PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than August 1, 1996, although solutions received after that date will also be considered until the time when a solution is published.

85. *Proposed by Robert E. Kennedy and Curtis Cooper, Central Missouri State University, Warrensburg, Missouri.*

A positive integer n is said to be a Niven number if it is divisible by its digital sum. Let A be the set of Niven numbers and $A(x)$ be the number of Niven numbers not exceeding x. Prove or disprove the following statement.

For every integer $m \geq 1$, there exists a positive integer n such that

$$\frac{n}{A(n)} = m.$$

86. *Proposed by Alan H. Rapoport, M.D., Ashford Medical Center, Santurce, Puerto Rico.*

An urn contains 20,000 balls consisting of 500 balls of each of 40 different colors. 100 balls consisting of exactly $1 \leq d \leq 40$ colors are selected at random without replacement from the urn. Let $P(d)$ denote the probability that the 100 balls consist of exactly d colors. Find d_0 such that

$$P(d_0) = \max_{1 \leq d \leq 40} P(d).$$

87*. *Proposed by James H. Taylor, Central Missouri State University, Warrensburg, Missouri.*

Let n be a nonnegative integer and $0 \le k \le n$. Let $J(\lambda)$ be the $(n+1)\times(n+1)$ matrix

$$\begin{pmatrix}
-\lambda & 1 & 0 & 0 & 0 & . & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\
n & -\lambda & 2 & 0 & 0 & . & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\
\lambda & n-2 & -\lambda & 3 & 0 & . & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\
-n & \lambda & n-4 & -\lambda & 4 & . & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\
0 & -n+1 & \lambda & n-6 & -\lambda & . & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & . & . & . & . & . & -\lambda & n-3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & . & . & . & . & . & 8-n & -\lambda & n-2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & . & . & . & . & . & \lambda & 6-n & -\lambda & n-1 & 0 \\
0 & 0 & 0 & 0 & 0 & . & . & . & . & . & -4 & \lambda & 4-n & -\lambda & n \\
0 & 0 & 0 & 0 & 0 & . & . & . & . & . & 0 & -3 & \lambda & 2-n & -\lambda
\end{pmatrix}.$$

Also, let v be the coefficient vector of the polynomial

$$(x+1)^{n-k}(x-1)^k.$$

Show that $J(n-2k)v = 0$.

88. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let m be a positive integer. Prove that

$$\prod_{i=1}^{m} \cos^2 \frac{i\pi}{2m+1} = \frac{1}{4^m}.$$