PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (∗) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than November 30, 1994, although solutions received after that date will also be considered until the time when a solution is published.

65. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Evaluate

$$\sum_{k=0}^{n} \left| \binom{n}{k} - 2^k \right|.$$

66. *Proposed by Alvin Beltramo (student), Central Missouri State University, Warrensburg, Missouri.*

Consider the following generalization of the car and the goats problem. A TV host shows you d doors, a car is hidden behind w doors and the rest of the doors are hiding goats. You get to pick a door, winning whatever is behind it. The host, who knows where the cars are, then opens s doors, in the process revealing x cars. The host invites you to switch your choice if you so wish. When should you switch?

67. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

Show that one more than four times the product of two consecutive even or odd numbered triangular numbers is a square.

68. *Proposed by Joseph B. Dence, University of Missouri - St. Louis, St. Louis, Missouri.*

Let the generalized Fibonacci sequence $\{U_n(P,Q)\}_{n=0}^{\infty}$ be defined by the Binet formula

$$U_n(P,Q) = \frac{\alpha^n - \beta^n}{\alpha - \beta},$$

where $P, Q \in \mathbb{Z}$, $P^2 - 4Q > 0$, and α and β are the unequal roots of $x^2 - Px + Q = 0$. For a particular case denote $U_n(2,-1)$ by P_n, the sequence of Pell numbers. Now consider the series of reciprocals:

$$S = \sum_{k=0}^{\infty} P_{2^k}^{-1}.$$

Show that S is irrational by finding its value explicitly.