Journal of Science of the Hiroshima University, Ser. A, Vol. 18, No. 1, June, 1954

A Note on the Relative 2-Dimensional Cohomology Group in Complete Fields with Respect to a Discrete Valuation[1]

By

Akira Kinohara

(Received January 31, 1954)

§ 1. Introduction

Let k, K $(k \subset K)$ be $\mathfrak{p}$-adic number fields, and $\mathfrak{o}$, $\mathfrak{O}$ its valuation ring respectively, and $\mathfrak{p}$, $\mathfrak{P}$ its prime ideal respectively. Let D be the relative different of K/k. Then Prof. Y. Kawada[2] has proved the following relations with respect to the relative 2-dimensional cohomology group:

$$(*)\qquad \begin{aligned} H(\mathfrak{O},\ \mathfrak{o}\,;\ \mathfrak{O}/\mathfrak{P}^r) &\cong \mathfrak{O}/\mathfrak{P}^r \quad \text{for} \quad \mathfrak{P}^r \supseteqq D \\ H(\mathfrak{O},\ \mathfrak{o}\,;\ \mathfrak{O}/\mathfrak{P}^r) &\cong \mathfrak{O}/D \quad \text{for} \quad \mathfrak{P}^r \subseteqq D. \end{aligned}$$

Now let k be a complete field (with respect to a discrete valuation) of such a quotient field as, in its integral domains, the fundamental theorem of the multiplicative ideal theory holds. Let K be a separable extension of finite degree over k. Let $\mathfrak{o}$, $\mathfrak{O}$ be its valuation ring respectively, and $\mathfrak{p}$, $\mathfrak{P}$ its prime ideal respectively. Let D be the relative different of K/k.

Our aim is to prove that $(*)$ relation is satisfied in the case of $\mathfrak{O} = \mathfrak{o}[\theta]$, and that, using this fact, we can give a definition of the relative different of K/k different from the usual one.

In the case of $\mathfrak{p}$-adic number fields,[3] let K' be the inertia field of K/k, and $\mathfrak{O} \cap K' = \mathfrak{O}'$. Then $\mathfrak{O}' = \mathfrak{o}[\zeta]$, and, moreover, since the number of the elements of the residue class field of K by $\mathfrak{P}$ is finite, we can take ζ such that $\zeta^{|N(\mathfrak{P})|-1} = 1$ where $|N(\mathfrak{P})|$ is the absolute value of the absolute norm of $\mathfrak{P}$. In addition, we can set $\mathfrak{O} = \mathfrak{O}'[\Pi]$ where Π is a prime element of $\mathfrak{P}$ in $\mathfrak{O}$. These two facts are essentially utilized for the proof of $(*)$ relation, but, in our case, they are not always true.

---

1) This note has been completed by the encouragement of Prof. M. Moriya. The author wishes to express here his hearty thanks to Prof. M. Moriya for his kindness.

2) Y. Kawada: On the derivations in number fields, Annals of Math. **54** (1951), pp. 310–314.

3) Y. Kawada, l. c.

§ 2. Relative different in complete fields with $\mathfrak{O}=\mathfrak{o}[\theta]$

We shall prove the following

LEMMA 1. *Let θ be an element in $\mathfrak{O}$ with the irreducible defining equation $F(x)=0$ in $\mathfrak{o}[x]$, then (A) is equivalent to (B):*

$$(A)\quad \mathfrak{O}=\mathfrak{o}[\theta], \qquad (B)\quad (F'(\theta))=D.$$

PROOF. First, we shall show $(A)\Rightarrow(B)$. Let us put $(F'(\theta))=D\mathfrak{F}$, then, as is well known, $\mathfrak{F}$ is the set of all elements $\zeta\in\mathfrak{O}$ with $\zeta\omega\in\mathfrak{o}[\theta]$ for all $\omega\in\mathfrak{O}$. Since $\mathfrak{O}=\mathfrak{o}[\theta]$, then $\mathfrak{F}=\mathfrak{O}$. Therefore, we have $(F'(\theta))=D$.

Next, we shall show $(B)\Rightarrow(A)$. Since $(F'(\theta))=D$, then $\mathfrak{F}=\mathfrak{O}$. Putting $\omega=1$, we get $\mathfrak{F}\subseteq\mathfrak{o}[\theta]$. Therefore, we have $\mathfrak{O}=\mathfrak{o}[\theta]$, q. e. d.

§ 3. On the relative 2-dimensional cohomology group in complete fields with $\mathfrak{O}=\mathfrak{o}[\theta]$

Here we shall give some necessary definitions. A mapping $f(\alpha, \beta)$ with domain $\alpha, \beta\in\mathfrak{O}$ into an $\mathfrak{O}$-module $\mathfrak{O}/\mathfrak{P}^r$ is called a *2-dimensional cocycle* of $\mathfrak{O}$ in $\mathfrak{O}/\mathfrak{P}^r$ if it satisfies:

(1) $$f(\alpha+\beta, \gamma)=f(\alpha, \gamma)+f(\beta, \gamma)$$

(2) $$f(\alpha, \beta)=f(\beta, \alpha)$$

(3) $$\alpha f(\beta, \gamma)+f(\alpha, \beta\gamma)=f(\alpha\beta, \gamma)+\gamma f(\alpha, \beta),$$

and for the two mappings f_1, f_2

(4) $$f_1(\alpha, \beta)+f_2(\alpha, \beta)=(f_1+f_2)(\alpha, \beta).$$

Next we shall consider a linear mapping $g(\alpha)$ from $\mathfrak{O}$ in $\mathfrak{O}/\mathfrak{P}^r$ and for the two linear mappings g_1, g_2 we put

(5) $$g_1(\alpha)+g_2(\alpha)=(g_1+g_2)(\alpha).$$

If f satisfies:

(6) $$f(\alpha, \beta)=\alpha g(\beta)+\beta g(\alpha)-g(\alpha\beta),$$

we shall call f a *2-dimensional coboundary* of $\mathfrak{O}$ in $\mathfrak{O}/\mathfrak{P}^r$, and f is denoted by δg.

We shall denote $Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ as the totality of all the *relative 2-dimensional cocycle* f of $\mathfrak{O}$ in $\mathfrak{O}/\mathfrak{P}^r$ with

(7) $$f(a, \alpha)=0 \qquad \text{for every} \qquad a\in\mathfrak{o},$$

and $B(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ as the totality of all the *relative 2-dimensional coboundary* δg with

$$(8) \qquad g(a)=0, \quad g(a\alpha)=ag(\alpha) \quad \text{for every} \quad a\in\mathfrak{o}.$$

Since $Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) \supseteqq B(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$, we shall define the *relative 2-dimensional cohomology group* $H(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ of $\mathfrak{O}$ in $\mathfrak{O}/\mathfrak{P}^r$ by

$$(9) \qquad H(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)=Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)/B(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r).$$

For $f\in Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ we have from (2), (3) and (7):

$$(10) \qquad f(a\alpha, \beta)=af(\alpha, \beta)=f(\alpha, a\beta) \quad \text{for every} \quad a\in\mathfrak{o}.$$

Now let $\mathfrak{O}=\mathfrak{o}[\theta]$, and θ an element in $\mathfrak{O}$ with the irreducible equation $F(x)=x^n+c_1x^{n-1}+\cdots+c_n=0$ in $\mathfrak{o}[x]$, then α, $\beta\in\mathfrak{O}$ may be represented in the form $\alpha=\sum_{i=0}^{n-1}a_i\theta^i$, $\beta=\sum_{i=0}^{n-1}b_i\theta^i$ $(a_i, b_i\in\mathfrak{o})$. From (10) we have:

$$(11) \qquad f(\alpha, \beta)=\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_i\, b_j\, f(\theta^i, \theta^j).$$

Therefore, a relative 2-dimensional cocycle f is uniquely determined by the values $f(\theta^i, \theta^j)$ $(i, j=1, 2, \cdots, n-1)$ with the conditions

$$(12) \qquad \theta^i f(\theta^j, \theta^k)+f(\theta^i, \theta^{j+k})\equiv f(\theta^{i+j}, \theta^k)+\theta^k f(\theta^i, \theta^j) \pmod{\mathfrak{P}^r}.$$

We shall prove the following

Lemma 2. *For every* $f\in Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ *we can take* $\delta g\in B(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ *such that* $f_1=f+\delta g$ *satisfies the condition:*

$$(13) \qquad f_1(\theta^i, \theta^j)\equiv 0 \pmod{\mathfrak{P}^r} \quad \text{for} \quad i+j\leqq n-1.$$

Proof. By mathematical induction on $i+j(\leqq n-1)$ we shall prove (13). If $i+j=2$, for an arbitrary $f\in Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ we can take a g such that

$$g(\theta)\equiv 0, \quad g(\theta^2)\equiv f(\theta, \theta), \quad g(\theta^3)\equiv 0, \quad \cdots, \quad g(\theta^{n-1})\equiv 0 \pmod{\mathfrak{P}^r},$$

then we have:

$$f_1(\theta, \theta)=(f+\delta g)(\theta, \theta)\equiv 0 \pmod{\mathfrak{P}^r}.$$

Now we shall assume that

$$f_1(\theta^i, \theta^j)\equiv 0 \pmod{\mathfrak{P}^r} \quad \text{for} \quad i+j\leqq l \ (<n-1),$$

then from

$$\theta^i f_1(\theta^j, \theta^k)+f_1(\theta^i, \theta^{j+k})\equiv f_1(\theta^{i+j}, \theta^k)+\theta^k f_1(\theta^i, \theta^j) \pmod{\mathfrak{P}^r},$$

we have $f_1(\theta^i, \theta^{j+k})\equiv f_1(\theta^{i+j}, \theta^k) \pmod{\mathfrak{P}^r}$ for $i+j+k=l+1$. Thus, if we take a

g^* such that

$$g^*(\theta)\equiv 0, \quad \cdots, \quad g^*(\theta^l)\equiv 0 \qquad (\text{mod } \mathfrak{P}^r),$$
$$g^*(\theta^{l+1})\equiv f_1(\theta^i, \theta^j) \qquad (\text{mod } \mathfrak{P}^r) \quad \text{for} \quad i+j=l+1 \ (\leqq n-1),$$
$$g^*(\theta^{l+2})\equiv 0, \quad \cdots, \quad g^*(\theta^{n-1})\equiv 0 \qquad (\text{mod } \mathfrak{P}^r),$$

we have $f_1^*(\theta^i, \theta^j)=(f_1+\delta g^*)(\theta^i, \theta^j)\equiv 0 \pmod{\mathfrak{P}^r}$ for $i+j\leqq l+1$ $(\leqq n-1)$. Here, let us denote f_1^*, $f_1+\delta g^*$ by f_1, $f+\delta g$ respectively, then we have:

$$f_1(\theta^i, \theta^j)=(f+\delta g)(\theta^i, \theta^j)\equiv 0 \pmod{\mathfrak{P}^r} \text{ for } i+j\leqq l+1(\leqq n-1), \text{ q. e. d.}$$

THEOREM 1. *If* $\mathfrak{O}=\mathfrak{o}[\theta]$, *we have:*

$$(*)\qquad \begin{aligned} &H(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)\cong\mathfrak{O}/\mathfrak{P}^r \quad \text{for} \quad \mathfrak{P}^r\supseteqq D \\ &H(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)\cong\mathfrak{O}/D \quad \text{for} \quad \mathfrak{P}^r\subseteqq D. \end{aligned}$$

PROOF. Let $Z_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ be the set of all the elements $f\in Z(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ with (13). Let $B_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)=Z_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)\cap B(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$. Then we have:

$$H(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)\cong Z_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)/B_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r).$$

Here, the condition $\delta g\in B_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ is

$$(14)\qquad \delta g(\theta^i, \theta^j)=\theta^i g(\theta^j)+\theta^j g(\theta^i)-g(\theta^{i+j})\equiv 0 \pmod{\mathfrak{P}^r} \quad \text{for} \quad 1\leqq i, j,\ i+j\leqq n-1.$$

Let us take as λ an arbitrary element of $\mathfrak{O}$, and let us put $g(\theta)\equiv\lambda \pmod{\mathfrak{P}^r}$, then (14) is equivalent to

$$(15)\qquad g(\theta^2)\equiv 2\theta\lambda, \quad g(\theta^3)\equiv 3\theta^2\lambda, \quad \cdots, \quad g(\theta^{n-1})\equiv(n-1)\theta^{n-2}\lambda \pmod{\mathfrak{P}^r}.$$

By (12) and (13) we have:

$$(16)\qquad f(\theta^i, \theta^{j+k})\equiv f(\theta^{i+j}, \theta^k) \pmod{\mathfrak{P}^r} \quad \text{for} \quad i+j, j+k\leqq n-1.$$

Hence we can put

$$(17)\qquad f(\theta^i, \theta^j)\equiv\mu_{i+j} \pmod{\mathfrak{P}^r} \quad \text{for} \quad i, j\leqq n-1.$$

Thus, let us put $j=1$, $k=n-1$, $i=1, 2, \cdots, n-2$ in (12), and using the relation $\theta^n=-c_1\theta^{n-1}-c_2\theta^{n-2}-\cdots-c_n$, we have:

$$(18)\qquad \theta\mu_n-c_1\mu_n\equiv\mu_{n+1}, \quad \theta^2\mu_n-(c_1\mu_{n+1}+c_2\mu_n)\equiv\mu_{n+2}, \quad \cdots,$$
$$\theta^{n-2}\mu_n-\left(\sum_{i=1}^{n-2}c_i\mu_{2n-2-i}\right)\equiv\mu_{2n-2} \pmod{\mathfrak{P}^r}.$$

Then $\mu_{n+1}, \cdots, \mu_{2n-2}$ are uniquely determined by μ_n recursively from (18). Therefore, a relative 2-dimensional cocycle $f \in Z_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ is uniquely determined by its value $\mu_n \equiv f(\theta^{n-1}, \theta) \pmod{\mathfrak{P}^r}$.

Now we shall consider the condition for $f = \delta g \in B_1(\mathfrak{O}, \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$. We have from (15)

$$\begin{aligned}\mu &\equiv f(\theta^{n-1}, \theta) \equiv \theta^{n-1} g(\theta) + \theta g(\theta^{n-1}) - g(\theta^n) \\ &\equiv n\theta^{n-1}\lambda + \sum_{i=1}^{n-1} (n-i) c_i \theta^{n-i-1}\lambda \equiv F'(\theta)\lambda \pmod{\mathfrak{P}^r}.\end{aligned}$$

On the other hand, since $(F'(\theta)) \equiv D$ by Lemma 1, we obtain easily the $(*)$ relation, q. e. d.

From this theorem, therefore, in the case of $\mathfrak{O} = \mathfrak{o}[\theta]$, we can give a definition different from the usual one for the relative different D of K/k. That is, the ideal D of $\mathfrak{O}$, which is uniquely defined in Theorem 1, is called the *relative different* of K/k.

Let $\mathfrak{k}$ be the residue class field of k by $\mathfrak{p}$, and $\mathfrak{K}$ the residue class field of K by $\mathfrak{P}$. Then we have the following two corollaries.

COROLLARY 1. *If $\mathfrak{K}$ is a separable extension over $\mathfrak{k}$, we have $(*)$ relation.*

COROLLARY 2. *If $\mathfrak{K}$ is a simple extension over $\mathfrak{k}$ and $\mathfrak{O}\mathfrak{p} = \mathfrak{P}$, we have $(*)$ relation.*

§ 4. On the relative 2-dimensional cohomology group in inertia fields

Let K' be the inertia field of K/k, and $\mathfrak{O}' = \mathfrak{O} \frown K'$. Then, for some $\eta \in \mathfrak{O}'$ we have $\mathfrak{O}' = \mathfrak{o}[\eta]$. Let $\varphi(x) = c_n x^n + \cdots + c_0$ be the irreducible defining polynomial of η over $\mathfrak{o}$. Then, it is well known that $\varphi'(\eta)$ is not divisible by $\mathfrak{P}$. Hence we have from Theorem 1

THEOREM 2. *In the inertia field K' of K/k we have:*

$$H(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) \simeq \mathfrak{O}/\mathfrak{O} \qquad \text{for} \quad r = 0, 1, \cdots.$$

Here we shall prove this theorem by a method[1] different from that of § 3.

PROOF of THEOREM 2. Now let us put for any $f \in Z(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ and every $\alpha \in \mathfrak{O}'$

$$g(\alpha) = \varphi'(\eta)^{-1} \sum_{\nu=2}^{n} c_\nu [\eta^{\nu-2} f(\alpha, \eta) + \cdots + \eta^{\nu-i-1} f(\alpha, \eta^i) + \cdots + f(\alpha, \eta^{\nu-1})],$$

then we have:

---

1) I am indebted to Prof. M. Moriya.

$$[\eta g(\alpha)+\alpha g(\eta)-g(\alpha\eta)]\varphi'(\eta)=\sum_{\nu=2}^{n}c_\nu\eta[\eta^{\nu-2}f(\alpha,\eta)+\cdots$$
$$+\eta^{\nu-i-1}f(\alpha,\eta^i)+\cdots+f(\alpha,\eta^{\nu-1})]+\sum_{\nu=2}^{n}c_\nu\alpha[\eta^{\nu-2}f(\eta,\eta)+\cdots$$
$$+\eta^{\nu-i-1}f(\eta,\eta^i)+\cdots+f(\eta,\eta^{\nu-1})]-\sum_{\nu=2}^{n}c_\nu[\eta^{\nu-2}f(\alpha\eta,\eta)+\cdots$$
$$+\eta^{\nu-i-1}f(\alpha\eta,\eta^i)+\cdots+f(\alpha\eta,\eta^{\nu-1})].$$

Here, let us put for a fixed i

$$T_i=c_\nu[\eta^{\nu-i}f(\alpha,\eta^i)+\alpha\eta^{\nu-i-1}f(\eta,\eta^i)-\eta^{\nu-i-1}f(\alpha\eta,\eta^i)],$$

then

$$T_i=c_\nu\eta^{\nu-i-1}[\eta f(\alpha,\eta^i)+\alpha f(\eta,\eta^i)-f(\alpha\eta,\eta^i)].$$

Since $\alpha f(\eta,\eta^i)-f(\alpha\eta,\eta^i)=-f(\alpha.\,\eta^{i+1})+\eta^i f(\alpha,\eta)$,

we obtain:

$$T_i=c_\nu\eta^{\nu-i-1}[\eta f(\alpha,\eta^i)-f(\alpha,\eta^{i+1})+\eta^i f(\alpha,\eta)].$$

Now let us put

$$S_\nu=\sum_{i=1}^{\nu-1}T_i=c_\nu\sum_{i=1}^{\nu-1}[\eta^{\nu-i}f(\alpha,\eta^i)-\eta^{\nu-i-1}f(\alpha,\eta^{i+1})]+c_\nu(\nu-1)\eta^{\nu-1}f(\alpha,\eta),$$

then

$$S_\nu=c_\nu[\eta^{\nu-1}f(\alpha,\eta)-f(\alpha,\eta^\nu)+(\nu-1)\eta^{\nu-1}f(\alpha,\eta)]$$
$$=\nu c_\nu\eta^{\nu-1}f(\alpha,\eta)-f(\alpha,c_\nu\eta^\nu).$$

Thus, we have:

$$\sum_{\nu=0}^{n}S_\nu=(\sum_{\nu=0}^{n}\nu c_\nu\eta^{\nu-1})f(\alpha,\eta)-\sum_{\nu=0}^{n}f(\alpha,c_\nu\eta^\nu)=\varphi'(\eta)f(\alpha,\eta).$$

On the other hand, since $\sum_{\nu=0}^{n}S_\nu=[\eta g(\alpha)+\alpha g(\eta)-g(\alpha\eta)]\varphi'(\eta)$ and $\varphi'(\eta)$ is not divisible by $\mathfrak{P}$, we have:

$$f(\alpha,\eta)\equiv\eta g(\alpha)+\alpha g(\eta)-g(\alpha,\eta)\qquad(\mathrm{mod}\ \mathfrak{P}^r).$$

From this fact, we can see that, for every $f\in Z(\mathfrak{O}',\mathfrak{o};\mathfrak{O}/\mathfrak{P}^r)$, there exists $\delta g\in B(\mathfrak{O}',\mathfrak{o};\mathfrak{O}/\mathfrak{P}^r)$ satisfying the following relations:

$$f_1(\eta,\alpha)=(f-\delta g)(\eta,\alpha)\equiv 0\qquad(\mathrm{mod}\ \mathfrak{P}^r)\quad\text{for all}\quad\alpha\in\mathfrak{O}'.$$

Thus, using the relations:

$$\eta f_1(\eta^i,\alpha)+f_1(\eta,\eta^i\alpha)\equiv f_1(\eta^{i+1},\alpha)+\alpha f_1(\eta,\eta^i)\quad(\mathrm{mod}\ \mathfrak{P}^r)\ \text{ for }\ i=0,\ 1,\ \cdots,$$

we have the following relations by the mathematical induction on i

$$f_1(\eta^i, \alpha) \equiv 0 \qquad (\text{mod } \mathfrak{P}^r) \quad \text{for} \quad i=0, 1, \cdots \text{ and all } \alpha \in \mathfrak{O}'.$$

Since any $\alpha, \beta \in \mathfrak{O}'$ are represented in the form $\alpha = \sum_{i=0}^{n-1} a_i \eta^i$, $\beta = \sum_{i=0}^{n-1} b_i \eta^i$ $(a_i, b_i \in \mathfrak{o})$, for $f_1 = f - \delta g$ the following relations are always satisfied:

$$f_1(\alpha, \beta) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_i b_j f(\eta^i, \eta^j) \equiv 0 \qquad (\text{mod } \mathfrak{P}^r).$$

Thus, for every $f \in Z(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ we can take $\delta g \in B(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$ such that $f = \delta g$. Hence we have $Z(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) \subseteqq B(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$. On the other hand, since $Z(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) \supseteqq B(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$, we have $Z(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) = B(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r)$, i. e.,

$$H(\mathfrak{O}', \mathfrak{o}; \mathfrak{O}/\mathfrak{P}^r) \cong \mathfrak{O}/\mathfrak{O} \qquad \text{for} \qquad r=0, 1, \cdots.$$

This completes the proof of our theorem.

Department of Mathematics,
Hiroshima University