Parallel Displacements in abstract Space.

By

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§ 1. Introduction.

Vector analysis hitherto studied seems to have been limited to those in the n-dimensional space or a functional space only, as far as I know. Here, going beyond such limitation, we intend to investigate the vector analysis in general topological spaces, including as special case all the vector-analysis hitherto considered. Namely, the space is composed of topological base space and vector space. As the first step, in this paper we shall put the definition for the vectors in our space by means of the transformations of the elements of the base space and thus attempt to investigate the general vector analysis.

§ 2. Notations used.

$\mathfrak{M}$ .... base space,
$\mathfrak{f}, \mathfrak{g}$, ... the elements of $\mathfrak{M}$,
$\mathfrak{K}, \mathfrak{L}$ ..... a vector space,
$\mathfrak{K}_P, \mathfrak{K}_Q$ ... vector spaces induced by operators $P, Q, \ldots$. respectively,
$V, W, \ldots$. elements of $\mathfrak{K}_Q$,
$v, w, \ldots$.. elements of $\mathfrak{K}_P$,
a, b, ... numbers,
operator: $H, P, Q, \ldots$. the correspondence between the elements of two different spaces,
Operator H belongs to a set $\mathfrak{H}$,
transformation: $T_1, T_2 \ldots$. the correspondences between the elements of the base space and belong to a group $\mathfrak{G}$,
transformation: $T'_{P,\mathfrak{f}} \ldots \bar{T}_{P,\mathfrak{f}} \ldots$. the correspondences between the elements in the same vector spaces,
$\mathfrak{A}$........ the totality of functions $f(t)$,
$\mathfrak{A}'$....... the totality of functions $\bar{f}(t)$,
$g(E)$ ..... the totality of images of $g(t)$ in $\mathfrak{M}$.

§ 3. Vector space.

In this section let $\mathfrak{M}$ be an abstract set and $\mathfrak{R}$ a linear space.

Generalizing from the definition of the vectors hitherto adopted, we shall introduce vectors in general space. For this purpose the following two methods may be considered.

1). Let $\mathfrak{H}$ be a set of certain operators which forms a linear manifold. Here we say that a set forms a linear manifold when

$H\mathfrak{f}$ forms a subspace $\mathfrak{R}_{\mathfrak{f}}$ of $\mathfrak{R}$ when H runs over $\mathfrak{H}$ for fixed $\mathfrak{f}$ and for $H_1, H_2 \in \mathfrak{R}$ $H_1+H_2 \to H_3$
where

$$(aH_1+bH_2)\mathfrak{f} \equiv aH_1\mathfrak{f}+bH_2\mathfrak{f}$$

If $H_1\mathfrak{f} = H_2\mathfrak{f}$ when $H_1T\mathfrak{f} = H_2T\mathfrak{f}$ for all T and H's, then there exists a transformation T'_f in $\mathfrak{R}$, for which the domain and range are $\mathfrak{R}_{\mathfrak{f}}$ and $\mathfrak{R}_{T\mathfrak{f}}$ respectively, satisfying the following relation

$$HT\mathfrak{f} = T'_{\mathfrak{f}}H\mathfrak{f}$$

In this case we can easily see that T and $T'_{\mathfrak{f}}$ are isomorphic to one another, the term "isomorphy" means that if

$$T_2T_1 = T_3 \qquad \text{then} \qquad T'_{2T_1\mathfrak{f}}T'_{1\mathfrak{f}} = T'_{3\mathfrak{f}}.$$

Especially when $T_{\mathfrak{f}}$ belongs to a definite transformation group of $\mathfrak{R}$, we call $H\mathfrak{f}$ or H a vector at $\mathfrak{f}$ in $\mathfrak{M}$.

2). In this case we put the following assumptions:

(1) $\mathfrak{M}$ has neighbourhoods at each element containing the element itself[(1)]

(2) In $\mathfrak{M}$ there exists a function $g(t)$[(2)] having the properties:

(a) The domain of $g(t)$ $[0, 1]$ contains 0 and is everywhere dense in the interval $(0, 1)$ and its range belongs to $\mathfrak{M}$,

(b) $g(t)$ is a continuous representation of $[0, 1]$.

We call such a range of $g(t)$ a "*pseud curve*" and we shall proceed our theory by means of these curves.

(3) $\mathfrak{R}$ is linear and has neighbourhoods at 0 containing itself and having the following properties:[(3)]

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(1) It can be put instead of the definition of neighbourhoods to define the definition of limiting points.

(2) For simplicity's sake, we will treat only the case when $f(t)$ is one parameter.

(3) The assumption (a), (b) of (3) are necessary only to explain the condition of linearity of $\overline{T}_{P,f(0)}$. (See p. 32)

(a) For small a, $aU_i(0) \subset U_i(0)$.

(b) Given $U_i(0)$, there is $U_j(0)$ such that $U_j(0) \subset U_i(0)$ and the sum of any two elements of $U_j(0)$ belongs to $U_i(0)$.

(4) $\mathfrak{G}$ is a topological transformation group.

We introduce the following notations:

P .... an operator whose domain is $\mathfrak{A}$ and range is $\mathfrak{R}_P$ in $\mathfrak{R}$,

$P[g(t) \mid t_1]$.... the image of a pseud-curve $g(t)$ in a domain $[0, t_1]$,

$\mathfrak{R}_{P, g(0)}$ .... the totality of ranges $P[f(t) \mid t_1]$ of pseud-curves having a common element at $t = 0$,

$\bar{\mathfrak{R}}_{P,f(0)}$ .... $\mathfrak{R}_{P,f(0)}$ with the transformation induced by T,

$\bar{f}(t)$.......... the pseud-curve in which $\lim_{t_1=0} \frac{P[\bar{f}(t) \mid t_1]}{t_1}$ exists,

$\mathfrak{R}'_{P,f(0)}$ .... the totality of the ranges $P[\bar{f}(t) \mid t_1]$ of the pseud-curves $\bar{f}(t)$'s having a common element at $t = 0$,

$v_{f(t)}$ .... $v_{f(t)} = \lim_{t_1=0} \frac{P[\bar{f}(t) \mid t_1]}{t_1}$.

From these assumptions we can conclude that: *For any T, when*

$$P[Tf_1(t) \mid t_1] = P[Tf_2(t) \mid t_2]^{(1)}$$

follows from the relation

$$P[f_1(t) \mid t_1] = P[f_2(t) \mid t_2]$$

providing that $f_1(0) = f_2(0)$, there always exists the transformation $T'_{P,f(0)}$ so that

$$P[Tf(t) \mid t_1] = T'_{P,f(0)} P[f(t) \mid t_1],$$

and $T'_{P,f(0)}$ is isomorphic (generally multiply) to T, the domain and range being $\mathfrak{R}_{P,f(0)}$ and $\mathfrak{R}_{P, Tf(0)}$ respectively.

Thus we can introduce the transformation $T'_{P,f(0)}$ in $\mathfrak{R}_{P,f(0)}$ from the transformation T in $\mathfrak{M}$ isomorphically.

Now let $\underset{P,f(0)}{T'(t)}$ be a transformation in $\mathfrak{R}_{P,f(0)}$ for a fixed t and, in the domain of t, it coincides with $T'_{P,f(0)}$ induced from T in the same domain of t. Further if there exists a transformation $\bar{T}_{P,f(0)}$ with

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(1) $Tf(t)$ express the pseud-curve composing of all element (in $\mathfrak{M}$) which correspond to any element of $f(E)$ by T.

the domain $\mathfrak{R}_{P,f(0)}$ or its sub-domain including the $\mathfrak{R}'_{P,f}$ and the range $\mathfrak{R}'_{P,Tf}$ such that for given $U_i(0)$ we can find t_i for which

$$\frac{\overline{T}_{P,f(0)}P[f(t)\mid t]-T'_{P,f(0)}P[f(t)\mid t]}{t}\in U_i(0)\quad\text{for all}\quad t\text{ in }[0,1]<t_i,$$

then we call it the "vector" at $f(0)$ *the quantity of* $\overline{\mathfrak{R}}_{P,f(0)}$ *which undergoes* $\overline{T}_{P,f(0)}$ *for a transformation* T *of the base space* $\mathfrak{M}$[1].

Such a transformation $\overline{T}_{P,\mathfrak{f}}$ always exists, because we can take $T'_{P,\mathfrak{f}}$ itself as $\overline{T}_{P,\mathfrak{f}}$ and we may easily find many of such tranformation $\overline{T}_{P,\mathfrak{f}}$ other than this $T'_{P,\mathfrak{f}}$. But it is desirable for $\overline{T}_{P,\mathfrak{f}}$ to be a unique transformation determined from T and to have a definite character such as linear contineous or projective and so on.

Especially, when $\mathfrak{R}'_{P,\mathfrak{f}}$ is everywhere dense in $\mathfrak{R}_{P,\mathfrak{f}}$, we will find the condition for which there exists the linear continuous transformation $\overline{T}_{P,\mathfrak{f}}$.

The condition is easily obtained as follows:

(1) *for any sequence of curves* $\{\bar{f}_i(t)\}$ *the existence of* $\lim_{i=\infty} v_{Tf_i(t)}$ *follows the existence of* $\lim_{i=\infty} v_{f_i(t)}$,

(2) *if* $v_{f_1(t)}=av_{f_2(t)}$ *then* $v_{Tf_1(t)}=av_{Tf_2(t)}$

(3) *if* $v_{f_3(t)}=v_{f_1(t)}+v_{f_2(t)}$ *then for any given* $U_i(0)$ *there exists* t_i *such that*

$$\frac{T'_{P,f(0)}\big(P[f_1(t)\mid t]+P[f_2(t)\mid t]\big)-T'_{P,f(0)}P[f_1(t)\mid t]-T'_{P,f(0)}P[f_2(t)\mid t]}{t}\in U_i(0)$$

for all $t<t_i$ in the domain.

In fact, in the case when $\overline{T}_{P,f}$ is linear, the vetor space $\overline{\mathfrak{R}}_{P,f}$ can be said to be the "tangential space at 0" in $\mathfrak{R}_P$; therefore the pseud-curve $g'(t)$ (if exists) corresponding to $P[g'(t)\mid t_2]=t_2\lim_{t_1=0}\frac{P[\bar{g}(t)\mid t_1]}{t_1}$ (t_2 varies in E) may be called the "*tangent*" to the curve $\bar{g}(t)$.

An example of vectors.

In the base space $\mathfrak{M}$ when the difference (not necessarily belonging

(1) The definition of vectors above mentioned includes the case of ordidary $p-$vectors and also that of the vectors depending not only of the quantities $\frac{dx^i}{dt}$ but of $\frac{d^mx^i}{dt^m}(m=1,2,\dots.)$ even when the base space is ordinary n-dimensional.

to the base space) of any two elements is defined "im kleinen", an operator P and a vector transformation $\overline{T}_{P,\mathfrak{f}}$ can be simply defined for a given transformation T as follows

$$P[f(t) \mid t_1] = f(t_1) - f(0) \equiv a(t_1) \qquad (\text{for} \quad t_1 \text{ in } [0, t] < t_0),$$

$$\overline{T}_{P,f(0)} = \lim_{t_1=0} T'_{P,f(0)}(t_1),$$

where

$$P[Tf(t) \mid t_1] = Tf(t_1) - Tf(0) \equiv a'(t_1) \qquad (\text{for} \quad t_1 \text{ in } [0, t] < t_0),$$

$$a'(t_1) = T'_{P,f(0)} a(t_1);$$

this definition of vector includes that of the ordinary case when $(dx^i) = a$, $(dx'^i) = a'$ and $(x'^i) = T(x^i)$

Remark. A). Relation between the definitions of vector **1)** and **2)**.

If $\overline{T}_{P,\mathfrak{f}} = T'_{P,\mathfrak{f}}$ in **2)**, then **2)** is included in **1)** when and only when there exists $f_3(t)$ for all given $f_1(t)$ and $f_2(t)$ providing $f_1(0) = f_2(0)$, such that

$$aP[f_1(t) \mid t_1] + bP[f_2(t) \mid t_2] = P[f_3(t) \mid t_3];$$

for, it is enough to take $P[f(t) \mid t_1]$ fixing $f(t)$ and t_1 as $H_{f(0)}$ in **1)**.

If $\mathfrak{A} \equiv \mathfrak{A}'$ and $\overline{T}_{P,\mathfrak{f}}$ is linear and we take $\overline{T}_{P,\mathfrak{f}}$ as the transformation of vectar space $\overline{\mathfrak{K}}_{P,f}$, then **2)** is included in **1)** when and only when there exists $f_3(t)$, such that

$$av_{f_1(t)} + bv_{f_2(t)} = v_{f_3(t)};$$

for, it is enough to take $v_{f(t)}$ fixing $\bar{f}_{(t)}$ and $\overline{\mathfrak{K}}_{P,f(0)}$ as $H_{f(0)}$ and $\mathfrak{H}_{\bar{f}(0)}$ in **1)** respectively.

B). Both in the cases **1)** and **2)**, the vector transformation group induced from a given group $\mathfrak{G}$ may differ from each other according as the different $\mathfrak{H}$'s in **1)** and P and $[0, 1]$ interval of t in **2)** differ but such different groups can be proved to be multiply isomorphic to each other.

§ 4. Derivatives.

Let by $\mathfrak{M}$ and P express the same space and operator as defined in § 3, and $\mathfrak{L}$ be a linear space satifying (a), (b) of (3) in § 3. And $V(\mathfrak{f})$ is a function whose domain is a sub-set $\mathfrak{M}_V$ of $\mathfrak{M}$ and whose range belongs to $\mathfrak{L}$ respectively.

As $\bar{f}(t)$ Consider a pseud-curve $\bar{f}(t)$ in § 3 having the following properties:

(1) $\bar{f}(0) \in \mathfrak{M}_V$,

(2) if $\bar{t}$ is a parameter taking the values of $\bar{t}$ which gives the commmon elements of $\bar{f}(t)$ and $\mathfrak{M}_V$, then 0 becomes a condensation point of $\bar{t}$.

If for a given $U_i(0)$ (in $\mathfrak{L}$) and $\bar{t}_i$ which is taken for $U_i(0)$ in a definite way, there exists an operator $D_{V,v,f(0)}$ independent of U_i and $\bar{t}_i$ such that

$$\frac{V(\bar{f}(\bar{t})) - V(\bar{f}(0))}{\bar{t}} - D_{V,v,f(0)} v_{\bar{f}(t)} \in U_i(0)^{(1)} \qquad \text{for all} \qquad \bar{t} < \bar{t}_i, \quad (4),$$

we call the operator $D_{V,v,f(0)}$ *"the derivative of* $V(\mathfrak{f})$*" at* $f(0)$ *with respect to* $v_{f(t)}$.

We can easily see $D_{V,v}$ *is also an element of a linear manifold having the same character* (a), (b) *of* (3) *in* § 3 *as in the case of* $V(\mathfrak{f})$.[2]

If $\mathfrak{A}_{P,f(t)}$ is an aggregate of $\bar{f}(t)$'s which have the common element at $t = 0$ and also the same vector $v_{f(t)}$, then for any two elements $\bar{f}_1(t)$, $\bar{f}_2(t)$ of $\mathfrak{A}_{P,f_1(t)}$ (4) can be replaced by the following condition:

$$\begin{cases} \lim \dfrac{V(\bar{f}_1(\bar{t})) - V(\bar{f}(0))}{\bar{t}} \text{ exists (in the same mean as above)} \\ \text{and} \\ \lim \dfrac{V(\bar{f}_1(\bar{t})) - V(\bar{f}_2(\bar{t}))}{\bar{t}} = 0.^{(3)} \end{cases} \quad (5)$$

The derivative depends of course not merely $V(f)$ but on the operator P, and we see that the derivatives of $V(f)$ with respect to the vectors induced by P, R have the same domain in $\mathfrak{M}$ when and only when

$$\mathfrak{A}_{P,f(t)} \equiv \mathfrak{A}_{R,f(t)}.$$

(1) In the case $f(t) \notin \mathfrak{A}'$ it seems to me to be possible to define derivatives by putting $\dfrac{P[g(t) \mid t_1]}{t_1}$ instead of $v_{g(t)}$ in (4).

(2) Such properties will be treated in latter papers precisely.

(3) It is natural that we got the second condition of (5) because each in $\mathfrak{A}_{P,g(t)}$ is in the relation something like in contact with each other, and in fact this is true in the case when contact of curves $\bar{g}(t)$'s is defined.

Some properties of derivatives.

From the definition of derivatives, we know easily the following properties of derivatives.

For the derivatives $D_{V,v}$, $D_{W,v}$ *of two vector fields* V, W, *we have the relation*

$$D_{V+W,v} = D_{V,v}+D_{W,v}\,.$$

But it must not be in general that

$$D_{V,v}v = D_{V,v_1}v_1+D_{V,v_2}v_2 \tag{6}$$

where

$$v = v_1+v_2\,.$$

The necessary and sufficient condition that (6) *may hold is that for a given* $U_i(0)$ *(in* $\mathfrak{L}$*),* $\bar{t}_j$ *exists such that if*

$$v_{f_3(t)} = v_{f_1(t)}+v_{f_2(t)}$$

then

$$\frac{V(\bar{f_3}(\bar{t}))-V(\bar{f_1}(\bar{t}))-V(\bar{f_2}(\bar{t}))+V(\bar{f_3}(0))}{\bar{t}} \in U_j(0)$$

$$\text{for all} \qquad \bar{t}<\bar{t}_j\,.$$

§ 5. Parallel displacements of vectors.

In this section, we consider an operator P having the same properties as Q in § 3, and take $\mathfrak{K}_Q$ as $\mathfrak{L}$ in § 4; for the other quantities we adopt the same notations as in § 4.

Let $C[\bar{f}(t) \mid \mathfrak{f}_2, \mathfrak{f}_1]$ be a given correspondence between the elements of the tangential space $\bar{\mathfrak{K}}_{Q,\mathfrak{f}_2}$, $\bar{\mathfrak{K}}_{Q,\mathfrak{f}_1}$ induced by an operator Q at any elements $\mathfrak{f}_2$, $\mathfrak{f}_1$ of $\mathfrak{M}$ depending on a pseud-curve $f(t)$ joining $\mathfrak{f}_1$ to $\mathfrak{f}_2$. And for $C[f(t) \mid \mathfrak{f}_2, \mathfrak{f}_1]$[1] we put the following assumption:

(1) Since such a one-parameter grop differs as the pseud-curve (the path), to consider the parallelism in the space is to consider a set of different one-parameter groups corresponding to all paths in space.

$$C[f(t)\,|\mathfrak{f}_2,\mathfrak{f}_1] = C[g(t)\mid \mathfrak{f}_2,\mathfrak{f}]C(h(t)\mid \mathfrak{f},\mathfrak{f}_1]$$

where

$$\mathfrak{f}\in f(E) \qquad \text{and} \qquad f(E) = g(E)\dot{+}h(E)\,, \qquad \mathfrak{f}_1\in h(E)\,, \qquad \mathfrak{f}_2\in g(E)\,.$$

Two elements corresponding by such $C[f(t)\mid \mathfrak{f}_2,\mathfrak{f}_1]$ are said to be parallel to each other along the pseud-curve $f(t)$, or in this case we say that one element displaces parallelly to the other along the pseud-curve.

In this case $C[f(t)\mid \mathfrak{f}_2,\mathfrak{f}_1]$ can be expressed as an operational product-integral of $c[f'(t)\mid \mathfrak{f}_2',\mathfrak{f}_1']$ along the pseud-curve

$$C = \widehat{\int_{f(t)}} c\, d\, f(E)$$

where $c[f'(t)\mid \mathfrak{f}_2'\,\mathfrak{f}_1']$ is an operator expressing the parallelism of vectors at two elements in a neighbourhood of an element in $\mathfrak{M}$ and $f'(t)$ is a part of $f(t)$ between the two elements $\mathfrak{f}_1'$, $\mathfrak{f}_2'$ on $f(t)$. For this reason we consider for a while the infinitesimal parallel displacement of vectors by $c[g(t)\mid \mathfrak{g}_2,\mathfrak{g}_1]$.

Let be a vector V at $\mathfrak{g}$ and be parallel to V a vector $\overline{V}$ at $\mathfrak{g}'$ in the neighbourhood of $\mathfrak{g}$, then

$$\overline{V} = cV\,.$$

Since all the elements of $\overline{\mathfrak{R}}_{Q,\mathfrak{g}}$ and $\overline{\mathfrak{R}}_{Q,\mathfrak{g}'}$ are originally those of $\mathfrak{R}$ (but differ only by their own transformations), c is regarded as a transformation in $\mathfrak{R}$.

If c is continuous and $\lim\limits_{f(E)=0} c = 1$, then we have for a vector field $V(\mathfrak{g})$ having the derivatives the following relation:

$$\overline{V}(\mathfrak{g}) = cV(\mathfrak{g}') = \big(I+F[\mathfrak{g}(t)\mid \mathfrak{g},\mathfrak{g}']\big)(V+D_{V v_{g(t)}}vt+\cdots\cdots)$$

or

$$\overline{V}(\mathfrak{g}) = V(\mathfrak{g})+D_{V,\,v_{g(t)}}vt+F[g(t)\mid \mathfrak{g},\mathfrak{g}']V+\cdots\cdots$$

Further, if $c[g(t) \mid \mathfrak{g}, \mathfrak{g}']$ is differentiable with respect to $v_{g(t)}$[1] at $g(0)$ i.e.

$$\lim_{t\to 0} \frac{c[g(t) \mid \mathfrak{g}, \mathfrak{g}'] - c[g(0) \mid \mathfrak{g}, \mathfrak{g}]}{t} = \Gamma_v v_{g(t)}$$

then we have

$$\begin{aligned} \overline{V}(\mathfrak{g}) - V(\mathfrak{g}) &= (D_{V,v} + \Gamma_v V, v)t + \dots. \\ &= (\nabla_v V. v)t + \dots. \end{aligned}$$

In order that the parallelism may be invariant by the transformation T in the base space, it must be that

$$\nabla'_v V. v = \overline{T}_{Q,\, g(0)} \nabla_v V. v$$

and

$$\Gamma'_v = \overline{T}_{Q,\, g(0)} \Gamma_v + \overline{T}'_{Q,\, v}$$

where $\overline{T}'_{Q,v}$ is the derivative of $\overline{T}_Q$. This shows that ∇_v is a vectorial operator in $\mathfrak{K}_Q$ and $\mathfrak{K}_P$.

§ 6. The curvature operator in linear parallelism.

Let V be a constant vector field[2] in $\mathfrak{M}$ and $v_{g_1(t)}$, $v_{g_2(t)}$ the two vectors in $\overline{\mathfrak{K}}_{P,\, g_1(0)}$ and $\overline{\mathfrak{K}}_{P,\, g_2(0)}$ corresponding to $g_1(t)$, $g_2(t)$ respectively and Γ, $\overline{T}_{P,f}$ and $\overline{T}_{Q,f}$ are differentiable with respect to every $g(t)$ of considering $g(t)$'s. We consider an operator

$$\begin{aligned} R_{v_1, v_2} V &= \nabla_{v_1}(\nabla_{v_2} V) - \nabla_{v_2}(\nabla_{v_1} V) \\ &= \Gamma_{v_2, v_1} V - \Gamma_{v_1, v_2} V + \Gamma_{v_2}(\Gamma_{v_1} V) - \Gamma_{v_1}(\Gamma_{v_2} V). \end{aligned}$$

The operator R_{v_1, v_2} is expressed it terms of Γ_v only and has a vectorial property, so that we call this R_{v_1, v_2} the "*curvature operator*".

Further, from the equation of transformation of Γ_v and from the fact that R_{v_1, v_2} is a vectorial operator, we have the following relations:

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(1) The operators in differentiation of V and c may be different, accordingly, the vector v_g in the both cases may be different, but we shall not treat these cases because the way of treatment is essentially the same.

(2) The term of constant vector field means that $V(\mathfrak{f}) = V(\mathfrak{g})$ for all element in $\mathfrak{M}_v$.

$$\left.\begin{aligned} \overline{T}''_{P,f,v_1,v_2} &= \overline{T}''_{P,f,v_2,v_1} \\ \overline{T}'_{Q,f,v_2} v_1 . v_2 &= \overline{T}'_{Q,f,v_1} v_2 . v_1 \end{aligned}\right\}.$$

This may be regarded as the expression for symmetricity of our generalized derivatives corresponding to the expressions

$$\frac{\partial^3 x'^l}{\partial x^i \partial x^j \partial x^k} = \frac{\partial^3 x'^l}{\partial x^j \partial x^i \partial x^k} \quad \text{and} \quad \frac{\partial^2 x'^l}{\partial x^i \partial x^j} = \frac{\partial^2 x'^l}{\partial x^j \partial x^i};$$

and the last equation seems to express that $\overline{T}_{Q,f}$ *behaves itself just like* $\dfrac{\partial x'^i}{\partial x^i}$ *in ordinary transformation.*

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