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On certain decomposition of bounded weak*-measurable functions taking their ranges in dual Banach spaces

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ABSTRACT. For a subset C of bidual Banach spaces, we introduce the notion of C-Pettis integrability of Gelfand integrable functions, a general notion of Pettis integrability of such ones. We show that a geometric property called weak*-C-dentability assures the decomposability of bounded weak*-measurable functions taking their ranges in dual Banach spaces into a C-Pettis integrable part and a weak* scalarly null part. Some related results follow from this.

1. Introduction

We begin with the requisite definitions. Throughout this paper, X denotes an arbitrary real Banach space, X^* and X^{**} its topological dual and bidual, respectively, and $B(X)$ the closed unit ball of X. The triple (S, Σ, μ) refers to a complete finite measure space and Σ^+ to the sets in Σ with positive μ-measure. In the following we always assume that S is endowed with Σ and μ. For each $E \in \Sigma^+$, denote $\Delta_E = \{\chi_F/\mu(F) : F \subset E, F \in \Sigma^+\}$. If C is a subset of X^{**}, a function $f : S \to X^*$ is said to be C-measurable if the real-valued function $(x^{**}, f(s))$ is μ-measurable for each $x^{**} \in C$. Especially, if $C = B(X)$ (resp. $C = B(X^{**})$), we say that f is weak*-measurable (resp. weakly measurable). A function $f : S \to X^*$ is said to be weak* scalarly null if $(x, f(s)) = 0$ μ-a.e. on S for every $x \in X$. We say that a function $f : S \to X^*$ is weak*-equivalent to a C-measurable function $g : S \to X^*$ if $f - g$ is weak* scalarly null. A weak*-measurable function $f : S \to X^*$ is called Gelfand integrable if $x \circ f \in L_1(S, \Sigma, \mu)$ for every $x \in X$, where $(x \circ f)(s) = (x, f(s))$ for every $s \in S$. If $f : S \to X^*$ is a Gelfand integrable function, by the closed graph theorem, we then obtain a bounded linear operator $T_f : X \to L_1(S, \Sigma, \mu)$ given by $T_f(x) = x \circ f$ for every $x \in X$, and the dual operator of T_f is denoted by $T_f^*(: L_\infty(S, \Sigma, \mu) \to X^*)$. If A is a bounded subset of X, $\bar{A}^*$ denotes the weak*-closure of A in X^{**}. If K is a subset of X^*, $\overline{\text{co}}^*(K)$ denotes the weak*-

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closed convex hull of K. For a subset C of X^{**} and a Gelfand integrable function $f : S \to X^*$, let us define C-Pettis integrability of f as follows. This is a generalization of Pettis integrability of such functions.

DEFINITION 1. Let C be a subset of X^{**} and $f : S \to X^*$ a Gelfand integrable function. Then f is called *C-Pettis integrable* if the following two conditions are satisfied.

(1) For every $x^{**} \in C$, $x^{**} \circ f \in L_1(S, \Sigma, \mu)$,

(2) For each $E \in \Sigma$, it holds that

$$(x^{**}, T_f^*(\chi_E)) = \int_E (x^{**}, f(s)) d\mu(s)$$

for every $x^{**} \in C$.

Especially, if $C = B(X^{**})$, such a function is called *Pettis integrable*. That is, a Gelfand integrable function $f : S \to X^*$ is called Pettis integrable if $x^{**} \circ f \in L_1(S, \Sigma, \mu)$ for every $x^{**} \in X^{**}$ and moreover for each $E \in \Sigma$ there exists an element x_E^* of X^* that satisfies

$$(x^{**}, x_E^*) = \int_E (x^{**}, f(s)) d\mu(s)$$

for every $x^{**} \in X^{**}$.

Note that C-Pettis integrability of Gelfand integrable functions f is a significant notion only when $X \not\supset C$, since it holds that $(x, T_f^*(\chi_E)) = \int_E (x, f(s)) d\mu(s)$ for every $E \in \Sigma$ and $x \in X$. So we always assume this condition. We then define a function to be C-Pettis decomposable as follows.

DEFINITION 2. Let C be a subset of X^{**} and $f : S \to X^*$ a weak*-measurable function. Then we say that f is *C-Pettis decomposable* if there exist functions g and h such that g is C-Pettis integrable and h is weak* scalarly null and $f = g + h$.

Now, in this paper, let us consider the following problem: Given a subset C of X^{**} and a bounded weak*-measurable function $f : S \to X^*$. What conditions assure the C-Pettis decomposability of f? In [10] and [2], such a type problem has been considered in the case where $C = B(X^{**})$. In fact, Talagrand has obtained a result (cf. (b) of Proposition (7-3-15) in [10] or Theorem 3 in [2]) concerning this by making use of a measure theoretic property, so-called the RS-property.

Here, suggested by an approach due to Girardi and Uhl [5] for a simpler proof of the fact that dentability implies the Radon-Nikodym property, we pay attention to a following geometric property of subsets in dual Banach spaces. This has been introduced in [3] and is a generalization of weak*-scalar dentability of subsets in dual Banach spaces.

DEFINITION 3. Let K be a bounded subset of X^*. A *weak*-open* (resp. *weak**) *slice* of K is a set of the form:

$$S(x,\alpha,K) = \left\{x^* \in K : (x,x^*) > \sup_{y^* \in K}(x,y^*) - \alpha\right\}$$

$$\left(\text{resp. } \bar{S}(x,\alpha,K) = \left\{x^* \in K : (x,x^*) \geqq \sup_{y^* \in K}(x,y^*) - \alpha\right\}\right)$$

where $x \in X$ and $\alpha > 0$.

Let C be a subset of X^{**}. Then the set K is said to be *weak*-C-dentable* if for every $\varepsilon > 0$ and $x^{**} \in C$, there exists a weak*-open slice $S(x,\alpha,K)$ of K such that $O(x^{**}|S(x,\alpha,K))$ $(= \sup\{|(x^{**},x^*) - (x^{**},y^*)| : x^*, y^* \in S(x,\alpha,K)\}$, the oscillation of x^{**} on $S(x,\alpha,K)) < \varepsilon$.

If K is weak*-$B(X^{**})$-dentable, then K is said to be *weak*-scalarly dentable*.

Note that the set K is weak*-C-dentable if and only if for every $\varepsilon > 0$ and $x^{**} \in C$, there exists a weak* slice $\bar{S}(x,\alpha,K)$ such that $O(x^{**}|\bar{S}(x,\alpha,K)) < \varepsilon$, since $\bar{S}(x,\alpha/2,K) \subset S(x,\alpha,K) \subset \bar{S}(x,\alpha,K)$ for every $x \in X$ and $\alpha > 0$.

Further, following [4], for a bounded weak*-measurable function $f : S \to X^*$ and $E \in \Sigma$ we define the *weak*-core* of f over E, denoted by $\mathrm{cor}_f^*(E)$, to be a subset of X^* given by

$$\mathrm{cor}_f^*(E) = \cap\{\overline{\mathrm{co}}^* f(E \backslash A) : \mu(A) = 0\}.$$

Then we know from Proposition 1 in [1] that $\mathrm{cor}_f^*(E)$ is a nonempty weak*-compact convex subset of X^* and $\mathrm{cor}_f^*(E) = \overline{\mathrm{co}}^*(T_f^*(\Delta_E))$ for every $E \in \Sigma^+$. Observing Andrews's way in [1] (or our way in [8]) carefully, we are aware that this geometric property (weak*-C-dentability) of $\mathrm{cor}_f^*(E)$ is an effective means to an end. Indeed, we can show that the notion of weak*-C-dentability presents a geometric sufficient condition insuring the C-Pettis decomposability of bounded weak*-measurable functions taking their ranges in dual Banach spaces. That is, we have the following Theorem, which is the main result of our paper.

THEOREM. *Let C be a subset of X^{**}, (S,Σ,μ) a complete finite measure space and $f : S \to X^*$ a bounded weak*-measurable function. Assume that the set $\mathrm{cor}_f^*(E)$ is weak*-C-dentable for every $E \in \Sigma^+$. Then f is C-Pettis decomposable. Especially, when $C = B(X^{**})$, f is Pettis decomposable if the set $\mathrm{cor}_f^*(E)$ is weak*-scalarly dentable for every $E \in \Sigma^+$.*

Our proof of Theorem is given by a more strict argument than in one of the fact that (c) implies (d) in Theorem of [8]. It heavily depends on the theory of lifting ([6]) and the exhaustion argument. §2 is devoted to the proof of Theorem. In §3, as an application of Theorem, we give new characterizations of K-weakly precompact sets A (see definition in §3) in terms of $\bar{A}^*$-Pettis decomposability.

2. Proof of Theorem

Let us recall the definition of the lifting on $L_\infty(S,\Sigma,\mu)$. A lifting ρ of $L_\infty(S,\Sigma,\mu)$ is a map: $L_\infty(S,\Sigma,\mu) \to M(S,\Sigma,\mu)$ (the set of all bounded μ-measurable functions on S) that is linear, multiplicative, positive such that $\rho(1)=1$, and such that $\rho(f)$ belongs to the class of f for each $f \in L_\infty(S,\Sigma,\mu)$. On (S,Σ,μ), such liftings exist and so we always take an arbitrary, but fixed lifting ρ. For each $E \in \Sigma$, $\rho(\chi_E)$ is the characteristic function of a uniquely determined set belonging to Σ, which is denoted by $\rho(E)$. So we have $\chi_{\rho(E)} = \rho(\chi_E)$ for each $E \in \Sigma$. The map $\rho : \Sigma \to \Sigma$ thus obtained satisfies that (1) $\rho(E) \equiv E$, (2) $\rho(E) = \rho(F)$ if $E \equiv F$, (3) $\rho(S) = S$, $\rho(\phi) = \phi$, (4) $\rho(E \cap F) = \rho(E) \cap \rho(F)$, and (5) $\rho(E \cup F) = \rho(E) \cup \rho(F)$. Here $E \equiv F(E, F \in \Sigma)$ means that $\mu((E \backslash F) \cup (F \backslash E)) = 0$.

Let $f : S \to X^*$ be a bounded weak*-measurable function such that $\|f(s)\| \leqq L$ on S. Then, in virtue of the lifting theory, we have a weak*-measurable function $\theta(f) : S \to X^*$ such that $(x, \theta(f)(s)) = \rho(x \circ f)(s)$ for every $x \in X$ and every $s \in S$. Note that $\sup_{s \in E}(x, \theta(f)(s)) = \text{ess} - \sup_{s \in E}(x, f(s))$ for every $E \in \Sigma^+$ with $\rho(E) = E$, $\|\theta(f)(s)\| \leqq L$ on S and $\theta(f)$ is weak*-equivalent to f. Hence, in order to prove Theorem, we have only to show that $\theta(f)$ is C-Pettis integrable under the assumption that the set $\mathrm{cor}_f^*(E)$ is weak*-C-dentable for every $E \in \Sigma^+$. To this end, we first prove the following Lemma 1, which is suggested by the part (iv) of proof of Theorem in [8] and whose proof is the almost same as in it. But, for the sake of completeness, we here wish to state the proof.

LEMMA 1. *Assume the same conditions as in Theorem. Then, for every* $x^{**} \in C$, $E \in \Sigma^+$ *and* $\varepsilon > 0$, *there exists an element* F *of* Σ^+ *with* $F \subset E$ *such that* $O(x^{**} | \overline{\mathrm{co}}^*(\theta(f)(F))) < \varepsilon$.

PROOF. Take $x^{**} \in C$, $E \in \Sigma^+$ and $\varepsilon > 0$, and set $D = \rho(E)$ $(\in \Sigma^+)$. Since M $(= \mathrm{cor}_f^*(D) = \overline{\mathrm{co}}^*(T_f^*(\Delta_D)))$ is weak*-C-dentable by the assumption, there exists a weak* slice $\bar{S}(x, \alpha, M)$ such that $O(x^{**} | \bar{S}(x, \alpha, M)) < \varepsilon$. Then we have that

$$
\begin{aligned}
&\bar{S}(x, \alpha, M) \\
&= \left\{ x^* \in M : (x, x^*) \geqq \sup_{y^* \in M} (x, y^*) - \alpha \right\} \\
&= \left\{ x^* \in M : (x, x^*) \geqq \sup_{y^* \in T_f^*(\Delta_D)} (x, y^*) - \alpha \right\} \\
&= \left\{ x^* \in M : (x, x^*) \geqq \sup\left(\int_G (x, f(s)) d\mu(s)/\mu(G) : G \subset D, G \in \Sigma^+ \right) - \alpha \right\} \\
&= \left\{ x^* \in M : (x, x^*) \geqq \text{ess} - \sup_{s \in D} (x, f(s)) - \alpha \right\} \\
&= \left\{ x^* \in M : (x, x^*) \geqq \sup_{s \in D} (x, \theta(f)(s)) - \alpha \right\}
\end{aligned}
$$

since $\rho(D) = D$. Set $F_0 = \left\{ t \in D : (x, \theta(f)(t)) \geqq \sup_{s \in D} (x, \theta(f)(s)) - \alpha \right\}$. Then $F_0 \in \Sigma^+$, since $\sup_{s \in D} (x, \theta(f)(s)) = \text{ess} - \sup_{s \in D} (x, \theta(f)(s))$. Furthermore we have that $\theta(f)(F_0) \subset M$. Indeed, suppose that there exists an element t of F_0 such that $\theta(f)(t) \bar{\in} M$. Then, by the separation theorem, there exists an element a of X such that

$$
\begin{aligned}
(a, \theta(f)(t)) > \beta &= \sup_{y^* \in M} (a, y^*) \\
&= \text{ess} - \sup_{s \in D} (a, f(s)) \\
&= \sup_{s \in D} (a, \theta(f)(s)),
\end{aligned}
$$

which is a contradiction. Thus we know that $\theta(f)(F_0) \subset \bar{S}(x, \alpha, M)$ and so $\overline{\text{co}}^*(\theta(f)(F_0)) \subset \bar{S}(x, \alpha, M)$, since $\bar{S}(x, \alpha, M)$ is a weak*-compact convex subset of X^*. Hence it holds that $O(x^{**} | \overline{\text{co}}^*(\theta(f)(F_0))) \leqq O(x^{**} | \bar{S}(x, \alpha, M)) < \varepsilon$. Finally, letting $F = F_0 \cap E$, we know that F is a desired set, as $D = \rho(E) \equiv E$. This completes the proof.

In virtue of Lemma 1 and the well-known exhaustion argument, we easily have:

LEMMA 2. *Assume the same conditions as in Theorem. Then, for every* $x^{**} \in C$, $E \in \Sigma^+$ *and* $\varepsilon > 0$, *there exists a disjoint sequence (possibly finite)* $\{F_n\}_{n \geqq 1}$ *of subsets of* E, *all of positive* μ*-measure, such that* $\mu(E \setminus \bigcup_{n \geqq 1} F_n) = 0$ *and* $O(x^{**} | \overline{\text{co}}^*(\theta(f)(F_n))) < \varepsilon$ *for every* $n \geqq 1$.

Now we are in a position to prove Theorem.

PROOF OF THEOREM. Set $\theta(f) = g$ and take an element x^{**} of C. In order to show that $x^{**} \circ g \in L_1(S, \Sigma, \mu)$, we have only to prove that it is μ-measurable, since g is a bounded function on S. This is easily proved as follows. Take any positive number ε. Then, in virtue of Lemma 2, there exists a disjoint sequence $\{F_n\}_{n \geqq 1} \subset \Sigma^+$ such that $\mu(S \setminus \bigcup_{n \geqq 1} F_n) = 0$ and $O(x^{**} | \overline{\mathrm{co}}^*(g(F_n))) < \varepsilon$ for every $n \geqq 1$. Hence, letting $h(s) = \sum_{n \geqq 1} x^{**}(x_n^*) \chi_{F_n}(s)$ (Here, $x_n^* \in g(F_n)$ for every $n \geqq 1$), we get that $|x^{**} \circ g(s) - h(s)| < \varepsilon$ μ-a.e. on S, since $O(x^{**} \circ g | F_n) = O(x^{**} | g(F_n)) \leqq O(x^{**} | \overline{\mathrm{co}}^*(g(F_n))) < \varepsilon$ for every $n \geqq 1$. This implies the μ-measurability of $x^{**} \circ g$.

Next let us show that

$$(x^{**}, T_g^*(\chi_E)) = \int_E (x^{**}, g(s)) d\mu(s)$$

for every $x^{**} \in C$ and $E \in \Sigma$. Take $x^{**} \in C$ and $E \in \Sigma$. Since this equality clearly holds if $\mu(E) = 0$, we may assume that $E \in \Sigma^+$. Let $\varepsilon > 0$. In virtue of Lemma 2 there exists a disjoint sequence $\{E_n\}_{n \geqq 1}$ of subsets of E, all of positive μ-measure such that $\mu(E \setminus \bigcup_{n \geqq 1} E_n) = 0$ and $O(x^{**} | \overline{\mathrm{co}}^*(g(E_n))) < \varepsilon$ for every $n \geqq 1$. So, letting $a_n^* = T_g^*(\chi_{E_n})/\mu(E_n)$ for every $n \geqq 1$, then we have that $a_n^* \in \overline{\mathrm{co}}^*(g(E_n))$, which is easily shown by the separation theorem. Further it holds that $\Sigma_{n \geqq 1} \mu(E_n) \cdot a_n^* = T_g^*(\chi_E)$, since for every $x \in X$ and every $k \geqq 1$

$$\begin{aligned} &|(x, \Sigma_{1 \leqq n \leqq k} \mu(E_n) \cdot a_n^* - T_g^*(\chi_E))| \\ &= \left| \Sigma_{1 \leqq n \leqq k} \int_{E_n} (x, g(s)) d\mu(s) - \int_E (x, g(s)) d\mu(s) \right| \\ &\leqq L \|x\| \cdot \mu(E \setminus \bigcup_{1 \leqq n \leqq k} E_n). \end{aligned}$$

So we have that

$$(x^{**}, T_g^*(\chi_E)) = (x^{**}, \Sigma_{n \geqq 1} \mu(E_n) \cdot a_n^*) = \Sigma_{n \geqq 1} \mu(E_n) \cdot (x^{**}, a_n^*).$$

On the other hand, we have by the dominated convergence theorem that

$$\int_E (x^{**}, g(s)) d\mu(s) = \Sigma_{n \geqq 1} \int_{E_n} (x^{**}, g(s)) d\mu(s).$$

Hence we have that

$$\begin{aligned} &\left| (x^{**}, T_g^*(\chi_E)) - \int_E (x^{**}, g(s)) d\mu(s) \right| \\ &\leqq \Sigma_{n \geqq 1} \left| \mu(E_n) \cdot (x^{**}, a_n^*) - \int_{E_n} (x^{**}, g(s)) d\mu(s) \right| \end{aligned}$$

$$\leqq \Sigma_{n\geqq 1}\int_{E_n}|(x^{**},a_n^*)-(x^{**},g(s))|d\mu(s)$$

$$\leqq \Sigma_{n\geqq 1}\int_{E_n}O(x^{**}|\overline{\mathrm{co}}^*(g(E_n)))d\mu(s)$$

$$\leqq \varepsilon\cdot\Sigma_{n\geqq 1}\mu(E_n)=\varepsilon\cdot\mu(E).$$

Since ε is arbitrary, we have that

$$(x^{**},T_g^*(\chi_E))=\int_E(x^{**},g(s))d\mu(s).$$

Thus the proof of Theorem is completed.

In virtue of Theorem, we easily have:

COROLLARY 1. *In Theorem, assume further that X is separable. Then f is C-Pettis integrable.*

3. K-weakly precompact sets

Let A be a bounded subset of X and K a weak*-compact (not necessarily convex) subset of X^*. Then we say that A is *K-weakly precompact* if every sequence $\{x_n\}_{n\geqq 1}$ in A has a pointwise convergent subsequence $\{x_{n(k)}\}_{k\geqq 1}$ on K, (that is, for every $x^*\in K$, $\lim_{k\to\infty}(x_{n(k)},x^*)$ exists). In the recent paper [8], we have made a study of K-weakly precompact sets A and have obtained various characterizations of such sets, which can be regarded as generalizations and refinements of some results in [3].

Now we are going to note in the following Corollary 2 that our Theorem also yields some more characterizations of K-weakly precompact sets A. The main part of this Corollary 2 is the equivalence among the statements (1), (2) and (3) (especially, (2) implies (3)). That is, we ought to take notice that characterizations of such sets in terms of $\bar{A}^*$-Pettis decomposability can be obtained by the medium of the notion of weak*-$\bar{A}^*$-dentability. In Corollary 2, $[0,1]$ is endowed with the σ-algebra of all Lebesgue measurable subsets of $[0,1]$ and the Lebesgue measure.

COROLLARY 2. *Let A be a bounded subset of X and K a weak*-compact subset of X^*. Then the following statements about A and K are equivalent.*

(1) *The set A is K-weakly precompact.*

(2) *For every (S,Σ,μ) and every weak*-measurable function $f:S\to K$, the set $\overline{\mathrm{co}}^*(T_f^*(\Delta_E))$ is weak*-$\bar{A}^*$-dentable for every $E\in\Sigma^+$.*

(3) *For every (S,Σ,μ) and every weak*-measurable function $f:S\to K$, f is $\bar{A}^*$-Pettis decomposable.*

(4) *Every weak*-measurable function* $f : [0,1] \to K$ *is* $\bar{A}^*$*-Pettis decomposable.*

(5) *For every* (S, Σ, μ) *and every weak*-measurable function* $f : S \to K$, f *is weak*-equivalent to a* $\bar{A}^*$*-measurable function.*

(6) *Every weak*-measurable function* $f : [0,1] \to K$ *is weak*-equivalent to a* $\bar{A}^*$*-measurable function.*

PROOF. $(1) \Rightarrow (2)$. This can be shown by the same argument as in the corresponding part of Theorem in [8].
$(2) \Rightarrow (3)$. This immediately follows from Theorem by setting $C = \bar{A}^*$.
$(3) \Rightarrow (4) \Rightarrow (6)$ and $(3) \Rightarrow (5) \Rightarrow (6)$. These are clear.
$(6) \Rightarrow (1)$. This has been obtained in Theorem of [8]. Hence the Corollary 2 holds.

REMARK. We wish to note here that the implication $(1) \Rightarrow (3)$ also can be proved by the effective use of the Bourgain property. The outline of its proof is as follows. Assume (1). Then, by the same argument as in Lemma 1 of [7], we first know that the family $\{x \circ \theta(f) : x \in A\}$ has the Bourgain property. Hence, by the same argument as in Theorem 13 of [9], $\theta(f)$ is $\bar{A}^*$-Pettis integrable. So (3) holds. Furthermore, we easily get by this observation and the argument developed in the proof of Theorems in [7] and [8] that every statement in Corollary 2 is equivalent to each of the following statements (α), (β), (γ) and (δ).

(α) For every (S, Σ, μ) and every weak*-measurable function $f : S \to K$, $\{x \circ \theta(f) : x \in A\}$ has the Bourgain property.

(β) For every weak*-measurable function $f : [0,1] \to K$, $\{x \circ \theta(f) : x \in A\}$ has the Bourgain property.

(γ) For every weak*-measurable function $f : [0,1] \to K$, $\{x \circ f_n : x \in A, n \geqq 1\}$ has the Bourgain property (Here f_n is the usual dyadic martingale associated with f).

(δ) For every weak*-measurable function $f : [0,1] \to K$,
$\inf_{n \geqq 1} \left\{ \sup_{x \in A} \|x \circ f_n - x \circ f_{n+1}\|_1 \right\} = 0$ (Here $\|\cdot\|_1$ denotes the L_1-norm).

In virtue of Corollary 2, we can obtain characterizations of Pettis sets, weak Radon-Nikodym sets and weakly precompact sets (see definitions of these notions in [8]) in terms of $\bar{A}^*$-Pettis decomposability, by varying A and K. For instance, setting $A = B(X)$, we have the following Corollary 3 on Pettis sets. As the other cases are analogous, we omit statements in them.

COROLLARY 3. *Let* K *be a weak*-compact subset of* X^*. *Then the following statements about* K *are equivalent.*

(1) *The set K is a Pettis set.*

(2) *For every (S, Σ, μ) and every weak*-measurable function $f : S \to K$, the set $\overline{\mathrm{co}}^*(T_f^*(\Delta_E))$ is weak*-dentable for every $E \in \Sigma^+$.*

(3) *For every (S, Σ, μ) and every weak*-measurable function $f : S \to K$, f is Pettis decomposable.*

(4) *Every weak*-measurable function $f : [0,1] \to K$ is Pettis decomposable.*

(5) *For every (S, Σ, μ) and every weak*-measurable function $f : S \to K$, f is weak*-equivalent to a weakly measurable function.*

(6) *Every weak*-measurable function $f : [0,1] \to K$ is weak*-equivalent to a weakly measurable function.*

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