PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than August 31, 1995, although solutions received after that date will also be considered until the time when a solution is published.

73. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let F_n and L_n denote the nth Fibonacci and Lucas numbers, respectively. Consider a right triangle such that the diameter of its circumcircle equals F_n and for its leg a, $a^2 = (L_{2n-1} + (-1)^n)/5$. Let p and q denote the segments formed on the hypotenuse by the footpoint of the height and let K denote the area of the triangle.

(a) Prove that the measures of p, q, and r (the inradius) are Fibonacci numbers.

(b) Prove that the squares of a, b, and $2K$ are products of Fibonacci numbers.

74. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let $s(n)$ denote the digital sum (base 10) of the positive integer n. Prove that if n is divisible by 41, then $s(n) \geq 5$.

75. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

"Prove that $n(n+1)$ is never a square for $n > 0$" is a problem in *Elementary Number Theory* by Underwood Dudley. Generalize this problem by showing that $n(n+1) \neq t^k$ for t an integer and $k \geq 2$.

76. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let the hypotenuse c of a right triangle ABC equal G^3 (where $G = (\sqrt{5}+1)/2$, the golden ratio) and $\overline{CF}$ (where F is the footpoint of the height on the hypotenuse) equals $\sqrt{c}$. Obtain the length of the shorter leg of triangle ABC and prove or disprove that its length equals $\max(\overline{AF}, \overline{FB})$.