said to be *α-continuous* [11, 16] if the preimage $f^{-1}(S) \in \tau^\alpha$ for every $S \in \sigma$. Each continuous function is α-continuous, but it is known that the converse can fail in general [11, Example 2.3].

Theorem 19. Let $(X,\tau),(Y,\sigma)$ be disconnected and an $f\colon(X,\tau)\to(Y,\sigma)$ be α-continuous. If an S is weakly connected in X, then $f(S)$ is weakly connected in Y.

Proof. Let $f(S)$ be weakly disconnected in Y. Then, for certain clopen (in Y) sets Y_1,Y_2 we have $Y=Y_1\cup Y_2$, $Y_1\cap Y_2=\emptyset$, and $Y_1\cap f(S)\neq\emptyset\neq Y_2\cap f(S)$. Hence, $X=f^{-1}(Y)=f^{-1}(Y_1)\cup f^{-1}(Y_2)$, $f^{-1}(Y_1)\cap f^{-1}(Y_2)=\emptyset$, and $f^{-1}(Y_1)\cap S\neq\emptyset\neq f^{-1}(Y_2)\cap S$. Since $f^{-1}(Y_1),f^{-1}(Y_2)\in\tau^\alpha$, the set S is weakly disconnected in X by Theorem 18.

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