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KŌDAI MATH. SEM. REP.
25 (1973), 32—33

SIMPLE PROOF OF A THEOREM ON TRANSVERSAL HYPERSURFACES OF A CERTAIN SASAKIAN MANIFOLD

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Introduction.

Goldberg and Yano [1] introduced a notion of noninvariant hypersurface of almost contact manifold, and proved that a noninvariant hypersurface of a Sasakian manifold admits a Kählerian structure. Since the Kählerian structure is quite natural, one may conjecture that if the ambient Sasakian manifold is of constant ϕ-holomorphic sectional curvature, then the Kählerian structure of the hypersurface is of constant holomorphic sectional curvature. In this direction, the present auther proved Theorem A. On the other hand, Yano, Eum and Ki [5] proved Theorem B, which seems to be the best result in this direction. In this note, we shall give a simple proof to Theorem B.

The present author wishes to express his hearty thanks to Professor S. Ishihara for his kind advices.

§1. Fundamentals.

Let $\tilde{M}^{2n+1}(\phi, \xi, \eta, \tilde{g})$ be a Sasakian manifold and let M^{2n} be a transversal hypersurface of $\tilde{M}^{2n+1}$, i.e., M^{2n} is a hypersurface of $\tilde{M}^{2n+1}$ such that ξ is not tangent to M^{2n} at each point. The notion of transversal hypersurface is essentially the same as that of noninvariant hypersurface. Let g and J be an induced metric and an induced almost complex structure, respectively. Let γ be a Riemannian metric on M^{2n}, defined by

$$\gamma = g - \eta \otimes \eta.$$

Then, it is known that (J, γ) is a Kählerian structure of M^{2n} ([1], [3]).

THEOREM A ([3]). *Let M^{2n} be a transversal hypersurface of a Sasakian manifold of constant ϕ-holomorphic sectional curvature. If M^{2n} is totally geodesic in the sense that the second fundamental form of M^{2n} with respect to ξ vanishes, then the Kählerian manifold $M^{2n}(J, \gamma)$ is of constant holomorphic sectional curvature.*

Received December 10, 1971.

In this note, we shall prove that Theorem A holds without assuming that M^{2n} is totally geodesic; namely,

THEOREM B. *Let M^{2n} be a transversal hypersurface of a Sasakian manifold of constant ϕ-holomorphic sectional curvature, then the Kählerian manifold $M^{2n}(J, \gamma)$ is of constant holomorphic sectional curvature.*

§2. The proof of Theorem B.

Let $\tilde{x}$ be an arbitrary point of $M^{2n}\cap\tilde{M}^{2n+1}$. Then we can find a small regular neighborhood $\tilde{U}$ of $\tilde{x}$ with respect to ξ, and $\tilde{U}$ has a fibering

$$\pi:\quad \tilde{U}\longrightarrow V=\tilde{U}/\xi.$$

Let $(\bar{J}, \bar{\gamma})$ be the induced Kählerian structure on V. Then we have

$$\tilde{g}=\pi^*\bar{\gamma}+\eta\otimes\eta$$

$$\phi X^*=(\bar{J}X)^*,$$

where X^* is the horizontal lift of a vector field X on V with respect to the contact form η which acts like an infinitesimal connection form ([4]). Let π_0 be the restriction of π to $U=\tilde{U}\cap M^{2n}$. Then π_0: $U\rightarrow V$ is a holomorphic isometry.

It is known that if the Sasakian manifold in consideration is of constant ϕ-holomorphic sectional curvature, then V is of constant holomorphic sectional curvature ([2]). Thus the argument above implies Theorem B.

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