On Soft Supra Compactness in Supra Soft Topological Spaces

A. M. Abd El-latif

Mathematics Department, Faculty of Arts and Science, Northern Border University, Rafha, KSA.
Mathematics Department, Faculty of Education, Ain Shams University, Roxy, 11341, Cairo, Egypt.

E-mail: Alaa.ali@nbu.edu.sa, Alaa_8560@yahoo.com

Abstract

In this paper we introduce anew form of soft supra compact spaces namely, soft supra compact spaces, soft supra closed spaces, soft supra lindelöf spaces and soft supra generalized compactness. Furthermore, we study its several properties and characterizations in detail. Also, the invariance of these kinds of soft supra compact spaces under some types of soft mapping and their hereditary properties are also investigated.

2010 Mathematics Subject Classification. **54A05**. 54A40, 54D20, 06D72
Keywords. Soft supra compactness, Soft supra lindelöf spaces, Soft supra generalized compactness, Soft functions.

1 Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties present in these problems. To overcome these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. Molodtsov [15] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which are free from the above difficulties. The notions of supra soft topological space were first introduced by El-Sheikh et al. [9], which extended in [1, 2, 3, 4, 5]

The purpose of this paper, is to introduce some types of soft compactness to supra soft topological spaces. Furthermore, several of their topological properties are investigated. The behavior of these concepts under various types of soft mappings has obtained.

2 Preliminaries

In this section, we present the basic definitions and results of (supra) soft set theory which will be needed in this paper. For more detail see [6, 8, 9, 10, 12, 15, 16, 17].

Definition 2.1. [16] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1) $\tilde{X}, \tilde{\varphi} \in \tau$, where $\tilde{\varphi}(e) = \varphi$ and $\tilde{X}(e) = X$, $\forall e \in E$,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

Tbilisi Mathematical Journal 11(1) (2018), pp. 169–178.
Tbilisi Centre for Mathematical Sciences.

Received by the editors: 10 April 2017.
Accepted for publication: 10 January 2018.

The triplet (X, τ, E) is called a soft topological space over X. A soft set (F, E) over X is said to be closed soft set in X, if its relative complement is an open soft set. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X)$.

Definition 2.2. [16] Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then, $\tilde{Y}$ denotes the soft set (Y, E) over X for which $\tilde{Y}(e) = Y \ \forall e \in E$.

Definition 2.3. [16] Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then, the sub soft set of (F, E) over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e) = Y \cap F(e) \ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y} \tilde{\cap} (F, E)$.

Definition 2.4. [16] Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then,

$$\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$$

is called the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.5. [7] Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets on X and Y respectively, $u : X \to Y$ and $p : E \to K$ be mappings. Let $f_{pu} : SS(X)_E \to SS(Y)_K$ be a mapping. Then;

(1) If $(F, E) \in SS(X)_E$. Then, the image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_K$ such that,
$f_{pu}(F)(k) = \begin{cases} \cup_{e \in p^{-1}(k) \cap E} \ u(F(e)), & p^{-1}(k) \cap E \neq \varphi, \\ \varphi, & otherwise. \end{cases}$
for all $k \in K$.

(2) If $(G, K) \in SS(Y)_K$. Then, the inverse image of (G, K) under f_{pu}, written as $f_{pu}^{-1}(G, K) = (f_{pu}^{-1}(G), p^{-1}(K))$, is soft set in $SS(X)_E$ such that,
$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(E))), & p(E) \in K, \\ \varphi, & otherwise. \end{cases}$
for all $e \in E$.

The soft function f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective).

Definition 2.6. [8] A family $\Psi = \{(U_i, E) : i \in \Lambda\}$ of soft sets is said to be a soft cover of a soft set (F, E), if $(F, E) \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda} (U_i, E)$. It is called open soft cover, if each member of Ψ is an open soft set. A subcover of Ψ is a subfamily of Ψ that is also a soft cover.

Definition 2.7. [18] A family Ψ of soft sets has the soft finite intersection property (SFIP), if the intersection of the members of each finite subfamily of Ψ is nonempty set.

Definition 2.8. [8] A soft topological space (X, τ, E) is called soft compact, if each open soft cover of $\tilde{X}$ has a finite subcover.

Definition 2.9. [17] A soft topological space (X, τ, E) is called soft lindelöf, if each open soft cover of $\tilde{X}$ has a countable subcover.

Definition 2.10. [9] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called supra soft topology on X if

(1) $\tilde{X}, \tilde{\varphi} \in \mu$,

(2) the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.11. [9] Let (X, τ, E) be a soft topological space and (X, μ, E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subset \mu$.

Definition 2.12. [9] Let (X, μ, E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X. We denote the set of all supra open soft sets over X by supra-$OS(X, \mu, E)$, or when there can be no confusion by supra-$OS(X)$ and the set of all supra closed soft sets by supra-$CS(X, \mu, E)$, or supra-$CS(X)$.

Definition 2.13. [9] Let (X, μ, E) be a supra soft topological space over X and $(F, E) \in SS(X)_E$. Then, the supra soft interior of (G, E), denoted by $int^s(G, E)$ is the soft union of all supra open soft subsets of (G, E) i.e
$int^s(G, E) = \tilde{\cup}\{(H, E) : (H, E) \text{ is supra open soft set and } (H, E) \tilde{\subseteq} (G, E)\}$.

Definition 2.14. [9] Let (X, μ, E) be a supra soft topological space over X and $(F, E) \in SS(X)_E$. Then, the supra soft closure of (F, E), denoted by $cl^s(F, E)$ is the soft intersection of all supra closed super soft sets of (F, E) i.e
$cl^s(F, E) = \tilde{\cap}\{(H, E) : (H, E) \text{ is supra closed soft set and } (F, E) \tilde{\subseteq} (H, E)\}$.

Definition 2.15. [2, 9] Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. The soft function $f_{pu} : SS(X)_E \rightarrow SS(Y)_K$ is called

1. Supra continuous soft function if $f_{pu}^{-1}(G, K) \in \mu \ \forall \ (G, K) \in \sigma$.
2. Supra open soft if $f_{pu}(F, E) \in \mu$ for each $(F, E) \in \tau$.
3. Supra irresolute soft if $f_{pu}^{-1}(G, K) \in \mu$ for each $(G, K) \in \nu$.
4. Supra irresolute open soft if $f_{pu}(F, E) \in \nu$ for each $(F, E) \in \mu$.

Definition 2.16. [13] A soft set (F, E) is called soft supra generalized closed set (soft supra g-closed) in a supra soft topological space (X, μ, E) if $cl^s(F, E) \tilde{\subseteq} (G, E)$ whenever $(F, E) \tilde{\subseteq} (G, E)$ and (G, E) is supra open soft in X. Also, it is called soft supra g-open set (soft supra g-open) if its relative complement $(F^{\tilde{c}}, E)$ is soft supra g-closed.

3 Soft Supra Compactness

In this section, we generalize the soft compact spaces [8] and soft lindelöf spaces [12, 17] to supra soft topological spaces [9] and investigate their properties in detail.

Definition 3.1. A family $\Psi = \{(U_i, E) : i \in \Lambda\}$ of soft sets is said to be a supra open soft cover, if each member of Ψ is supra open soft set.

Definition 3.2. A soft subset (F,E) of the space (X,μ,E) is said to be soft supra compact (resp. soft supra lindelöf), if every supra open soft cover $\{(U_i,E) : i\in\Lambda\}$ of (F,E) has a finite (resp. countable) subfamily Λ_o of Λ such that

$$(F,E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(U_i,E).$$

The space (X,μ,E) is said to be soft supra compact if $\tilde{X}$ is soft supra compact as a soft subset.

Proposition 3.3. If X is finite (resp. countable), then (X,μ,E) is soft supra compact (resp. soft supra lindelöf) for any supra soft topology μ on X.

Proof. Obvious. Q.E.D.

Definition 3.4. Let (X,μ,E) be a supra soft topological space and Y be a non null subset of X. Then,

$$\mu_Y = \{(F_Y,E) : (F,E)\in\mu\}$$

is called the supra soft relative topology on Y and (Y,μ_Y,E) is called a supra soft subspace of (X,μ,E).

Theorem 3.5. Every supra closed soft subspace of a soft supra compact space is soft supra compact.

Proof. Let (A,E) be a supra closed soft subspace of soft supra compact space (X,μ,E) and $\{(U_i,E) : i\in\Lambda\}$ be a supra open soft cover of (A,E). Then, $\{(U_i,E) : i\in\Lambda\}\tilde{\cup}(A^{\tilde{c}},E)$ is a supra open soft cover of $\tilde{X}$ and for $\tilde{X}$ is soft supra compact, there exists a finite subcover $\{(U_i,E) : i\in\Lambda_o\}\tilde{\cup}(A^{\tilde{c}},E)$ for $\tilde{X}$. Now, $[\{(U_i,E) : i\in\Lambda_o\}\tilde{\cup}(A^{\tilde{c}},E)]-(A^{\tilde{c}},E)$ is a finite subcover of $\{(U_i,E) : i\in\Lambda\}$ for (A,E). So, (A,E) is soft supra compact. Q.E.D.

Theorem 3.6. Every supra closed soft subspace of a soft supra lindelöf space is soft supra lindelöf.

Proof. It is similar to the proof of the above theorem. Q.E.D.

Theorem 3.7. A supra soft topological space (X,μ,E) is soft supra compact if and only if each family of supra closed soft sets in $\tilde{X}$ with the SFIP has a nonempty intersection.

Proof. Let $\{(F_i,E) : i\in\Lambda\}$ be a family of supra closed soft sets with the SFIP and $\tilde{\cap}_{i\in\Lambda}(F_i,E)=\tilde{\varphi}$. Then, $\tilde{\cup}_{i\in\Lambda}(F_i,E)^{\tilde{c}}=\tilde{X}$. Hence, the family $\{(F_i,E)^{\tilde{c}} : i\in\Lambda\}$ forms a supra open soft cover of $\tilde{X}$. Since (X,μ,E) is soft supra compact space. Then, there exists a finite subfamily Λ_o of Λ which also covers $\tilde{X}$. i.e., $\tilde{\cup}_{i\in\Lambda_o}(F_i,E)^{\tilde{c}}=\tilde{X}$. Consequently, $\tilde{\cap}_{i\in\Lambda_o}(F_i,E)=\tilde{\varphi}$, which is a contradiction. On the other hand, if (X,μ,E) is not soft supra compact space, then there exists a supra closed soft cover $\{(F_i,E) : i\in\Lambda\}$ of $\tilde{X}$ such that for every finite subfamily Λ_o of Λ we have $\tilde{\cup}_{i\in\Lambda_o}(F_i,E)\neq\tilde{X}$. Hence, $\tilde{\cap}_{i\in\Lambda_o}(F_i,E)^{\tilde{c}}\neq\tilde{\varphi}$, and so $\{(F_i,E)^{\tilde{c}} : i\in\Lambda\}$ has the SFIP. By hypothesis, $\tilde{\cap}_{i\in\Lambda}(F_i,E)^{\tilde{c}}\neq\tilde{\varphi}$. Hence, $\tilde{\cup}_{i\in\Lambda}(F_i,E)\neq\tilde{X}$, which is a contradiction. Q.E.D.

Theorem 3.8. A supra soft topological space (X,μ,E) is soft supra compact if and only if every family Ψ of supra soft sets with SFIP, $\tilde{\cap}\{cl^s(F,E) : (F,E)\in\Psi\}\neq\tilde{\varphi}$.

Proof. **Necessity:** Suppose that there exists a family Ψ of supra soft sets with the SFIP such that $\tilde{\cap}\{cl^s(F,E):(F,E)\tilde{\in}\Psi\}=\tilde{\varphi}$. Hence, $\tilde{\cup}\{(cl^s(F,E))^{\tilde{c}}:(F,E)\tilde{\in}\Psi\}=\tilde{X}$. Consequently, $\{(cl^s(F,E))^{\tilde{c}}:(F,E)\tilde{\in}\Psi\}$ is a supra open soft cover of $\tilde{X}$ and for (X,μ,E) is soft supra compact space, there exists a finite soft subcover Ψ_o of Ψ such that, $\tilde{\cup}\{(cl^s(F,E))^{\tilde{c}}:(F,E)\tilde{\in}\Psi_o\}=\tilde{X}$. This implies that, $\tilde{\cup}\{(F,E)^{\tilde{c}}:(F,E)\tilde{\in}\Psi_o\}=\tilde{X}$, and so $\tilde{\cap}\{(F,E):(F,E)\tilde{\in}\Psi_o\}=\tilde{\varphi}$, which is a contradiction. Hence, $\tilde{\cap}\{cl^s(F,E):(F,E)\tilde{\in}\Psi\}\neq\tilde{\varphi}$.

Sufficient: Assume that (X,μ,E) is not soft supra compact space. Then, there exists a family Υ of supra open soft sets covering $\tilde{X}$ without a finite soft subcover. So, for every finite subfamily Υ_o of Υ we have $\tilde{\cup}\{(F,E):(F,E)\tilde{\in}\Upsilon_o\}\neq\tilde{X}$. Hence, $\tilde{\cap}\{(F,E)^{\tilde{c}}:(F,E)\tilde{\in}\Upsilon_o\}\neq\tilde{\varphi}$, and therefore $\{(F,E)^{\tilde{c}}:(F,E)\tilde{\in}\Upsilon_o\}$ is a family of supra soft sets with SFIP. Now, $\tilde{\cup}\{(F,E):(F,E)\tilde{\in}\Upsilon\}=\tilde{X}$, and hence $\tilde{\cap}\{(F,E)^{\tilde{c}}:(F,E)\tilde{\in}\Upsilon\}=\tilde{\varphi}$. It follows that, $\tilde{\cap}\{(cl^s(F,E))^{\tilde{c}}:(F,E)\tilde{\in}\Upsilon\}=\tilde{\varphi}$, which is a contradiction. Q.E.D.

Theorem 3.9. In a supra soft topological space (X,μ,E), if (F,E) is soft supra compact and (K,E) is supra open soft set contained in (F,E). Then, $((F,E)-(K,E))$ is soft supra compact set.

Proof. Let $\{(U_i,E):i\tilde{\in}\Lambda\}$ be a supra open soft cover of $((F,E)-(K,E))$. Then, $((F,E)-(K,E))\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda}(U_i,E)$. Since $(K,E)\tilde{\subseteq}(F,E)$ and (K,E) is supra open soft,

$$(F,E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda}(U_i,E)\tilde{\cup}(K,E).$$

Since (F,E) is soft supra compact, there exists a finite subfamily Λ_o of Λ such that

$$(F,E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(U_i,E)\tilde{\cup}(K,E).$$

Therefore, $((F,E)-(K,E))\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(U_i,E)\tilde{\cap}(K^{\tilde{c}},E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(U_i,E)$, and hence $(F,E)-(K,E)$ is a soft supra compact.

Q.E.D.

Theorem 3.10. The finite (resp. countable) soft union of soft supra compact (resp. soft supra lindelöf) sets is soft supra compact (resp. soft supra lindelöf).

Proof. We will show the case when (F,E) and (H,E) are two soft supra compact sets, the other case is similar. Assume that $\{(U_i,E):i\tilde{\in}\Lambda\}$ is a supra open soft cover of $(F,E)\tilde{\cup}(H,E)$. Then, $\{(U_i,E):i\in\Lambda\}$ is a supra open soft cover of (F,E) and (H,E). Since (F,E) and (H,E) are soft supra compact, there exist finite subfamilies Λ_o and Λ_1 of Λ such that $(F,E)\tilde{\subseteq}\tilde{\cup}_{ir\in\Lambda_o}(U_{ir},E)$ and $(H,E)\tilde{\subseteq}\tilde{\cup}_{ij\in\Lambda_1}(U_{ij},E)$. Hence, $(F,E)\tilde{\cup}(H,E)\tilde{\subseteq}(\tilde{\cup}_{ir\in\Lambda_o}(U_{ir},E))\tilde{\cup}(\tilde{\cup}_{ij\in\Lambda_1}(U_{ij},E))$. It follows that,

$$(F,E)\tilde{\cup}(H,E)\tilde{\subseteq}\tilde{\cup}_{ir\in\Lambda_o,ij\in\Lambda_1}[(U_{ir},E)\tilde{\cup}(U_{ij},E)]\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda}(U_i,E).$$

Thus, $(F,E)\tilde{\cup}(H,E)$ is soft supra compact.

Q.E.D.

Theorem 3.11. Let (A,E) be a soft supra compact (resp. soft supra lindelöf) subset of supra soft topological space (X,μ,E) and (B,E) be a supra closed soft subset of $\tilde{X}$. Then, $(A,E)\tilde{\cap}(B,E)$ is soft supra compact (resp. soft supra lindelöf).

Proof. We will show the case when (A, E) is soft supra compact subset of $\tilde{X}$, the other case is similar. Suppose that $(G, E) = \{(G_i, E) : i\in\Lambda\}$ is a supra open soft cover of $(A, E)\tilde{\cap}(B, E)$. Then, $(O, E) = \{(G_i, E) : i \in \Lambda\}\tilde{\cup}(B^{\tilde{c}}, E)$ is a supra open soft cover of (A, E). But, (A, E) is soft supra compact. So, there exists a finite subfamily Λ_o of Λ such that $(A, E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(G_i, E)\tilde{\cup}(B^c, E)$. Hence, $(A, E)\tilde{\cap}(B, E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(G_i, E)\tilde{\cap}(B, E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(G_i, E)$. Therefore, $(A, E)\tilde{\cap}(B, E)$ is soft supra compact.

Q.E.D.

Theorem 3.12. Every supra soft subspace of a supra soft topological space (X, μ, E) is soft supra compact if and only if every supra open soft subspace of $\tilde{X}$ is soft supra compact.

Proof. For the necessity, it is obvious. Conversely, let (Y, μ_Y, E) be a supra open soft subspace of supra soft topological space (X, μ, E) and $\{(U_\alpha, E) : \alpha\in\Lambda\}$ be a supra open soft cover of $\tilde{Y}$. Assume that $(V, E) = \tilde{\cup}_{\alpha\in\Lambda}(U_\alpha, E)$. Hence (V, E) is supra open soft subspace of $\tilde{X}$. By hypothesis, (V, E) is soft supra compact. So, $\{(U_\alpha, E) : \alpha \in \Lambda_o, \Lambda_o \; is \; finite\}$ is a finite subcover of (V, E). It is follows, $(V, E)\tilde{\subseteq}\tilde{\cup}_{\alpha\in\Lambda_o}(U_\alpha, E)$ and hence $(Y, E)\tilde{\subseteq}(V, E)\tilde{\subseteq}\tilde{\cup}_{\alpha\in\Lambda_o}(U_\alpha, E)$. Therefore, (Y, μ_Y, E) is soft supra compact.

Q.E.D.

Theorem 3.13. Every supra soft subspace of a supra soft topological space (X, μ, E) is soft supra lindelöf if and only if every supra open soft subspace of $\tilde{X}$ is soft supra lindelöf.

Proof. It similar to the proof of Theorem 3.12. Q.E.D.

Theorem 3.14. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let f_{pu}:$(X_1, \tau_1, E) \longrightarrow (X_2, \tau_2, K)$ be a supra soft continuous surjective function. If (X_1, τ_1, E) is soft supra compact, then (X_2, τ_2, K) is soft compact.

Proof. Let $\{(U_i, K) : i\in\Lambda\}$ be a τ_2-open soft cover of $\tilde{X}_2$. Since f_{pu} is supra soft continuous function, $\{f_{pu}^{-1}((U_i, K)) : i\in\Lambda\}$ is a μ-supra open soft cover of $\tilde{X}_1$ and for $\tilde{X}_1$ is soft supra compact, there exists a finite subfamily Λ_o of Λ such that $\{f_{pu}^{-1}((U_i, K)) : i\in\Lambda_o\}$ also forms a μ-supra open soft cover of $\tilde{X}_1$. Since f_{pu} is surjective, $\{f_{pu}(f_{pu}^{-1}((U_i, K))) : i\in\Lambda_o\} = \{(U_i, K) : i\in\Lambda_o\}$ forms a finite τ_2-open soft cover of $\tilde{X}_2$. The proof is completed.

Q.E.D.

Theorem 3.15. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let f_{pu}:$(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra soft continuous surjective function. If (X_1, τ_1, E) is soft supra lindelöf, then (X_2, τ_2, K) is soft lindelöf.

Proof. Similar to the proof of the above theorem. Q.E.D.

Theorem 3.16. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let f_{pu}:$(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra open soft injective function. If (G, K) is soft supra compact subset of $\tilde{X}_2$, then $f_{pu}^{-1}(G, K)$ is soft compact in $\tilde{X}_1$.

Proof. Let $\{(H_i, E) : i\in\Lambda\}$ be a τ_1-open soft cover of $f_{pu}^{-1}(G, K)$. Then,

$$(G, K)\tilde{\subseteq} f_{pu}(f_{pu}^{-1}(G, K))\tilde{\subseteq} f_{pu}(\tilde{\cup}_{i\in\Lambda}(H_i, E)) = \tilde{\cup}_{i\in\Lambda} f_{pu}(H_i, E)),$$

where f_{pu} is supra open soft injective function. Since (G, K) is soft supra compact in $\tilde{X}_2$, there exists a finite subfamily Λ_o of Λ such that $(G, K)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o} f_{pu}(H_i, E)$. Therefore,

$$f_{pu}^{-1}(G, K)\tilde{\subseteq} f_{pu}^{-1}(\tilde{\cup}_{i\in\Lambda_o} f_{pu}(H_i, E)) = \tilde{\cup}_{i\in\Lambda_o} f_{pu}^{-1}(f_{pu}(H_i, E)) = \tilde{\cup}_{i\in\Lambda_o}(H_i, E).$$

This completes the proof. Q.E.D.

Theorem 3.17. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra open soft injective function. If (G, K) is soft supra lindelöf subset of $\tilde{X}_2$, then $f_{pu}^{-1}(G, K)$ is soft lindelöf in $\tilde{X}_1$.

Proof. Similar to the proof of the above theorem. Q.E.D.

The proof of the following theorems are straightforward and thus omitted.

Theorem 3.18. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra irresolute soft function. If (Z, E) is soft supra compact (resp. soft supra lindelöf) subset of $\tilde{X}_2$, then $f_{pu}^{-1}(Z, K)$ is soft supra compact (resp. soft supra lindelöf) in $\tilde{X}_1$.

Theorem 3.19. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra irresolute soft surjective function. If $\tilde{X}_2$ is soft supra compact (resp. soft supra lindelöf) , then so $\tilde{X}_1$.

Theorem 3.20. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra irresolute open soft injective function. If (F, E) is soft supra compact (resp. soft supra lindelöf) subset of $\tilde{X}_1$, then $f_{pu}(F, E)$ is soft supra compact (resp. soft supra lindelöf) in $\tilde{X}_2$.

Theorem 3.21. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra irresolute open soft surjective function. If X_1 is soft supra compact (resp. soft supra lindelöf), then so X_2.

4 Soft Supra Closed Spaces

In this section, we introduce and study the concepts of soft supra closed spaces and soft supra generalized compact spaces.

Definition 4.1. A soft subset (F, E) of the space (X, μ, E) is said to be soft supra closed, if every supra open soft cover $\{(U_i, E) : i\in\Lambda\}$ of (F, E) has a finite subfamily Λ_o of Λ such that

$$(F, E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o} cl^s(U_i, E).$$

The space (X, μ, E) is said to be soft supra closed if $\tilde{X}$ is soft supra closed as a soft subset.

Remark 4.2. Every soft supra compact space is soft supra closed space, but the converse is not true.

Theorem 4.3. A soft supra topological space (X, μ, E) is soft supra closed if and only if every family Ψ of supra soft sets with SFIP, $\tilde{\cap}\{cl^s(F,E) : (F,E)\in\Psi\}\neq\tilde{\varphi}$.

Proof. **Necessity:** Let Ψ be a family of supra soft sets with the SFIP such that $\tilde{\cap}\{cl^s(F,E) : (F,E)\in\Psi\}=\tilde{\varphi}$. Hence, $\tilde{\cup}\{(cl^s(F,E))^{\tilde{c}} : (F,E)\in\Psi\}=\tilde{X}$. Consequently, $\{(cl^s(F,E))^{\tilde{c}} : (F,E)\in\Psi\} = \{(G,E) : (G,E)\in\Psi\}$ is a supra open soft cover of $\tilde{X}$ and for (X, μ, E) is soft supra closed space, there exists a finite soft subcover Ψ_o of Ψ such that, $\tilde{\cup}\{cl^s(cl^s(F,E))^{\tilde{c}} : (F,E) \in \Psi_o\} = \tilde{X}$, and so $\tilde{\cap}\{[cl^s(cl^s(F,E))^{\tilde{c}}]^{\tilde{c}} : (F,E) \in \Psi_o\}= \tilde{\cap}\{(G,E) : (G,E) \in \Psi_o\} = \tilde{\varphi}$, which is a contradiction. Hence, $\tilde{\cap}\{cl^s(F,E) : (F,E) \in \Psi\} \neq \tilde{\varphi}$.

Sufficient: Let $\{(cl^s(G,E))^{\tilde{c}} : (G,E)\in\Psi\} = \{(F,E) : (F,E)\in\Psi\}$ be a supra open soft cover of $\tilde{X}$ such that for every finite collection $\{(F,E) : (F,E)\in\Psi_o\}$, we have $\tilde{\cup}\{(F,E) : (F,E)\in\Psi_o\}\neq \tilde{X}$. Hence, $\tilde{\cap}\{(F,E)^{\tilde{c}} : (F,E)\in\Psi_o\}\neq \tilde{\varphi}$, so $\tilde{\cap}\{cl^s(G,E) : (G,E)\in\Psi_o\}\neq \tilde{\varphi}$. Therefore, $\{cl^s(G,E) : (G,E)\in\Psi_o\}$ is a family of supra soft sets with SFIP in $\tilde{X}$. Also, for $\{(F,E) : (F,E)\in\Psi\}$ is supra open soft cover of $\tilde{X}$, then $\tilde{X} = \tilde{\cup}\{(F,E) : (F,E)\in\Psi\}$ and so $\tilde{\cap}\{(F,E)^{\tilde{c}} : (F,E)\in\Psi\} = \tilde{\cap}\{cl^s(G,E) : (G,E)\in\Psi\} = \tilde{\varphi}$, which is a contradiction. It follows, every supra open soft cover $\{(F,E) : (F,E)\in\Psi\}$ of $\tilde{X}$ has a finite collection $\{(F,E) : (F,E) \in \Psi_o\}$ such that $\tilde{X} = \tilde{\cup}_{(F,E)\in\Psi_o} cl^s(F,E)$. Therefore, $\tilde{X}$ is soft supra closed space.

Q.E.D.

Theorem 4.4. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra irresolute soft surjective function. If $\tilde{X}_1$ is soft supra closed space, then so $\tilde{X}_2$.

Proof. Let $\{(B_i, K) : i\in\Lambda\}$ be a ν-supra open soft cover of $\tilde{X}_2$. Since f_{pu} is supra irresolute soft function, $\{f_{pu}^{-1}(B_i, K) : i\in\Lambda\}$ is μ-supra open soft cover of $\tilde{X}_1$. By hypothesis, there exists a finite subfamily Λ_o of Λ such that $\tilde{\cup}_{i\in\Lambda_o} cl^s(f_{pu}^{-1}(B_i, K)) = \tilde{X}_1$. Since f_{pu} is surjective, $f_{pu}(\tilde{X}_1) = \tilde{X}_2 = f_{pu}[\tilde{\cup}_{i\in\Lambda_o} cl^s$
$(f_{pu}^{-1}(B_i, K))]\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o} cl^s(f_{pu}(f_{pu}^{-1}(B_i, K))) = \tilde{\cup}_{i\in\Lambda_o} cl^s(B_i, K)]$. Therefore, $\tilde{X}_2$ is soft supra closed space. Q.E.D.

Definition 4.5. A family $\Psi = \{(U_i, E) : i\in\Lambda\}$ of soft sets is said to be a supra generalized open soft cover (supra g-open soft cover, for short), if each member of Ψ is soft supra g-open set.

Definition 4.6. A soft subset (F, E) of the space (X, μ, E) is said to be soft supra generalized compact , if every supra g-open soft cover $\{(V_i, E) : i\in\Lambda\}$ of (F, E) has a finite subfamily Λ_o of Λ such that

$$(F,E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(V_i, E).$$

The space (X, μ, E) is said to be soft supra generalized compact if $\tilde{X}$ is soft supra generalized compact as a soft subset.

Theorem 4.7. Every supra g-closed soft subspace of a soft supra generalized compact space is soft supra generalized compact.

Proof. Let (G, E) be a supra g-closed soft subspace of soft supra generalized compact space (X, μ, E) and $\{(V_i, E) : i\in\Lambda\}$ be a supra g-open soft cover of $\tilde{X}$. Then, $\{(V_i, E) : i\in\Lambda\}\tilde{\cup}(G^{\tilde{c}}, E)$ is a finite subfamily Λ_o of Λ such that $\tilde{X} = \tilde{\cup}_{i\in\Lambda_o}(V_i, E)\tilde{\cup}(G^{\tilde{c}}, E)$. Hence, $(G, E)\tilde{\subseteq}\tilde{\cup}_{i\in\Lambda_o}(V_i, E)$. Therefore, (G, E) is soft supra generalized compact. Q.E.D.

Definition 4.8. Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. The soft function $f_{pu} : SS(X)_E \rightarrow SS(Y)_K$ is called supra soft generalized continuous function if the preimage of every ν-supra generalized open soft set under f_{pu} is $\mu-$supra generalized open soft.

Theorem 4.9. Let (X_1, τ_1, E) and (X_2, τ_2, K) be two soft topological spaces and μ and ν be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:(X_1, \tau_1, E)\longrightarrow(X_2, \tau_2, K)$ be a supra soft generalized continuous surjective function. If X_1 is soft supra generalized compact, then so X_2.

Proof. Let $\{(H_i, K) : i\in\Lambda\}$ be a ν-supra g-open soft cover of $\tilde{X}_2$. Since f_{pu} is supra soft generalized continuous function, $\{f_{pu}^{-1}((H_i, K)) : i\in\Lambda\}$ is a μ-supra g-open soft cover of $\tilde{X}_1$ and for $\tilde{X}_1$ is soft supra generalized compact, there exists a finite subfamily Λ_o of Λ such that $\{f_{pu}^{-1}((H_i, K)) : i\in\Lambda_o\}$ also forms a μ-supra g-open soft cover of $\tilde{X}_1$. Since f_{pu} is surjective, $\{f_{pu}(f_{pu}^{-1}((H_i, K))) : i\in\Lambda_o\} = \{(H_i, K): i\in\Lambda_o\}$ is a finite ν-supra g-open soft cover of $\tilde{X}_2$. Therefore, X_2 is soft supra generalized compact. Q.E.D.

5 Conclusion

It is an interesting exercise to work on soft supra compact spaces and soft supra closed spaces. Therefore, a new class of supra generalized open soft sets in supra soft topological spaces as a generalization of soft compact spaces, called soft supra compact spaces, is introduced and studied. In future, the generalization of these concepts by using the soft ideal notions [11] and some types of supra open soft sets [9] will be introduced and the future research will be undertaken in this direction.

References

[1] A. M. Abd El-latif, On some subsets of soft weak structures, Facta Universitatis: Ser. Math. Inform., 32 (1) (2017), 59-72.

[2] A. M. Abd El-latif and and Rodyna A. Hosny, Supra open soft sets and associated soft separation axioms, International Journal of Advances in Mathematics, 6 (2017), 68-81.

[3] A. M. Abd El-latif and Rodyna A. Hosny, Soft supra extra strongly generalized closed, An. Univ. Oradea Fasc. Mat., 24 (1) (2017), 103-112.

[4] A. M. Abd El-latif, Soft extra strongly semi star generalized closed sets, Gazi University Journal of Science (GUJS), 33 (4) (2017), 421-430.

[5] A. M. Abd El-latif, Soft supra strongly generalized closed sets, Journal of Intelligent & Fuzzy Systems, 31 (3) (2016), 1311-1317.

[6] M. E. Abd El-Monsef and A. E. Ramadan, On fuzzy supra topological spaces, Indian J. Pure and Appl. Math., 4 (18) (1987), 322-329.

[7] B. Ahmad and A. Kharal, Mappings on soft classes, New Math. Nat. Comput., 7 (3) (2011), 471-481.

[8] A. Aygunoglu and H. Aygun, Some notes on soft topological spaces, Neural Computing and Applications, 21 (1) (2012), 113–119

[9] S. A. El-Sheikh and A. M. Abd El-latif, Decompositions of some types of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology, 9 (1) (2014), 37-56.

[10] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform., 7 (2) (2014), 181-196.

[11] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014), 1595-1603.

[12] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi compactness via soft ideals , Appl. Math. Inf. Sci., 8 (5) (2014), 2297-2306.

[13] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014), 1731-1740.

[14] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, On supra topological spaces, Indian J. Pure and Appl. Math., 14 (4) (1983), 502-510.

[15] D. A. Molodtsov, Soft set theory-firs tresults, Comput. Math. Appl., 37 (1999), 19-31.

[16] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl., 61 (2011), 1786-1799.

[17] Weijian Rong, The countabilities of soft topological spaces, International Journal of Computational and Mathematical Sciences, 6 (2012), 159-162.

[18] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, remarks on soft topological spaces, Ann. Fuzzy Math. Inform., 3 (2) (2012), 171-185.