PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 1, 1996, although solutions received after that date will also be considered until the time when a solution is published.

Comment by the editor. Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin pointed out that the statement of Problem 73 was incorrect. A corrected version of the problem is printed below.

73. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let F_n and L_n denote the nth Fibonacci and Lucas numbers, respectively. Consider a right triangle such that the diameter of its circumcircle equals F_n and for its leg a, $a^2 = (L_{2n-1} + (-1)^n)/5$. Let p and q denote the segments formed on the hypotenuse by the footpoint of the height and let K denote the area of the triangle.

(a) Prove that the measures of p, q, and c (the hypotenuse) are Fibonacci numbers.

(b) Prove that the squares of a, b, and $2K$ are products of Fibonacci numbers.

77. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let $P(k,n)$ denote the nth polygonal number of k "dimensions." For example, $P(3,5)$ denotes the 5th triangular number and $P(4,2)$ denotes the second square number.

(a) Find all polygonal numbers (that is, all k values) for which

$$4P(k,n)P(k,n+2)+1$$

is a square number for all n.

(b) Find all polygonal numbers for which

$$P(k,n)P(k,n+2)+1$$

is a square number for all n.

78. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let $P(k,n)$ denote the nth polygonal number of k "dimensions." For example, $P(3,5)$ denotes the 5th triangular number and $P(4,2)$ denotes the second square number.

Let $k \geq 3$, $n \geq 1$ and let $A(k,n)$ denote the 3rd order determinant such that for $1 \leq r, s \leq 3$,

$$a_{r,s} = P(k+r-1, n+s-1).$$

Prove or disprove that for all $k \geq 3$ and $n \geq 1$, $A(k,n) = 0$.

79. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let a_i denote the leading digit of 2^i. Is the sequence $\{a_i\}_{i=0}^{\infty}$ eventually periodic?

80. *Proposed by Larry Hoehn, Austin Peay State University, Clarksville, Tennessee.*

Show that

$$x^n + y^n = z^{n+1}$$

has a non-trivial integral solution for $n > 2$.