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Corrections to the papers on finite H-spaces

Yutaka HEMMI

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1. In [2], Proposition 4.3 is incorrect in case that $H^*(X; Z)$ has torsions, e.g., $X=G_2$ (the exceptional Lie group). To correct [2], we must add the assumption $(*)$ and Theorem 1.4 in [2] should be replaced by the following

THEOREM 1.4′. *For a 3-connected finite H-space X, assume that*
$(*)$ *$H^*(X; Z)$ has no 2-torsion,*
(1.5) *$H^*(X; G)$ are primitively generated for $G=Z_2$ and Q, and*
(1.6) *the indecomposable module $QH^n(X; Z_2)$ vanishes for $n=15$.*
Then, X has the homotopy type of $(S^7)^l$ for some $l \geqq 0$.

(We note that $(*)$ and (1.5) for $G=Q$ imply (1.5) for $G=Z_2$, which can be proved by using Theorem 2.2 of Hodgkin [11] in the references of [2].)

Corollary 1.7 in [2] is valid by the proof given in [2; p. 56], because $(*)$ for $\tilde{X}$ is proved there and so Theorem 1.4′ can be applied to $\tilde{X}$.

We can prove Theorem 1.4′ by correcting [2; §§2, 4–5] as follows:
In Lemma 2.4 and §4, the assumption $(*)$ should be added. In §§4–5,

$K^*(\ )$ and Z in the coefficient should be replaced by $K^*(\ )\otimes Z_{(2)}$ and $Z_{(2)}$,

respectively, ($Z_{(2)}$ is the ring of integers localized at 2), $K^*(\ )\otimes Q$ in line -5 of p. 60 by $K^*(\ )\otimes Z_{(2)}$, and the isomorphism in line -4 of p. 60 by

$$F_{2p-1}K^1(X) \otimes Z_{(2)}/F_{2p}K^1(X) \otimes Z_{(2)} \cong H^{2p-1}(X; Z_{(2)})\,;$$

and the Adams operation ψ^n in Proposition 4.5 and Lemma 4.7 (i) should mean the one $\psi^n\otimes\mathrm{id}$ localized at 2. Furthermore, 'integers A and B' in line -5 of p. 62 and 'A is even or add' in §5 should mean 'coefficients A and B in $Z_{(2)}$' and '$A\equiv 0$ mod 2 or not', respectively.

2. In [1], Lemma 7.8 is incorrect (see (b) below); and it should be replaced by the following

LEMMA 7.8′. *Let $m \geqq 2$ and E be an exponential sequence with $|E|=2p^m(p-1)$ and $E \neq p^m\Delta_1$. Then*

$$r_E \equiv \sum r_{E_s}\theta_s \bmod (p^2, v_1, v_2, \cdots),$$

where $\theta_s \in BP^*BP$, and E_s satisfies (1) *for $m \geqq 2$ and* (2) *in Proposition* 7.7.

PROOF. Let $E=(e_1, e_2, \cdots)$ satisfy $|E|=2\sum e_i(p^i-1)=2p^m(p-1)$. Then, $e_i=0$ $(i>m)$ and $e_m<p$; and $e_m=p-1$ if and only if $E=E_0=\Delta_1+(p-1)\Delta_m$. Since $(p-2)(p^m-1)+\sum_{i=1}^{m-1}(p^i-1)<p^m(p-1)$, these show that

(a) E_0 is the least one, and $e_t \geqq 2$ for some $1 \leqq t < m$ if $E \neq E_0$.

Now, put $E_1=2\Delta_1$, $E_2=p\Delta_{m-1}$, $F=E_1+E_2+(p-2)\Delta_m$, $F_s=F-E_s$ $(s=1, 2)$, $b_2=(p+2)(p+1)/2$ and $b_m=1$ if $m \geqq 3$. Then (7.4) in [1] shows that $r_{E_1}r_{F_1} \equiv b_m r_F+(p-1)r_{E_0}$ and $r_{E_2}r_{F_2} \equiv b_m r_F \mod (v_1, v_2, \cdots)$. Thus, we see the following (b) which is the lemma for $E=E_0$:

(b) $r_{E_0} \equiv r_{E_1}\theta_1 + r_{E_2}\theta_2 \mod (p^2, v_1, v_2, \cdots)$ where $\theta_s = (-1)^s(p+1)r_{F_s}$.

When $E \neq p^m\Delta_1$ and $E \neq E_0$, according to (a) and (b), we see Lemma 7.8 in [1] by the proof given in [1; pp. 466–7] where t should be taken to satisfy (a) so that $|2\Delta_t|<2p^m$ and the two '$2\Delta_1$' in line -1 of p. 466 should be replaced by '$2\Delta_t$'. Q.E.D.

References

[1] Y. Hemmi, On finite H-spaces given by sphere extensions of classical groups, Hiroshima Math. J. **14** (1984), 451–470.

[2] Y. Hemmi, On 3-connected finite H-spaces, Hiroshima Math. J. **15** (1985), 55–67.

Department of Mathematics,
Faculty of Science,
Kochi University