Uniqueness for the Difference Monomials of P-Adic Entire Functions

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Abstract

The aim of this paper is to discuss the uniqueness of p-adic difference monomials $f^n f(z+c)$. The results obtained in this paper are the p-adic analogues and supplements of the theorems given by Qi, Yang and Liu [Uniqueness and periodicity of meromorphic functions concerning the difference operator, Comput. Math. Appl. 60(2010), 1739-1746], Wang, Han and Wen [Uniqueness theorems on difference monomials of entire functions, Abstract Appl. Anal. 2012(2012), Article ID 407351], Yang and Hua [Uniqueness and value-sharing of meromorphic functions, Ann. Acad. Sci. Fenn. Math. 22(1997), 395-406].

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1 Introduction and main results

W.K. Hayman proposed the following well-known conjecture.

Hayman's Conjecture [10]. If an entire function satisties $f^n f' \neq 1$ for all positive integers $n \in N$, then f is a constant.

It has been verified by Hayman himself in [11] for the case $n > 1$ and Clunie in [9] for the case $n \geq 1$, respectively. In 1997, corresponding to the above famous conjecture of Hayman, Yang and Hua studied the unicity of differential monomials and obtained the following theorem.

Theorem 1.1. [24] Let f and g be two nonconstant entire functions, $n \geq 6$ a positive integer. If $f^n f'$ and $g^n g'$ share 1 CM, then either $f(z) = c_1 e^{cz}$, $g(z) = c_2 e^{-cz}$, where c_1, c_2, c are three constants satisfying $(c_1 c_2)^{n+1} c^2 = -1$, or $f = tg$ for a constant t such that $t^{n+1} = 1$.

In 2010, Qi, Yang and Liu studied the uniqueness of difference monomials and obtained the following result.

Theorem 1.2. [21] Let f and g be transcendental entire functions with finite order, c a nonzero complex constant, and $n \geq 6$ an integer. If $f^n f(z+c)$ and $g^n g(z+c)$ share 1 CM, then $fg = t_1$ or $f = t_2 g$ for some constants t_1 and t_2 which satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

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In 2012, Wang, Han and Wen proved the following theorem.

Theorem 1.3. [22] Let f and g be transcendental entire functions with finite order, c a nonzero complex constant, and $n \geq 6$ an integer. If $E_{3)}(1, f^n f(z+c)) = E_{3)}(1, g^n g(z+c))$, then $fg = t_1$ or $f = t_2 g$ for some constants t_1 and t_2 which satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

In recent years, similar problems are investigated in non-Archimedean fields. Now let K be an algebraically closed field of characteristic zero, complete for a non-Archimedean absolute value. We denote by $A(K)$ the ring of entire functions in K and by $M(K)$ the field of meromorphic functions. The value sharing problems for meromorphic functions in K was investigated first in [1] and [13]. In recent years, numerous interesting results were obtained in the investigation of the value-sharing problem for meromorphic function in K [2]-[4], [6]-[8], [16]-[18], [19][20][23].

Let us recall some basic definitions. For $f \in M(K)$ and $S \subset \hat{K}$, we define

$$E_f(S) = \bigcup_{a \in S} \{(z,m) | f(z) = a \quad with\, multiplicity \quad m\},$$

and we denote by $E_f^k(a)$ the set of all a-points of f where an a-point with mutiplicity m is counted m times if $m \leq k$ and $k+1$ times if $m > k$. It's obvious that if $E_f^k(a) = E_g^k(a)$, then z_0 is a zero of $f - a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\leq k)$ and z_0 is a zero of $f - a$ with multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$, where m is not necessarily equal to n.

Let F be a nonempty subset of $M(K)$. Two functions f, g of F are said to share S, counting multiplicity(share S CM), if $E_f(S) = E_g(S)$.

In the present paper, we discuss the uniqueness problem of p-adic difference monomials $f^n f(z+c)$ and prove the following theorems.

Theorem 1.4. Let f and g be nonconstant p-adic entire functions, $n \geq 8$ an integer. If $E^2_{f^n f(z+c)}(1) = E^2_{g^n g(z+c)}(1)$, then $f = tg$, where t is a constant and $t^{n+1} = 1$.

Theorem 1.5. Let f and g be nonconstant p-adic entire functions, $n \geq 8$ an integer. If $E_{f^n f(z+c)}(1) = E_{g^n g(z+c)}(1)$, then $f = tg$, where t is a constant and $t^{n+1} = 1$.

The main tool of the proof is the p-adic Nevanlinna theory [12][13][14][15]. So in the next section, we establish the basic properties of the characteristic functions of p-adic meromorphic functions.

2 Counting functions and Characteristic functions of p-adic meromorphic functions

Let f be a nonconstant entire function on K and $b \in K$. Then we can write f in the following form

$$f = \sum_{n=q}^{\infty} b_n (z-b)^n,$$

where $b_q \neq 0$ and we denote $\omega_f^0(b) = q$. For a point $a \in K$, we define the function $\omega_f^a : K \to N$ by $\omega_f^a(b) = \omega_{f-a}^0(b)$.

For a real number ρ with $0 < \rho \leq r$. Take $a \in K$ and we set

$$N_f(a, r) = \frac{1}{ln\rho} \int_\rho^r \frac{n_f(a, x)}{x} dx \,,$$

where $n_f(a, x)$ is the number of solutions of the equation $f(z) = a$(counting multiplicities) in the disk $D_x = \{z \in K : |z| \leq x\}$. If $a = 0$, the we set $N_f(r) = N_f(0, r)$.

If l is a positive integer, then we define

$$N_{l,f}(a, r) = \frac{1}{ln\rho} \int_\rho^r \frac{n_{l,f}(a, x)}{x} dx \,,$$

where $n_{l,f}(a, x) = \sum_{|z| \leq r} \min\{\omega_{f-a}(z), l\}$.

Let k be a positive integer. Define the function ω_f^k from K into N by $\omega_f^k(z) = 0$ if $\omega_f^0(z) > k$ and $\omega_f^k(z) = \omega_f^0(z)$ if $\omega_f^0(z) \leq k$. And $n_f^{\leq k}(r) = \sum_{|z| \leq r} \omega_f^{\leq k}(z)$, $n_f^{\leq k}(a, r) = n_{f-a}^{\leq k}(r)$.

Define

$$N_f^{\leq k}(a, r) = \frac{1}{ln\rho} \int_\rho^r \frac{n_f^{\leq k}(a, x)}{x} dx \,,$$

If $a = 0$, then we set $N_f^{\leq k}(r) = N_f^{\leq k}(0, r)$. Set

$$N_{l,f}^{\leq k}(a, r) = \frac{1}{ln\rho} \int_\rho^r \frac{n_{l,f}^{\leq k}(a, x)}{x} dx \,,$$

where $n_{l,f}^{\leq k}(a, x) = \sum_{|z| \leq r} \min\{\omega_{f-a}^{\leq k}(z), l\}$. In a similar way, we can define $N_f^{<k}(a, r)$, $N_{l,f}^{<k}(a, r)$, $N_f^{>k}(a, r)$, $N_f^{\geq k}(a, r)$, $N_{l,f}^{\geq k}(a, r)$ and $N_{l,f}^{>k}(a, r)$.

Recall that for a nonconstant entire function $f(z)$ on K, represented by the power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

for each $r > 0$, we define $|f|_r = \max\{|a_n| r^n, 0 \leq n < \infty\}$.

Now let $f = \frac{f_1}{f_2}$ be a nonconstant meromorphic function on K, where f_1 and f_2 are entire functions on K having no common zeros. We set $|f|_r = \frac{|f_1|}{|f_2|}$. For a point $a \in K \cup \{\infty\}$, we define the function $\omega_f^a : K \to N$ by $\omega_f^a(b) = \omega_{f_1 - af_2}^0(b)$ with $a \neq \infty$ and $\omega_f^\infty(b) = \omega_{f_2}^0(b)$.

Taking $a \in K$, we denote the counting function of zeros of $f-a$, counting multiplicity, in the disk $D_r = \{z \in K : |z| \le r\}$, i.e. we set $N_f(a,r) = N_{f_1 - af_2}(r)$ and set $N_f(\infty, r) = N_{f_2}(r)$. In a similar way, for nonconstant meromorphic function on K, we can define $N_f^{<k}(a,r)$, $N_{l,f}^{<k}(a,r)$, $N_f^{>k}(a,r)$, $N_f^{\ge k}(a,r)$, $N_{l,f}^{\ge k}(a,r)$ and $N_{l,f}^{>k}(a,r)$.

We define

$$m_f(\infty, r) = \max\{0, log|f|_r\}, \quad m_f(a,r) = m_{\frac{1}{f-a}}(\infty, r)\,,$$

and then characteristic function of f by

$$T_f(r) = m_f(\infty, r) + N_f(\infty, r)\,.$$

Thus we get

$$N_f(a,r) + m_f(a,r) = T_f(r) + O(1)\,,$$

where $a \in K \cup \{\infty\}$ and

$$T_f(r) = T_{\frac{1}{f}}(r) + O(1), \quad m_{\frac{f^{(k)}}{f}}(\infty, r) = O(1)\,.$$

3 Some Lemmas

In this section, we present some lemmas which will be needed in the sequel.

Lemma 3.1. [12][5] Let f be a nonconstant meromorphic function on K and let a_1, a_2,...,a_q be distinct points of K. Then

$$(q-1)T_f(r) \le N_{1,f}(\infty, r) + \sum_{i=1}^{q} N_{1,f}(a_i, r) - N_{0,f'}(r) - logr + O(1)\,.$$

Lemma 3.2. Let f and g be nonconstant meromorphic functions on K. If $E_f^2(1) = E_g^2(1)$, then one of the following three cases holds:

$$\begin{aligned}(i) \quad T_f(r) \le\ & N_{1,f}(0,r) + N_{1,f}^{\ge 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\ge 2}(0,r) + N_{1,f}(\infty, r) \\ & + N_{1,f}^{\ge 2}(\infty, r) + N_{1,g}(\infty, r) + N_{1,g}^{\ge 2}(\infty, r) - logr + O(1)\,,\end{aligned}$$

$$(ii) \quad f = g\,, \quad (iii) \quad fg = 1\,.$$

Proof. Set

$$H = \left(\frac{f''}{f'} - \frac{2f'}{f-1}\right) - \left(\frac{g''}{g'} - \frac{2g'}{g-1}\right)\,.$$

First we suppose that $H \not\equiv 0$. It's obvious that $m_H(\infty, r) = O(1)$, and

$$\begin{aligned} N_f^{\le 1}(1,r) &\le N_H(0,r) \le T_H(r) + O(1) \le N_H(\infty, r) + O(1) \\ &\le N_{1,f}^{\ge 2}(0,r) + N_{1,g}^{\ge 2}(0,r) + N_{1,f}^{\ge 2}(\infty, r) + N_{1,g}^{\ge 2}(\infty, r) \\ &\quad + N_{1,0,f'}(r) + N_{1,0,g'}(r) + O(1)\,, \end{aligned} \tag{1.1}$$

where $N_{1,0,f'}(r)$ is the counting function of those zeros of f' that are not zeros of $f(f-1)$, while each zero is counted with multiplicity 1.
On the other hand, by Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty,r) + N_{1,f}(0,r) + N_{1,f}(1,r) - N_{0,f'}(r) - logr + O(1)\,. \tag{1.2}$$

Since $E_f^2(1) = E_g^2(1)$, we note that

$$N_{1,f}(1,r) = N_f^{\leq 1}(1,r) + N_{1,f}^{\geq 2}(1,r) = N_f^{\leq 1}(1,r) + N_{1,g}^{\geq 2}(1,r)\,, \tag{1.3}$$

Then

$$\begin{aligned} T_f(r) \leq\ & N_{1,f}(\infty,r) + N_{1,f}(0,r) + N_f^{\leq 1}(1,r) \\ & + N_{1,g}^{\geq 2}(1,r) - N_{0,f'}(r) - logr + O(1)\,. \end{aligned} \tag{1.4}$$

Next we consider $N_{1,g}^{\geq 2}(1,r)$.

$$\begin{aligned} & N_{g'}(0,r) - N_g(0,r) + N_{1,g}(0,r) = N_{\frac{g'}{g}}(0,r) \leq T_{\frac{g'}{g}}(r) + O(1) \\ & = N_{\frac{g'}{g}}(\infty,r) + m_{\frac{g'}{g}}(\infty,r) + O(1) = N_{1,g}(\infty,r) + N_{1,g}(0,r) + O(1)\,. \end{aligned} \tag{1.5}$$

So

$$N_{g'}(0,r) \leq N_{1,g}(\infty,r) + N_g(0,r) + O(1)\,. \tag{1.6}$$

Moreover

$$N_{0,g'}(r) + N_{1,g}^{\geq 2}(1,r) + N_g^{\geq 2}(0,r) - N_{1,g}^{\geq 2}(0,r) \leq N_{g'}(0,r)\,, \tag{1.7}$$

where $N_{0,g'}(r)$ is the counting function of those zeros of g' that are not zeros of $g(g-1)$. From (6) and (7) , we get

$$N_{0,g'}(r) + N_{1,g}^{\geq 2}(1,r) \leq N_{1,g}(\infty,r) + N_{1,g}(0,r) + O(1)\,. \tag{1.8}$$

Combining (1), (4) and (8), we obtain

$$\begin{aligned} T_f(r) \leq\ & N_{1,f}(0,r) + N_{1,f}^{\geq 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\geq 2}(0,r) + N_{1,f}(\infty,r) \\ & + N_{1,f}^{\geq 2}(\infty,r) + N_{1,g}(\infty,r) + N_{1,g}^{\geq 2}(\infty,r) - logr + O(1)\,. \end{aligned}$$

Suppose $H \equiv 0$. Then by integration we get

$$f \equiv \frac{ag+b}{cg+d}\,, \tag{1.9}$$

where a, b, c and d are constants and $ad - bc \neq 0$. So $T_f(r) = T_g(r) + O(1)$.

We now consider the following cases.

Case 1. Let $ac \neq 0$. Then

$$f - \frac{a}{c} = \frac{bc - ad}{c(cg + d)} . \tag{1.10}$$

So, By Lemma 3.1, we get

$$\begin{aligned} T_f(r) &\leq N_{1,f}(\infty, r) + N_{1,f-\frac{a}{c}}(0, r) + N_{1,f}(0, r) + O(1) \\ &= N_{1,f}(0, r) + N_{1,f}(\infty, r) + N_{1,g}(\infty, r) + O(1) , \end{aligned}$$

which implies (i).

Case 2. $a \neq 0$ and $c = 0$. Then $f = \frac{a}{d}g + \frac{b}{d}$. If $b \neq 0$, by Lemma 3.1,

$$\begin{aligned} T_f(r) &\leq N_{1,f}(\infty, r) + N_{1,f-\frac{b}{d}}(0, r) + N_{1,f}(0, r) + O(1) \\ &= N_{1,f}(\infty, r) + N_{1,g}(0, r) + N_{1,f}(0, r) + O(1) , \end{aligned}$$

which implies (i).

If $b = 0$, then $f = \frac{ag}{d}$. If $\frac{a}{d} = 1$, we obtain (ii). If $\frac{a}{d} \neq 1$, then by $E_f^2(1) = E_g^2(1)$ we get $f \neq 1$ and $f \neq \frac{a}{d}$. According to Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty, r) + N_{1,f}(1, r) + N_{1,f}(\frac{a}{d}, r) + O(1) = N_{1,f}(\infty, r) + O(1) ,$$

which implies (i).

Case 3. $a = 0$ and $c \neq 0$. Then $f = \frac{b}{cg+d}$. If $d \neq 0$, by Lemma 3.1,

$$\begin{aligned} T_f(r) &\leq N_{1,f}(\infty, r) + N_{1,f-\frac{b}{d}}(0, r) + N_{1,f}(0, r) + O(1) \\ &= N_{1,f}(\infty, r) + N_{1,g}(0, r) + N_{1,f}(0, r) + O(1) , \end{aligned}$$

which implies (i).

If $d = 0$, then $f = \frac{b}{cg}$. If $\frac{b}{c} = 1$, we obtain (iii). If $\frac{b}{c} \neq 1$, then by $E_f^2(1) = E_g^2(1)$ we get $f \neq 1$ and $f \neq \frac{b}{c}$. According to Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty, r) + N_{1,f}(1, r) + N_{1,f}(\frac{b}{c}, r) + O(1) = N_{1,f}(\infty, r) + O(1) ,$$

which implies (i). The proof of Lemma 3.2 is complete.

Lemma 3.3. [16] Let f and g be nonconstant meromorphic functions on K. If $E_f(1) = E_g(1)$, then one of the following three cases holds:

$$\begin{aligned} (i) \quad T_f(r) \leq{}& N_{1,f}(0, r) + N_{1,f}^{\geq 2}(0, r) + N_{1,g}(0, r) + N_{1,g}^{\geq 2}(0, r) + N_{1,f}(\infty, r) \\ &+ N_{1,f}^{\geq 2}(\infty, r) + N_{1,g}(\infty, r) + N_{1,g}^{\geq 2}(\infty, r) - logr + O(1) , \end{aligned}$$

$$(ii) \quad f = g , \quad (iii) \quad fg = 1 .$$

Lemma 3.4. [2] Let f be a nonconstant p-adic meromorphic function. Then

$$m_{\frac{f(z+c)}{f}}(\infty, r) = O(1);\ T_{f(z+c)}(r) = T_{f(z)}(r) + O(1)\,.$$

Lemma 3.5. Let f be a p-adic entire function, $c \in K$. If $F(z) = f^n(z)f(z+c)$, then

$$T(r, F) = (n+1)T(r, f) + O(1)\,.$$

Proof. We can deduce form Lemma 3.4 that

$$\begin{aligned}(n+1)T_f(r) = T_{f^{n+1}}(r) + O(1) = m_{f^{n+1}}(r) + O(1)\\ \le m_{\frac{f^{n+1}}{F}}(r) + m_F(r) + O(1) = m_{\frac{f}{f(z+c)}}(r) + m_F(r) + O(1)\\ \le T_F(r) + O(1)\,.\end{aligned}$$

Therefore

$$(n+1)T_f(r) \le T_F(r) + O(1)\,.$$

On the other hand, Lemma 3.4 implies

$$T_F(r) \le T_{f^n}(r) + T_{f(z+c)}(r) = nT_f(r) + T_f(r) + O(1) = (n+1)T_f(r) + O(1)\,.$$

We obtain the conclusion of Lemma 3.5.

4 Proof of Theorem 1.4

Let

$$F = f^n f(z+c),\ G = g^n g(z+c)\,. \tag{1.11}$$

Then it is easy to verify $E_F^2(1) = E_G^2(1)$. Suppose the Case (i) in Lemma 3.2 holds

$$T_F(r) \le N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) + N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) - logr + O(1)\,. \tag{1.12}$$

From Lemma 3.4, we have

$$\begin{aligned}N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) \le 2N_{1,F}(0,r)\\ = 2N_{1,f}(0,r) + 2N_{1,f(z+c)}(0,r) \le 4T_f(r)\,,\end{aligned} \tag{1.13}$$

and

$$\begin{aligned}N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) \le 2N_{1,G}(0,r)\\ = 2N_{1,g}(0,r) + 2N_{1,g(z+c)}(0,r) \le 4T_g(r)\,.\end{aligned} \tag{1.14}$$

From (12), (13), (14) and Lemma 3.5, we deduce

$$T_F(r) = (n+1)T_f(r) \le 4T_f(r) + 4T_g(r) + O(1)\,, \tag{1.15}$$

that is,

$$(n-3)T_f(r) \le 4T_g(r) + O(1)\,. \tag{1.16}$$

Similarly we can deduce

$$(n-3)T_g(r) \le 4T_f(r) + O(1)\,. \tag{1.17}$$

Combining (16) and (17), we have

$$(n-7)T_f(r) + (n-7)T_g(r) \le O(1)\,, \tag{1.18}$$

which contradicts the hypothesis $n \ge 8$. Therefore $F = G$ or $FG = 1$.

If $F = G$, that is

$$f^n(z)f(z+c) = g^n(z)g(z+c)\,. \tag{1.19}$$

Let $h(z) = \frac{f(z)}{g(z)}$. We have

$$h^n(z)h(z+c) = 1\,. \tag{1.20}$$

If $h(z)$ is not a constant, then Lemma 3.4 implies

$$nT_h(r) = T_{h(z+c)}(r) + O(1) = T_h(r) + O(1)\,, \tag{1.21}$$

which is a contadiction with $n \ge 8$. Thus $h(z) = t$, where t is a constant. From (20) we have $f = tg$ and $t^{n+1} = 1$.

If $FG = 1$, that is

$$f^n(z)f(z+c)g^n(z)g(z+c) = 1\,. \tag{1.22}$$

Let $\omega(z) = f(z)g(z)$. We have

$$\omega^n(z)\omega(z+c) = 1\,. \tag{1.23}$$

By a similar discussion, we can show that ω is a constant. Therefore $fg = \omega$ and $\omega^{n+1} = 1$. This is a contradiction because nonconstant entire function on K have at least one zero and hence, if fg is a constant, at least one of the two functions f or g is meromorphic, but not entire. This completes the proof of Theorem 1.4.

5 Proof of Theorem 1.5

Let

$$F = f^n f(z+c),\ G = g^n g(z+c)\,. \tag{1.24}$$

Then it is easy to verify $E_F(1) = E_G(1)$. Suppose the Case (i) in Lemma 3.3 holds

$$T_F(r) \le N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) + N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) - logr + O(1)\,. \tag{1.25}$$

Similar to the arguments in Theorem 1.4, we see that Theorem 1.5 holds.

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