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RELATIVE EVANS POTENTIALS

Dedicated to Professor Yûsaku Komatu on his 60th birthday

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Consider a 2-form $P=P(z)dxdy$ on an open Riemann surface R such that coefficients $P(z)$ are nonnegative Hölder continuous functions of local parameters $z=x+iy$ on R. Let Ω be the complement of the closure of a regular subregion of R and U be a positive solution of the equation $\Delta u=Pu$ on Ω continuous on $\bar{\Omega}$. A function E on Ω will be referred to as an *Evans potential relative to* (P, U) on Ω if E is a solution of $\Delta u=Pu$ on Ω with boundary values zero on $\partial\Omega$ such that

(1) $$\lim_{z\to\infty} E(z)/U(z)=+\infty$$

where ∞ is the point at infinity of R. We are interested in finding the condition on U assuring the existence of such an E. For this purpose we consider a linear operator L_{Ω}^{P} from $C(\partial\Omega)$ into the class of solutions of $\Delta u=Pu$ on Ω given as follows. Let $\{S\}$ be a directed net of regular subregions of R such that $S\supset R-\Omega$, and let $L_{\Omega\cap S}^{P}\varphi$ be the solution of $\Delta u=Pu$ on $\Omega\cap S$ with boundary values zero on ∂S and $\varphi\in C(\partial\Omega)$ on $\partial\Omega$. Then the limit

$$L_{\Omega}^{P}\varphi=\lim_{S\to R} L_{\Omega\cap S}^{P}\varphi$$

is a bounded solution of $\Delta u=Pu$ on Ω with boundary values φ on $\partial\Omega$. We say that U has the *ideal boundary values zero* if $L_{\Omega}^{P}U=U$. The main purpose of this paper is to prove the following

THEOREM. *An Evans potential E relative to (P, U) exists for any P on Ω if and only if U has the ideal boundary values zero.*

Suppose the existence of E and set $V=U-L_{\Omega}^{P}U$ which is a nonnegative solution of $\Delta u=Pu$ on Ω with boundary values zero on $\partial\Omega$. In view of $\varepsilon E-V=(\varepsilon E/U-V/U)\cdot U$ and $V/U\leqq 1$, (1) implies that the inferior limit of $\varepsilon E-V$ as $z\to\infty$ or $\partial\Omega$ is nonnegative for any positive number ε and therefore $\varepsilon E-V\geqq 0$ on Ω which in turn implies $V=0$, i.e. $L_{\Omega}^{P}U=U$. Thus the essential part of our proof is to show the existence of E under the assumption $L_{\Omega}^{P}U=U$. The proof will be given in nos. **1-6**. Before doing this we mention several direct con-

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sequences of our theorem, most of which are earlier published:

a) An *Evans-Selberg potential* $q(z, z_0)$ on R is a harmonic function on $R-\{z_0\}$ such that $\lim_{z\to\infty}q(z, z_0)=+\infty$ and $q(z, z_0)-\log|z-z_0|=O(1)$ as $z\to z_0$. Kuramochi [2, cf. also 3] proved that such $q(z, z_0)$ exists on R if and only if R is parabolic (cf. [5]). Let $z_0\in R-\bar{\Omega}$ and D be a parametric disk about z_0 with $\bar{D}\subset R-\bar{\Omega}$. Observe that $L_{\Omega}^{0}1=1$ if and only if R is parabolic. Therefore by our theorem an Evans potential E relative to $(0, 1)$ exists on Ω if and only if R is parabolic. If $q(z, z_0)$ exists, then $q(\cdot, z_0)-L_{\Omega}^{0}q(\cdot, z_0)$ is an E. Conversely suppose E exists. Set $\Omega_0=\Omega\cup(D-\{z_0\})$ and let $s(z)$ be $2\pi\left(\int_{\partial\Omega}*dE\right)^{-1}E(z)$ on Ω and $\log|z-z_0|$ on D. By the Sario theorem [12], the equation $L_{\Omega_0}^{0}(q-s)=q-s$ has a harmonic solution q on $R-\{z_0\}$ if and only if $\int_{\partial\Omega_0}*ds=0$, which is actually the case for the present s, and the solution q is a required $q(z, z_0)$.

b) An *Evans solution* v of $\Delta u=Pu$ on R is a solution on R such that $\lim_{z\to\infty}v(z)=+\infty$. Let $\omega=L_{\Omega}^{P}1$. Since $L_{\Omega}^{P}\omega=\omega$, there exists an Evans potential E relative to (P, ω). The equation $L_{\Omega}^{P}(v-E)=v-E$ has a solution v which is a solution of $\Delta u=Pu$ on R provided $P\not\equiv 0$ on R (Sario-Nakai [13]). Since $|v-E| \leqq(\sup_{\partial\Omega}|v-E|)\cdot\omega$ on Ω, $\lim_{z\to\infty}v(z)/\omega(z)=+\infty$. Therefore an Evans solution of $\Delta u=Pu$ $(P\not\equiv 0)$ exists on R if $\inf_{\Omega}\omega>0$ (Nakai [6]).

c) We denote by O_B the class of pairs (R, P) $(P\not\equiv 0$ on $R)$ such that the only bounded solution of $\Delta u=Pu$ on R is zero. It has been conjectured that $(R, P)\in O_B$ is characterized by the existence of an Evans solution of $\Delta u=Pu$ on R (cf. [6; p. 92]). Recently the author ([9]) proved the existence of a *singular* P on any R, i.e. a P such that any nonnegative solution of $\Delta u=Pu$ on R has zero infimum, which completely negates the conjecture. However the conjecture sounds so natural that we still feel that it must be 'almost true'. The *P-unit* e_{Ω}^{P} on Ω is given by $\sup u(z)$ where u runs over the class of solutions of $\Delta u=Pu$ on Ω with $u\leqq 1$. Then the conjecture is true if it is modified as follows:

The pair (R, P) *belongs to* O_B *if and only if there exists a solution* v *of* $\Delta u=Pu$ *on* R *such that* $\lim_{z\to\infty}v(z)/e_{\Omega}^{P}(z)=+\infty$ *for one and hence for every admissible* Ω.

An Evans potential E relative to (P, e_{Ω}^{P}) exists on Ω if and only if $L_{\Omega}^{P}e_{\Omega}^{P}=e_{\Omega}^{P}$, which is, by Ozawa [10]-Royden [11], equivalent to $(R, P)\in O_B$. From this E we can construct a required v by the entirely same method as in **b**.

d) Let R be a hyperbolic Riemann surface and $G^0(z, \zeta)$ be the harmonic Green's function on R. An *Evans harmonic function* h on R is a positive harmonic function on R such that $\lim_{n\to+\infty}h(z_n)=+\infty$ for every sequence $\{z_n\}$ of points in R converging to the point at infinity of R with $\liminf_{n\to+\infty}G^0(z_n, \zeta)>0$ for one and hence for every $\zeta\in R$. The existence of such an h was shown in Nakai [7]. This is also a direct consequence of our theorem. Let $\zeta\in R-\bar{\Omega}$.

Since $L_\Omega^0 G^0(\cdot, \zeta)=G^0(\cdot, \zeta)$, an Evans potential E relative to $(0, G^0(\cdot, \zeta))$ exists on Ω. The hyperbolicity of R assures the existence of a harmonic solution h of $L_\Omega^0(h-E)=h-E$ on R (cf. [8]), which is the required h.

e) Assume that $P\not\equiv 0$ on R. Then the Green's function $G(z, \zeta)$ of the equation $\Delta u=Pu$ always exists on R (Myrberg [4]). As a counter part of the result in **d**, Kawai [1] proved that there exists a positive solution u of $\Delta u=Pu$ on R such that $\lim_{n\to+\infty}u(z_n)=+\infty$ for every sequence $\{z_n\}$ converging to the point at infinity of R with $\liminf_{n\to+\infty}G(z_n, \zeta)>0$ for one and hence for every $\zeta\in R$. By the entirely same observation as in **d** we can derive this result from our theorem.

1. We proceed to the existence proof of an Evans potential E relative to (P, U) on Ω under the assumption $L_\Omega^P U=U$ on Ω. Throughout our proof we fix a regular exhaustion $\{R_n\}$ $(n=1, 2, \cdots)$ of R such that $R_1\supset\bar{\Omega}_0$ with $\Omega_0=R-\bar{\Omega}$ and set $\Omega_n=R_n\cap\Omega$. Then $\{\Omega_n\}$ $(n=1, 2, \cdots)$ is an 'exhaustion' of Ω. We also set $\beta_0=\partial\Omega=\partial\Omega_0$ and $\beta_n=\partial R_n$, i.e. $\partial\Omega_n=\beta_0\cup\beta_n$. Let R^* be the Čech compactification of R, $\beta=R^*-R$ the Čech ideal boundary of R, and $\Omega^*=R^*-(R-\bar{\Omega})=R^*-\Omega_0$, i.e. $\Omega^*=\beta_0\cup\Omega\cup\beta$. Then any $[-\infty, +\infty]$-valued continuous function on $\beta_0\cup\Omega$ can be uniquely extended to a $[-\infty, +\infty]$-valued continuous function on Ω^*. We denote by $U_{n,m}$ $(n<m)$ the solution of $\Delta u=Pu$ on $\Omega_m-\bar{\Omega}_n$ with boundary values U on β_n and zero on β_m. Since $L_\Omega^P U=U$, $\lim_m U_{0,m}=U$. In view of $U_{0,m}\leqq U_{n,m}\leqq U$, we also have

$$\lim_{m\to+\infty} U_{n,m}=U \qquad (n=0, 1, \cdots). \tag{2}$$

For our purpose it is convenient to consider the elliptic equation

$$\Delta v(z)+2\nabla \log U(z)\cdot\nabla v(z)=0 \tag{3}$$

instead of $\Delta u=Pu$, where $\nabla\varphi$ is the gradient vector field $(\partial\varphi/\partial x, \partial\varphi/\partial y)$. It is easily verified that the operator $v\to Tv=U\cdot v$ is a bijective correspondence between the class of solutions of (3) and that of $\Delta u=Pu$. The merit of considering (3) is the validity of the following

Minimum principle. *Let v be a $[0, +\infty]$-valued continuous function on $\Omega-\Omega_n$ such that v is a solution of* (3) *on $\Omega-\bar{\Omega}_n$. Then v takes its minimum on β_n.*

Set $c=\min\{v(z)\,;\, z\in\beta_n\}$. Since $Tv-cU_{n,m}$ is a solution of $\Delta u=Pu$ on $\Omega_m-\bar{\Omega}_n$ with boundary values $U\cdot v-cU_{n,m}=(v-c)\cdot U\geqq 0$ on β_n and $U\cdot v-cU_{n,m}=U\cdot v\geqq 0$ on β_m, we see that $Tv\geqq cU_{n,m}$ on $\Omega_m-\bar{\Omega}_n$. By (2) we deduce that $Tv\geqq cU$, i.e. $v\geqq c$ on $\Omega-\bar{\Omega}_n$.

2. Modifying the Green's function $G(z, \zeta)$ ($G_n(z, \zeta)$, resp.) of $\Delta u=Pu$ on Ω (Ω_n, resp.) we consider a new kernel $K(z, \zeta)$ on Ω defined by

$$K(z, \zeta)=U(z)^{-1}G(z, \zeta)U(\zeta)^{-1} \tag{4}$$

which is also a positive symmetric kernel on Ω and $TK(\cdot, \zeta)=G(\cdot, \zeta)U(\zeta)^{-1}$. Needless to say $K(z, \zeta)$ is *not* the Green's function of (3) unless $P\equiv 0$ and $U\equiv 1$. First we give an *analytic* proof of the existence of the *K-capacitary measure* μ_n on β_n characterized by the identity

$$\int_{\beta_n} K(z, \zeta)d\mu_n(\zeta)=1 \tag{5}$$

for every $z\in\beta_n$. Fix a point $z\in\Omega_n$. Apply the Green formula to functions $G(z, \cdot)$ and $U-U_{0,n}$ for the region Ω_n less the small δ-disk about z. Then on letting $\delta\to 0$ we obtain

$$2\pi(U(z)-U_{0,n}(z))=-\int_{\beta_n} G(z, \zeta)^*d_\zeta(U(\zeta)-U_{0,n}(\zeta))+\int_{\beta_n} U(\zeta)^*d_\zeta G(z, \zeta)\,. \tag{6}$$

Still fixing the same point $z\in\Omega_n$ we again apply the Green formula to functions $G_m(z, \cdot)$ and $U_{n,m}$ for the region $\Omega_m-\bar{\Omega}_n$ to obtain

$$0=-\int_{\beta_n} G_m(z, \zeta)^*d_\zeta U_{n,m}(\zeta)+\int_{\beta_n} U(\zeta)^*d_\zeta G_m(z, \zeta)\,.$$

In view of the limit property (2), on letting $m\to+\infty$, we conclude that

$$0=-\int_{\beta_n} G(z, \zeta)^*d_\zeta U(\zeta)+\int_{\beta_n} U(\zeta)^*d_\zeta G(z, \zeta)\,.$$

Subtraction of the above identity from (6) implies

$$U(z)-U_{0,n}(z)=\frac{1}{2\pi}\int_{\beta_n} G(z, \zeta)^*d_\zeta U_{0,n}(\zeta)\,. \tag{7}$$

It is not hard to see that both sides of the above are continuous functions of z on $\Omega_n\cup\beta_n$. Tnerefore the validity of (7) on Ω_n implies that on $\Omega_n\cup\beta_n$. For $z\in\Omega_n\cup\beta_n$, (7) takes on the form

$$1-U_{0,n}(z)/U(z)=\frac{1}{2\pi}\int_{\beta_n} U(z)^{-1}G(z, \zeta)U(\zeta)^{-1}U(\zeta)^*d_\zeta U_{0,n}(\zeta)\,.$$

Observe that

$$d\mu_n(\zeta)=\frac{1}{2\pi} U(\zeta)^*d_\zeta U_{0,n}(\zeta)$$

is a nonnegative Borel measure on β_n and we have

$$1-U_{0,n}(z)/U(z)=\int_{\beta_n} K(z, \zeta)d\mu_n(\zeta) \tag{8}$$

for $z\in\Omega_n\cup\beta_n$ and in particular (5) is valid for $z\in\beta_n$.

3. The kernel $K(z, \zeta)$ on Ω has a natural extension to Ω^*:

$$K(z^*, \zeta^*)=\lim_{z\in\Omega, z\to z^*}\left(\lim_{\zeta\in\Omega, \zeta\to\zeta^*} K(z, \zeta)\right) \tag{9}$$

for $(z^*, \zeta^*)\in\Omega^*\times\Omega^*$. Since $K(z, \zeta^*)=\lim_{\zeta\in\Omega,\zeta\to\zeta^*}K(z, \zeta)$ is a positive solution of (3) on $\Omega-\{\zeta^*\}$ with boundary values zero on β_0, we could define $K(z^*, \zeta^*)=$

$\lim_{z\in\Omega, z\to z^*}K(z, \zeta^*)$ in (9). For a compact set X in Ω^* we consider three quantities $\varepsilon(X)$, $\delta(X)$, and $\tau(X)$ as follows. First let $M(X)$ be the class of unit Borel measures on X and set

$$\varepsilon(X)=\inf_{\mu\in N(X)}[\mu, \mu] \tag{10}$$

where $[\mu, \nu]=\int K(z, \zeta)d\mu(z)d\nu(\zeta)$ for two Borel measures μ and ν. The quantity $1/\varepsilon(X)$ is referred to as the *K-capacity* of X. Next let

$$\binom{k}{2}\delta_k(X)=\inf_{z_1^*,\cdots,z_k^*\in X}\sum_{i<j}^{1,\cdots,k}K(z_i^*, z_j^*)\,.$$

Then we know (cf. e.g. [6, 7, 13]) the existence of the limit

$$\delta(X)=\lim_{k\to+\infty}\delta_k(X)\in[0, +\infty] \tag{11}$$

which is referred to as the *K-transfinite diameter* of X. Finally let

$$k\tau_k(X)=\sup_{z_1^*,\cdots,z_k^*\in X}\left(\inf_{z^*\in X}\sum_{i=1}^{k}K(z^*, z_i^*)\right).$$

Then we know (cf. e.g. [6, 7, 13]) the existence of the limit

$$\tau(X)=\lim_{k\to+\infty}\tau_k(X)\in[0, +\infty] \tag{12}$$

which is referred to as the *K-Tchebycheff constant* of X. We have (cf. e.g. [6, 7, 13]) the following inequality:

$$\delta(X)\leqq\tau(X)\,, \tag{13}$$

which is the half of the Fékete identity. We do not know to what extent the following is true for general X in Ω^* but at least for X in Ω and in particular for β_n we have (cf. e.g. [7, 13])

$$\varepsilon(\beta_n)=\delta(\beta_n)=\tau(\beta_n)\,. \tag{14}$$

4. Based on the minimum principle in no. **1** we prove

$$\delta(\beta_n)\leqq\delta(\beta)\,. \tag{15}$$

Fix an arbitrary system $z_1^*, \cdots, z_k^*$ of k points in β. We set

$$v(\zeta_1^*, \cdots, \zeta_k^*)=\sum_{i<j}^{1,\cdots,k}K(\zeta_i^*, \zeta_j^*)\,.$$

First observe the function $z\to v(z, z_2^*, \cdots, z_k^*)$ is a positive solution of (3) on $\Omega-\bar{\Omega}_n$ continuous on $(\Omega-\bar{\Omega}_n)\cup\beta_n$. Thus the function takes its minimum on β_n, say at $z_1\in\beta_n$, and in particular we have

$$v(z_1, z_2^*, \cdots, z_k^*)\leqq v(z_1^*, z_2^*, \cdots, z_k^*)\,.$$

Assume that l points $z_1, \cdots, z_l\in\beta_n$ $(1\leqq l<k)$ are chosen so as to satisfy

$$v(z_1, \cdots, z_l, z_{l+1}^*, \cdots, z_k^*)\leqq v(z_1, \cdots, z_{l-1}, z_l^*, \cdots, z_k^*)\,.$$

Then since $z \to v(z_1, \cdots, z_l, z, z_{l+2}^*, \cdots, z_k^*)$ is a positive solution of (3) on $\Omega - \bar{\Omega}_n$ which is $(0, +\infty]$-valued continuous on $(\Omega - \bar{\Omega}_n) \cup \beta_n$, the function takes its minimum on β_n, say at $z_{l+1} \in \beta_n$. Therefore

$$v(z_1, \cdots, z_{l+1}, z_{l+2}^*, \cdots, z_k^*) \leqq v(z_1, \cdots, z_l, z_{l+1}^*, \cdots, z_k^*).$$

We can thus find k points $z_1, \cdots, z_k$ in β_n such that

$$v(z_1, \cdots, z_k) \leqq v(z_1^*, \cdots, z_k^*).$$

By the definition of $\delta_k(\beta_n)$, the left hand side of the above dominates $\binom{k}{2}\delta_k(\beta_n)$, and since $z_1^*, \cdots, z_k^*$ are arbitrarily chosen in β, we conclude that $\delta_k(\beta_n) \leqq \delta_k(\beta)$. On making $k \to +\infty$ we obtain (15).

5. Fix a point $z_0 \in \Omega_1$. Observe that the function $\zeta \to K(z_0, \zeta)$ is a positive solution on $\Omega - \Omega_1$. Let $a = \min\{K(z_0, \zeta);\ \zeta \in \beta_1\} > 0$. Once more we use the minimum principle in no. **1** to conclude

(16) $$K(z_0, \zeta) \geqq a$$

for every $\zeta \in \Omega - \Omega_1$. From (8) and (16) it follows that

$$1 - U_{0,n}(z_0)/U(z_0) \geqq \int_{\beta_n} a\, d\mu_n(\zeta) = a\mu_n(\beta_n).$$

We conclude by (2) that

(17) $$\lim_{n \to +\infty} \mu_n(\beta_n) = 0.$$

Set $(\mu, \nu) = \int G(z, \zeta) d\mu(z) d\nu(\zeta)$. Then $[\mu, \nu] = (U^{-1} \cdot \mu, U^{-1} \cdot \nu)$. We know that the Schwarz inequality $(\mu, \nu)^2 \leqq (\mu, \mu) \cdot (\nu, \nu)$ is valid (cf. e.g. [6]). Therefore the same is true of $[\mu, \nu]$: $[\mu, \nu]^2 \leqq [\mu, \mu] \cdot [\nu, \nu]$. Let $\mu \in M(\beta_n)$. By (5), $[\mu, \mu_n] = 1$. Hence $1 \leqq [\mu, \mu][\mu_n, \mu_n]$, and again by (5), $[\mu_n, \mu_n] = \mu_n(\beta_n)$. A fortiori $[\mu, \mu] \geqq 1/\mu_n(\beta_n)$ for every $\mu \in M(\beta_n)$. By the definition (10) we conclude that

(18) $$\varepsilon(\beta_n) \geqq 1/\mu_n(\beta_n).$$

Using (13), (15), (14), (18) and (17) successively in this order we deduce

(19) $$\tau(\beta) = +\infty.$$

6. In view of (19) and $\tau(\beta) = \lim_{k \to +\infty} \tau_k(\beta)$ we can find a subsequence $\{k_m\}$ $(m = 1, 2, \cdots)$ of positive integers such that $\tau_{k_m}(\beta) > 2^m$. By the definition of τ_{k_m} there exists k_m points $z_{m,\imath}^*$ $(\imath = 1, \cdots, k_m)$ in β such that

$$\inf_{z^* \in \beta} \sum_{\imath=1}^{k_m} K(z^*, z_{m,\imath}^*) > 2^m k_m.$$

Denoting by ε_p the point measure at $p \in \Omega^*$, set

$$\nu_m = (2^m k_m)^{-1} \sum_{\imath=1}^{k_m} \varepsilon_{z_{m,\imath}^*}$$

and

$$\nu=\sum_{m=1}^{+\infty}\nu_m .$$

Then $\int K(z^*, \zeta^*)d\nu_m(\zeta^*)>1$ for $z^*\in\beta$ and a fortiori $\int K(z^*, \zeta^*)d\nu(\zeta^*)=+\infty$ for $z^*\in\beta$. Since $\nu(\beta)=1$, $F(z^*)=\int K(z^*, \zeta^*)d\nu(\zeta^*)$ is $[0, +\infty]$-valued continuous on Ω^* and $F(z)$ is a solution of (3) on Ω such that $F=0$ on β_0 and $F=+\infty$ on β. Set $E=TF=U\cdot F$. Then E is a solution of $\Delta u=Pu$ on Ω with boundary values zero on β_0 and $E/U=+\infty$ on β. Therefore E satisfies (1) and E is an Evans potential relative to (P, U) on Ω.

The proof is herewith complete.

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