BEGINNING ALGEBRA: ARITHMETIC IN DISGUISE

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I come to you with good news! Your students need not fear Beginning Algebra! It is only Arithmetic in Disguise. By this I mean that it is the rules of arithmetic in symbolic form which are applied to the solution of verbal problems. These problems are, of course, arithmetic problems.

Since it is possible that you may not believe this, I have three arguments that will help to convince you. I will also give four conclusions that I deduce from it. But before I do this, let me put my remarks in perspective.

Algebra can be seen from different perspectives. Its meaning depends on your professional goals when you study it and has taken on different meanings in certain historical periods.

To the beginner it is a maze to be traversed and from which he hopefully will gain some useful knowledge. To the non-mathematics professional it is a tool to be mastered and used occasionally. To the mathematician it is an essential part of his training that has strong connections with Topology, Analysis, and Applied Mathematics.

Similar observations may be made about the historical development of the discipline. In its earlier stages, it is little more than an arithmetic devise for solving practical problems. In the Renaissance, it is developed for its own sake. Its practitioners of this period solved the cubic and quartic equations in terms of radicals. This will lead their followers to ask if the same can be accomplished for those of degree five and higher and will ultimately help pave the way for the blooming of Abstract Algebra in the nineteenth and twentieth centuries. In the seventeenth and eighteenth centuries it is a tool to be used in the development of Analysis which will be used to solve many problems in Physics.

I wish to look at the subject through the eyes of the beginner which historically means that we need only concern ourselves with the earliest history of Algebra. This point of view will be defended in terms of the content of the course, the historical development of the subject, and on pedagogical grounds.

First, I shall discuss the content of the course. It deals with problems that lead either to linear equations and inequalities in one and two variables and problems that can be solved by quadratic equations solved either by factoring or the quadratic formula. In each of the topics treated in this course there is a cycle. The cycle consists of mastering arithmetic skills in symbolic form, applying these skills to the solution of equations or inequalities, and finally the solution of the verbal problems. Sometimes the cycle is not completed, as is the case in solving systems of linear inequalities in two variables. The reason for this is that the natural application occurs in connection with the graphical solution of linear programming problems which, understandably, are regarded as beyond the scope of the course.

The first two parts of this cycle are algorithmic. They involve the processes of arithmetic and include the arithmetic of signed numbers, collecting terms, multiplying and dividing polynomials, factoring polynomials, and the manipulation of formulas. The verbal problems are all arithmetic problems and they fall into fixed categories (in order to simplify matters for the student). Standard examples are problems that, with the appropriate vocabulary, are translations from verbal to symbolic form, interest problems, mixture problems, distance rate time problems, work problems, and geometry problems. In the translation problems the student must know about integers and fractions. In the remaining problems the student either makes direct use of a formula or does simple model building, as in the case of mixture, distance rate time, and work problems. As teachers, we supply the necessary assumptions to complete the models for our students.

Algebra has a long history, most of which is not relevant to this discussion. Its earliest use occurs with the Babylonians, Chinese, and Greeks (Euclid and Diophantus). The father of algebra in its present form is al-Kwhowarizmi and his famous book, *Al–jabr wa′l Majabalah*. His algebra is rhetorical, which means that it is entirely verbal as opposed to Diophantus syncopated, or partially symbolic form. The aim of his work is stated as follows:

> . . . compose a short work on Calculating by (the rules of) Completion and Reduction confining it to what is easiest, and most useful in arithmetic such as men constantly require in cases of inheritance, legacies, partitions, law suits, and trade, and in all their dealing with one another or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts

and kinds are concerned.

By Completion and Reduction he means transposition and multiplicative cancellation respectively. The problems to which he refers are to be solved by linear or quadratic equations. This is close to the content of the course outlined above with some difference in verbal problems. It was not until the time of Descartes that we find modern algebraic symbols.

Rectangular coordinates occur when we solve linear systems of equations or inequalities in two variables. They are due to Nicholas Oresme, not Descartes. Oresme was concerned with the graphing of functions.

Finally, note that the cycle becomes a learning spiral in the classroom. The problems solved by a single linear equation are encountered a second time using the two variable technique. The same can be said of problems solved by quadratic equations using factoring and later by the formula. The student now sees Beginning Algebra as a vibrant and useful tool with relevance to his life.

Since Beginning Algebra is Arithmetic in Disguise, the following conclusions can be drawn concerning how it is taught:

1. Verbal problems are central to the course. They are its reason for being and must be taught at the risk of frustrating our students and ourselves and at the expense of angering parents and school administrators.

2. Axiomatics are neither desirable or necessary to accomplish this goal. It is not necessary to know laws that go by the names of commutative, associative, or distributive to gain understanding in solving these verbal problems. This vocabulary is excess baggage and certainly does not aid in the student's comprehension of number. I do not wish to be misunderstood. The student must understand that the distributive property is used when multiplying polynomials, since he will use it repeatedly. The concepts of additive and multiplicative inverse are also important. What is not necessary is that these concepts be presented as part of an axiomatic scheme. Teaching the students to recognize the various properties is not a central part of the course and is a waste of your time and the student's.

3. Concepts concerning number should be kept to a minimum. Of course, the student must know the meaning of the terms

integer, fraction, and square root. The concept of irrational numbers is not meaningful at this point of the student's career and should be avoided. The terminology should be introduced as it is needed and not all at once.

4. The language of sets should be abolished from the subject. It is not necessary to use this language to discuss solutions of equations or inequalities. In fact, the concept of set and set operations adds a higher level of abstraction than is required for solving the verbal problems in the course. It adds an unnecessary vocabulary and in no way clarifies the subject.

Thus, the statement, "Beginning Algebra is Arithmetic in Disguise", is full of meaning for all of us that teach it. Beginning Algebra is a practical subject of great value.

References

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2. M. L. Lial and C. D. Miller, *Beginning Algebra*, Fifth Edition, Scott, Foresman and Company, Glenview, Illinois, 1988.