PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than May 15, 2004, although solutions received after that date will also be considered until the time when a solution is published.

145. *Proposed by José Luis Diaz-Barrero, Universidad Politécnica de Cataluña, Barcelona, Spain.*

Let F_n denote the nth Fibonacci number ($F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$) and let L_n denote the nth Lucas number ($L_0 = 2$, $L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$). Prove that

$$F_{n+1} > \frac{1}{3}\left(\frac{L_n^{L_n}}{F_n^{F_n}}\right)^{\frac{1}{L_n - F_n}}$$

holds for all positive integer $n \geq 2$.

146. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.*

Find all (x, y) with $0 \leq x < 2\pi$ and $0 \leq y < 2\pi$ such that

$$\cos^2 y = 2(\sin x + \cos x \cos y - 1).$$

147. *Proposed by Zdravko F. Starc, Vršac, Serbia and Montenegro.*

Let F_n be the Fibonacci numbers defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that for $n \geq 1$

$$(F_1^4 + F_{n+1}^4)(F_2^4 + F_{n+2}^4) \cdots (F_n^4 + F_{2n}^4) < \left(\frac{2^{4n-1}}{n}\right)^{2n}.$$

148. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Show that

$$\prod_{i=1}^{\infty}\left(\frac{\cos\frac{x}{4^i} + \cos\frac{3x}{4^i}}{2}\right) = \prod_{i=1}^{\infty}\left(\frac{1 + 2\cos\frac{2x}{5^i} + 2\cos\frac{4x}{5^i}}{5}\right),$$

where x is any real or complex number.