PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to the problem editor, whose address appears on the inside back cover. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 31, 1992, although solutions received after that date will also be considered until the time when a solution is published.

33. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

If A, B and C are the angles of a triangle, prove that

$$\cot A + \cot B + \cot C \geq \sqrt{3}\ .$$

Under what conditions will equality hold?

34. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let m and k be positive integers and $1 \leq k \leq m$. Evaluate

$$\sum_{\substack{n_1+2n_2+\cdots+m\cdot n_m=m \\ n_1+n_2+\cdots+n_m=k}} \frac{m!}{(1!)^{n_1}n_1!(2!)^{n_2}n_2!\cdots(m!)^{n_m}n_m!}\ ,$$

where n_1, n_2, …, n_m are non-negative integers.

35. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Let F_n denote the nth Fibonacci number ($F_1 = F_2 = 1$ and $F_n = F_{n-2} + F_{n-1}$ for $n \geq 3$) and let L_n denote the nth Lucas number ($L_1 = 1$, $L_2 = 3$ and $L_n = L_{n-2} + L_{n-1}$ for $n \geq 3$). Express L_n^2 as a polynomial in F_n.

36. *Proposed by James Taylor, Central Missouri State University, Warrensburg, Missouri.*

Show the following relation between an elliptic integral of the third kind with a modulus of a special form and elliptic integrals of the first and third kind with a simpler modulus.

$$\prod(\alpha^2, \frac{2\sqrt{l}}{1+l}) = \operatorname{sgn}\left([1-\alpha^2]\left[(1-\alpha^2)^2 - k'^2\right]\right)\frac{(1+k')\sqrt{(1-\alpha_1^2)(k_1^2-\alpha_1^2)}}{(k^2-\alpha^2)}\prod(\alpha_1^2, k_1)$$

$$+\frac{k^2(1+k_1)}{2(k^2-\alpha^2)}K(k_1)$$

where

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} \ ,$$

$$\prod(n,k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{(1+n\sin^2\phi)\sqrt{1-k^2\sin^2\phi}} \ ,$$

and

$$k = \frac{2\sqrt{l}}{1+l} \ , \ k' = \sqrt{1-k^2} \ , \ k_1 = \frac{1-k'}{1+k'} \ ,$$

$$\alpha^2 = \frac{(1+k')^2}{2}\left[k_1 + \alpha_1^2 - \sqrt{(1-\alpha_1^2)(k_1^2-\alpha_1^2)}\right] .$$