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# ON A $K$-SPACE OF CONSTANT HOLOMORPHIC SECTIONAL CURVATURE

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## 1. Introduction.

It is well known that a Kähler space of constant holomorphic sectional curvature is an Einstein space (Yano [13]). The main purpose of the present paper is to generalize this result to a $K$-space (Theorem 4. 4). Preliminary facts will be given in § 2. In § 3, we shall prepare some lemmas for the proof of main theorem. Particularly, we shall prove some lemmas about the $K$-space of constant holomorphic sectional curvature. In § 4, we shall prove the main theorem and some related theorems on the generalized Chern-form.

## 2. Preliminaries.

Let $M$ be an $n$-dimensional $(n>2)$ almost Hermitian manifold with Hermitian structure $(F_j{}^i, g_{ji})$, i.e., with an almost complex structure $F_j{}^i$ and a positive definite Riemannian metric tensor $g_{ji}$ satisfying

(2. 1) $$F_j{}^a F_a{}^i = -\delta_j^i,$$

(2. 2) $$g_{ts} F_j{}^t F_i{}^s = g_{ji}.$$

If an almost Hermitian structure satisfies

(2. 3) $$\nabla_k F_{ji} + \nabla_j F_{ki} = 0,$$

where $\nabla_j$ denotes the operator of covariant differentiation with respect to the Riemannian connection and $F_{ji} = F_j{}^t g_{ti}$, then the manifold is called a *$K$-space* (or *Tachibana space*, or *nearly Kähler manifold*).

Now, in a $K$-space let $R_{kji}{}^h$, $R_{ji} = R_{hji}{}^h$ and $R = g^{ji} R_{ji}$ be Riemannian curvature tensor, Ricci tensor and scalar curvature respectively. Then we have the following identities [7], [8]:

(2. 4) $$\nabla_j F_{ih} + F_j{}^b F_i{}^a \nabla_b F_{ah} = 0,$$

(2. 5) $$F_{hk} \nabla^t \nabla_t F_j{}^h = R_{kj} - R^*{}_{jk}, \quad \text{or} \quad \nabla^t \nabla_t F_j{}^h = F^{hl}(R_{jl} - R^*{}_{lj}),$$

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where $\nabla^t = g^{ta}\nabla_a$ and $R^*_{ji} = (1/2)F^{ba}R_{bati}F_j{}^t$,

$$R_{ji} = F_j{}^b F_i{}^a R_{ba}, \qquad R^*_{ji} = F_j{}^b F_i{}^a R^*_{ba}, \tag{2.6}$$

$$R^*_{ji} = R^*_{ij}, \tag{2.7}$$

$$\nabla_j F_{ts}(\nabla_i F^{ts}) = R_{ji} - R^*_{ji}, \tag{2.8}$$

where $F^{ji} = F_t{}^i g^{tj}$; and

$$\nabla_j F_{ih}(\nabla^j F^{ih}) = R - R^* = \text{constant} > 0, \tag{2.9}$$

where $R^* = g^{ji}R^*_{ji}$.

Since we have $T^{kji}\nabla_k R_{jihl} = 0$ for any skew-symmetric tensor $T^{kji}$, we get

$$(\nabla^k F^{ji})\nabla_k R_{jihl} = 0. \tag{2.10}$$

As $(1/2)\nabla_i R^* = \nabla^j R^*_{ji}$ in a $K$-space [5], we have

$$\nabla^k (R_{ik} - R^*_{ik}) = \frac{1}{2}\nabla_i (R - R^*) = 0. \tag{2.11}$$

## 3. Some lemmas.

In general, it is well known that the differential form

$$\hat{K} = \hat{K}_{ji} dx^j \wedge dx^i$$

is closed in any almost Hermitian manifold, where

$$\hat{K}_{ji} = 2R_{jir}{}^t F_t{}^r - F_s{}^t(\nabla_j F_r{}^s)\nabla_i F_t{}^r,$$

which is called the generalized Chern-form [8].

In a $K$-space, taking account of (2. 8), we have

$$\hat{K}_{ji} = F_j{}^r (5R^*_{ri} - R_{ri}). \tag{3.1}$$

Recently, A. Gray proved the following

LEMMA 3. 1. (Gray [1]) *In a K-space, the relation*

$$R_{jist} - R_{bast}F_j{}^b F_i{}^a = -(\nabla_j F_i{}^r)\nabla_s F_{tr}, \tag{3.2}$$

$$R_{jihk} - F_j{}^a F_i{}^b F_h{}^c F_k{}^d R_{abcd} = 0 \tag{3.3}$$

*hold good.*

Moreover, the present authors proved the following

LEMMA 3. 2. (Takamatsu and Watanabe [11]) *In a K-space, we have*

(3. 4) $$\nabla_h S_{si} = -\frac{1}{2}\{S_{ri}(\nabla^r F_{sm})F_h{}^m + S_{rs}(\nabla_i F_m{}^r)F_h{}^m\},$$

*where* $S_{ji} = R_{ji} - R^*{}_{ji}$.

Now we have the following

LEMMA 3. 3. *In a K-space, the relation*

(3. 5) $$S^{ji}(R_{kjih} - 5R_{kjba}F_i{}^b F_h{}^a) = 0$$

*holds good.*

*Proof*. If we transvect (3. 2) with $\nabla_h F^{ji}$, then we have, taking account of (2. 4) and (2. 8),

$$2(\nabla_h F^{ji})R_{jist} = -S_h{}^r \nabla_s F_{tr}.$$

Applying $\nabla^h$ to this equation, we have, by (2. 10) and (2. 11),

$$2(\nabla^h \nabla_h F^{ji})R_{jist} = -S_h{}^r \nabla^h \nabla_s F_{tr},$$

or by Ricci's identity and $S^{hr}\nabla_s \nabla_h F_{tr} = 0$,

(3. 6) $$\begin{aligned} 2(\nabla^h \nabla_h F^{ji})R_{jist} &= S_h{}^r(-\nabla_s \nabla^h F_{tr} + R^h{}_{st}{}^a F_{ar} + R^h{}_{sr}{}^a F_{ta}) \\ &= S_h{}^r(R^h{}_{sat}F_r{}^a - R^h{}_{sar}F_t{}^a). \end{aligned}$$

Transvecting (3. 6) with $F_k{}^t$, we have,

(3. 7) $$2F_k{}^t(\nabla^h \nabla_h F^{ji})R_{jist} = S^{hr}(R_{hskr} - R_{hsta}F_k{}^t F_r{}^a).$$

Substituting (2. 5) into (3. 7), we obtain

$$2F_k{}^t F_a{}^j S^{ai} R_{jist} = S^{hr}(R_{hskr} - R_{hsta}F_k{}^t F_r{}^a),$$

or by Bianchi's identity,

(3. 8) $$2F_k{}^t F_a{}^j S^{ai}(R_{itjs} + R_{tjis}) = S^{hr}(R_{hskr} - R_{hsta}F_k{}^t F_r{}^a).$$

Moreover, making use of $F_a{}^j S^{ai} = -F_a{}^i S^{aj}$, (3. 8) reduces to

(3. 9) $$4F_k{}^t F_a{}^j R_{tjis} S^{ai} = S^{hr}(R_{hskr} - R_{hsta}F_k{}^t F_r{}^a),$$

from which we have (3. 5).

Transvecting (3. 5) with $g^{kh}$, we have the following

LEMMA 3. 4. (Takamatsu [10]) *In a K-space, the relation*

(3. 10) $$S^{ji}(R_{ji} - 5R^*{}_{ji}) = 0$$

*holds good.*

LEMMA 3.5. *In a K-space, we have*

$$(R_{kjih}-R_{kjba}F_i^{\ b}F_h^{\ a})S^{ji}=\frac{1}{4}(3R_{kr}+R^*_{\ kr})S_h^{\ r}. \tag{3.11}$$

*Proof.* By Ricci's identity, we have

$$\nabla_h\nabla_i S_{sl}-\nabla_i\nabla_h S_{sl}=R_{hims}S_l^{\ m}+R_{himl}S_s^{\ m}. \tag{3.12}$$

Transvecting (3.12) with $g^{il}$, we have, by virture of (2.11),

$$R_{hims}S^{im}=R_{hm}S_s^{\ m}-\nabla^i\nabla_h S_{si}. \tag{3.13}$$

Now, applying $\nabla^i$ to (3.4), we have

$$\begin{aligned}-2\nabla^i\nabla_h S_{si}&=S_{ri}(\nabla^i\nabla^r F_{sm})F_h^{\ m}+S_{ri}(\nabla^r F_{sm})\nabla^i F_h^{\ m}\\&\quad+\nabla^i S_{sr}(\nabla_i F_m^{\ r})F_h^{\ m}+S_{rs}(\nabla^i\nabla_i F_m^{\ r})F_h^{\ m}+S_{sr}(\nabla_i F_m^{\ r})\nabla^i F_h^{\ m}.\end{aligned} \tag{3.14}$$

Let us calculate the right hand side of (3.14). By Ricci's identity and (3.2), we have

$$\begin{aligned}\nabla_i\nabla_s F_m^{\ r}-\nabla_s\nabla_i F_m^{\ r}&=R_{isa}^{\ \ \ r}F_m^{\ a}-R_{ism}^{\ \ \ b}F_b^{\ r}\\&=F_m^{\ a}(R_{isa}^{\ \ \ r}-R_{istb}F^{rb}F_a^{\ t})\\&=F_m^{\ a}(-\nabla_i F_{st})\nabla_a F^{rt}.\end{aligned} \tag{3.15}$$

Multiplying both sides of (3.15) by $S^i_{\ r}F_h^{\ m}$, we have

$$S^i_{\ r}F_h^{\ m}\nabla_i\nabla_s F_m^{\ r}=S^i_{\ r}(\nabla_i F_{st})\nabla_h F^{rt}.$$

Thus, the first term of the right hand side of (3.14) reduces to

$$S_{ri}(\nabla^i\nabla^r F_{sm})F_h^{\ m}=-\nabla^i F_s^{\ t}(\nabla^r F_{ht})S_{ri}. \tag{3.16}$$

For the third term, by (3.2), (3.4), (3.9) and $(\nabla^i F_m^{\ r})F_h^{\ m}=(\nabla^r F_h^{\ m})F_m^{\ i}$, we have

$$\begin{aligned}\nabla_i S_{rs}(\nabla^i F_m^{\ r})F_h^{\ m}&=-\frac{1}{2}\{S_{ts}(\nabla^t F_{rl})F_i^{\ l}+S_{tr}(\nabla_s F_l^{\ t})F_i^{\ l}\}(\nabla^i F_m^{\ r})F_h^{\ m}\\&=-\frac{1}{2}\{S_{ts}(\nabla_r F^t_{\ m})-S_{tr}(\nabla_s F_m^{\ t})\}\nabla^r F_h^{\ m}\\&=-\frac{1}{2}\{S_{ts}S_h^{\ t}+S^{tr}(\nabla_t F_{sm})\nabla_h F_r^{\ m}\}\\&=-\frac{1}{2}\{S_{ts}S_h^{\ t}-S^{tr}(R_{tshr}-R_{tsba}F_r^{\ a}F_h^{\ b})\}\\&=-\frac{1}{2}S_{ts}S_h^{\ t}+2F_h^{\ t}F_a^{\ j}R_{tjbs}S^{ab}.\end{aligned} \tag{3.17}$$

The fourth term, making use of (2. 5), reduces to

(3. 18) $$S_{rs}(\nabla^i\nabla_i F_m{}^r)F_h{}^m = S_{rs}S_h{}^r.$$

Substituting (3. 16), (3. 17) and (3. 18) into (3. 14), we have

(3. 19) $$\nabla^i\nabla_h S_{si} = \frac{1}{4}S_s{}^r S_{hr} - F_h{}^t F_a{}^j R_{tjis}S^{ai}.$$

Thus, making use of (3. 19), from (3. 13), we have

$$R_{hmis}S^{im} = R_{hm}S_s{}^m - \frac{1}{4}S_{hm}S_s{}^m + F_h{}^t F_m{}^j R_{tjis}S^{mi},$$

or

$$(R_{hmis} - F_h{}^t F_m{}^j R_{tjis})S^{im} = \left(R_{hm} - \frac{1}{4}S_{hm}\right)S_s{}^m,$$

from which we have (3. 11).

LEMMA 3. 6. *In a K-space, we have*

(3. 20) $$S^{ji}R_{kjih} = \frac{5}{16}T_{kh},$$

(3. 21) $$S^{ji}R_{kjba}F_i{}^b F_h{}^a = \frac{1}{16}T_{kh},$$

(3. 22) $$S^{hj}F_h{}^r F_k{}^q R_{rjiq} = \frac{1}{8}T_{ik},$$

*where* $T_{kh} = T_{hk} = (3R_{kr} + R^*{}_{kr})S_h{}^r$.

*Proof.* First, by (3. 11) and $S_{ji} = S_{ij}$, $T_{kh} = T_{hk}$ is easily verified. (3. 20) and (3. 21) immediately follow from simultaneous equation (3. 5) and (3. 11) with respect to $S^{ji}R_{kjih}$, $S^{ji}R_{kjba}F_i{}^b F_h{}^a$. For (3. 22), making use of Bianchi's identity, (2. 6) and (3. 21), we have

(3. 23)
$$\begin{aligned}
S^{hj}F_h{}^r F_k{}^q R_{rjiq} &= S^{hj}F_h{}^r F_k{}^q(-R_{jirq} - R_{irjq}) \\
&= S^{hj}F_h{}^r F_k{}^q R_{ijrq} - S^{hj}F_h{}^r F_k{}^q R_{qjri} \\
&= S^{hj}F_h{}^r F_k{}^q R_{ijrq} + S^{hr}F_h{}^j F_k{}^q R_{qjri} \\
&= \frac{1}{16}T_{ik} + \frac{1}{16}T_{ik} = \frac{1}{8}T_{ik}.
\end{aligned}$$

LEMMA 3. 7. *In a K-space, we have*

(3. 24) $$\nabla_k S_{ji}(\nabla^k S^{ji}) = \frac{1}{8}(R_{bh} - 5R^*{}_{bh})S^{bi}S^h{}_i.$$

*Proof.* Calculating the square of both sides of (3. 4), we have

$$2\nabla_k S_{ji}(\nabla^k S^{ji}) = S^h{}_i S^{bi}(\nabla_h F_{jm})\nabla_b F^{jm} + S^{hi}S^{bj}(\nabla_h F_{jm})\nabla_b F_i{}^m. \tag{3. 25}$$

Substituting (2. 8) and (3. 2) into (3. 25), by (3. 20) and (3. 21), we obtain

$$\begin{aligned}2\nabla_k S_{ji}(\nabla^k S^{ji}) &= S^h{}_i S^{bi} S_{hb} - S^{hi}S^{bj}(R_{hjbi} - R_{hjpq}F_b{}^p F_i{}^q)\\ &= S^h{}_i S^{bi} S_{hb} - S^{hi}\left(\frac{5}{16}T_{hi} - \frac{1}{16}T_{hi}\right)\\ &= S^h{}_i S^{bi} S_{hb} - \frac{1}{4}T_{hi}S^{hi}\\ &= \frac{1}{4}(4R_{bh} - 4R^*{}_{bh} - 3R_{hb} - R^*{}_{hb})S^{bi}S_i{}^h\\ &= \frac{1}{4}(R_{bh} - 5R^*{}_{bh})S^{bi}S^h{}_i.\end{aligned}$$

Lastly, we shall state some lemmas for a $K$-space of constant holomorphic sectional curvature.

Let $k$ be a holomorphic sectional curvature of an almost Hermitian space with respect to a vector $X^h$, that is,

$$k = -\frac{R_{mjrh}F_q{}^m X^q F_p{}^r X^p X^j X^h}{g_{kj}X^k X^j g_{ih} X^i X^h}. \tag{3. 26}$$

If $k$=constant with respect to any vector at any point of the space, then the space is called a space of constant holomorphic sectional curvature. We know the following

LEMMA 3. 8. (Mizusawa and Kotō [4]) *If an almost Hermitian space is a space of constant holomorphic sectional curvature, then the curvature tensor of the space satisfies the following relation*:

$$\begin{aligned}&R_{kjih} + R_{ijkh} - F_k{}^q F_h{}^l(R_{ljiq} + R_{ijlq})\\ &- F_i{}^p F_h{}^m(R_{kjmp} + R_{mjkp}) - F_j{}^m F_i{}^p(R_{mhkp} + R_{khmp})\\ &- F_k{}^q F_j{}^l(R_{ihlq} + R_{lhiq}) + F_k{}^q F_i{}^p F_j{}^m F_h{}^l(R_{mqlp} + R_{lqmp})\\ &= -4kF_k{}^q F_i{}^p(g_{qp}g_{hj} + g_{qh}g_{jp} + g_{qj}g_{ph}).\end{aligned} \tag{3. 27}$$

For a $K$-space of constant holomorphic sectional curvature, we know the following

LEMMA 3. 9. (Kotō [2], Takamatsu [9]) *In a K-space of constant holomorphic sectional curvature, we have*

$$R_{kr} + 3R^*{}_{kr} = (n+2)kg_{kr}, \tag{3. 28}$$

*or*

$$(3.29)\qquad R_{kr}+3R^*{}_{kr}=\frac{R+3R^*}{n}g_{kr},$$

*where $k$ is positive constant.*

LEMMA 3.10. *In a K-space of constant holomorphic sectional curvature $k$, we have*

$$(3.30)\qquad (R_{kr}-5R^*{}_{kr})S_i{}^rS^{ki}=2k(S^2-nS_{ki}S^{ki}),$$

*where $S=R-R^*$.*

*Proof.* Transvecting (3.27) with $S^{hj}$ and making use of (3.3), we have,

$$(3.31)\qquad \begin{aligned}&(R_{kjih}+R_{ijkh})S^{hj}-F_k{}^qF_h{}^l(R_{ljiq}+R_{ijlq})S^{hj}\\ &-F_i{}^pF_h{}^m(R_{kjmp}+R_{mjkp})S^{hj}-F_j{}^mF_i{}^p(R_{mhkp}+R_{khmp})S^{hj}\\ &-F_k{}^qF_j{}^l(R_{ihlq}+R_{lhiq})S^{hj}+(R_{jkhi}+R_{hkji})S^{hj}\\ =&-4k(g_{ki}g_{hj}+F_{kh}F_{ij}+F_{kj}F_{ih})S^{hj}.\end{aligned}$$

Then, substituting (3.20), (3.21) and (3.22) into (3.31) and making use of (2.6), we have

$$(3.32)\qquad \begin{aligned}&-\left(\frac{5}{16}T_{ki}+\frac{5}{16}T_{ik}\right)-\left(\frac{1}{8}T_{ik}+\frac{1}{16}T_{ik}\right)\\ &-\left(\frac{1}{16}T_{ki}+\frac{1}{8}T_{ki}\right)-\left(\frac{1}{8}T_{ki}+\frac{1}{16}T_{ki}\right)-\left(\frac{1}{16}T_{ik}+\frac{1}{8}T_{ik}\right)\\ &-\frac{10}{16}T_{ki}=-4k(Sg_{ki}+2S_{ki}).\end{aligned}$$

From (3.32), by $T_{ki}=T_{ik}$, we have

$$-2T_{ki}=-4k(Sg_{ki}+2S_{ki}),$$

i.e.

$$(3.33)\qquad (3R_{kr}+R^*{}_{kr})S_i{}^r=2k(Sg_{ki}+2S_{ki}).$$

Then forming $(3.33)-2S_i{}^r\times(3.28)$, we have

$$(R_{kr}-5R^*{}_{kr})S_i{}^r=2k(Sg_{ki}-nS_{ki}).$$

Thus, transvecting this last equation with $S^{ki}$, we have

$$(R_{kr}-5R^*{}_{kr})S_i{}^rS^{ki}=2k(S^2-nS_{ki}S^{ki}).$$

Moreover, we know the following

LEMMA 3. 11. (Sawaki and Yamagata [6]) *In a K-space of constant holomorphic sectional curvature, we have*

$$S_{ji}S^{ji}=\frac{(R+3R^*)(R-R^*)}{2n}, \tag{3. 34}$$

$$(R_{ji}-5R^*{}_{ji})(R^{ji}-5R^{*ji})=\frac{1}{n}(R+3R^*)(5R^*-R), \tag{3. 35}$$

*and*

$$R^*<R\leqq 5R^*. \tag{3. 36}$$

LEMMA 3. 12. *Let $M$ be a K-space of constant holomorphic sectional curvature. Then, in order that $M$ is an Einstein space, it is necessary and sufficient that $R=5R^*$.*

*Proof.* For sufficiency, taking account of (3. 29) and (3. 35), we can easily see that $M$ is an Einstein space.

Conversely, if $M$ is an Einstein space, then from (3. 29), we have $R^*{}_{ji}=(R^*/n)g_{ji}$. Therefore we have $R_{ji}-R^*{}_{ji}=((R-R^*)/n)g_{ji}$, from which we get $R=5R^*$, by virture of (3. 10).

## 4. Theorems.

THEOREM 4. 1. *Let $M$ be a K-space such that*

$$S_{ji}=R_{ji}-R^*{}_{ji}=ag_{ji}. \tag{4. 1}$$

*Then $M$ is an Einstein space.*

*Proof.* Substituting (4. 1) into (3. 5), we have

$$R_{ji}=5R^*{}_{ji}. \tag{4. 2}$$

Transvecting (4. 1) and (4. 2) with $g^{ji}$, we have

$$R-R^*=na, \qquad R=5R^*, \tag{4. 3}$$

respectively.

Substituting (4. 2) and (4. 3) into (4. 1), we have

$$R_{ji}=\frac{R}{n}g_{ji}.$$

Since a 6-dimensional $K$-space satisfies the condition (4. 1) [10], we have

COROLLARY 4. 2. (Matsumoto [3]) *A 6-dimensional K-space is an Einstein space.*

If the generalized Chern-form $\hat{K}$ vanishes, then from (3. 1), we have

$$R_{ji}=5R^{*}{}_{ji}.$$

Therefore from (3. 24), we get $\nabla_k S_{ji}=0$. But, we know that if a symmetric tensor $E_{ji}$ is parallel in an irreducible Riemannian space, then $E_{ji}=cg_{ji}$ where $c$ is constant.

Consequently, by virtue of Theorem 4. 1, we have

COROLLARY 4. 3. *An irreducible K-space with vanishing generalized Chern-form $\hat{K}$ is an Einstein space.*

THEOREM 4. 4. *A K-space of constant holomorphic sectional curvature is an Einstein space.*

*Proof.* Substituting (3. 34) into (3. 30), we have

$$\begin{aligned}(R_{kr}-5R^{*}{}_{kr})S^{r}{}_{i}S^{ki}&=2k(S^{2}-nS_{ki}S^{ki})\\&=2k\left\{(R-R^{*})^{2}-\frac{n(R+3R^{*})(R-R^{*})}{2n}\right\}\\&=k(R-R^{*})(R-5R^{*}).\end{aligned}\tag{4. 4}$$

On the other hand, making use of (4. 4), from (3. 24), we have

$$(\nabla_k S_{ji})\nabla^{k}S^{ji}=\frac{1}{8}k(R-R^{*})(R-5R^{*}).\tag{4. 5}$$

In the above equations (4. 5), since $k>0$ and $R-R^{*}>0$, we have

$$R-5R^{*}\geqq 0.\tag{4. 6}$$

Comparing (4. 6) with (3. 36), we have $R=5R^{*}$.

Consequently, by virtue of Lemma 3. 12, the proof of the theorem is complete.

COROLLARY 4. 5. *The generalized Chern-form $\hat{K}$ of a K-space of constant holomorphic sectional curvature vanishes.*

*Proof.* By Theorem 4. 4 and Lemma 3. 12, we have $R=5R^{*}$. Hence, from (3. 35), we have $R_{ji}=5R^{*}{}_{ji}$ which shows that $\hat{K}=0$.

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