SUPPLEMENT AND CORRECTION TO THE PRECEDING PAPER[1] 'A FUNCTIONAL METHOD FOR STATIONARY CHANNELS'

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In the second part of Theorem 6 of the paper in the title, it was stated that, under (C4), the capacity C_s can be achieved on $P(X, S)$. In the proof of this, we shaw that $H(p)$, $H'(p)$ and $H''(p)$ are weakly* upper-semicontinuous on $P(X, S)$, by which we have concluded impatiently that the weak* upper-semicontinuity of $R(p)$. The proof of this conclusion is incomplete, and hence the achieving of C_s on $P(X, S)$ is yet unkown for such channel distribution. Therefore, in Theorem 6, the statement relative to the part for (C4) should be putout.

In order to study the theorem for the imput X and the output Y being general discrete dynamical systems which have not clopen bases or more generally clopen partitions, we shall give the following theorem which corresponds to the first half of Theorem 6.

THEOREM. *If both the input X and the output Y are compact metric spaces with the homeomorphisms S and T respectively, in which the channel distribution $\nu(\cdot, \cdot)$ is defined by* (C1), (C2) *and* (C3). *Then, for every finite Borel measurable partitions*[2] $\mathfrak{X}$ *of X and* $\mathfrak{Y}$ *of Y, the transmission functional* $\mathfrak{R}(\cdot;\ \mathfrak{X}, \mathfrak{Y})$ *is defined and the equality* (19) $(C_s(\mathfrak{X}, \mathfrak{Y})=\sup\{\mathfrak{R}(p;\ \mathfrak{X}, \mathfrak{Y});\ p\in P_e(X)\})$ *holds.*

Now, we remark that the entropy functionals $H(\cdot, \mathfrak{X}, S)$ and $H(\cdot, \mathfrak{Y}, T)$ are defined and so is $\mathfrak{R}(\cdot;\ \mathfrak{X}, \mathfrak{Y})$ for every $\mathfrak{X}$ and $\mathfrak{Y}$ which are not necessarily clopen, and also remark that theorems (Theorems 2, 4 and the first part for (C′1) of Theorem 6) given in the paper mentioned in the title (and also Theorem 5 in the paper [3]) are available for the present $(X, \mathfrak{X}, S)$ and $(Y, \mathfrak{Y}, T)$.

Under these considerations, the theorem in this paper is proved, along the method of Parthasarathy [2], by the similar way of Theorem 6 for (C′1). Indeed, denote R $(\subset X)$ the set of all transitive points $r\in X$ in the sense of [1] (this is the case that R is the set of all regular points $r\in X$ relative to $C(X)$ in the notation of [2] or [3]) which satisfies $p(R)=1$ for every $p\in P(X, S)$. Then for any bounded Borel measurable function $f(x)$ on X

$$\int_X f(x)dp(x)=\int_R\int_X f(x)dm_r(x)dp(r)$$

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1) Kōdai Math. Sem. Rep. **16** (1964) 27–39. The notations given in that paper are used without any explanations.

2) That is, $\mathfrak{X}$ and $\mathfrak{Y}$ are the partitions of X and Y consisting of finite number of Borel measurable sets of X and Y respectively.

for every $p \in P(X, S)$, where m_r is the ergodic Borel measure on (X, S) corresponding to $r \in R$ (cf. [1], (2. 6)). This formula and Theorem 2 imply the Parthasarathy's formula

$$\mathfrak{R}(p;\ \mathfrak{X},\ \mathfrak{Y})=\int_R \mathfrak{R}(m_r;\ \mathfrak{X},\ \mathfrak{Y})dp(r)$$

for every $p \in P(X, S)$ and for every pair $\mathfrak{X}$, $\mathfrak{Y}$, and hence the theorem is proved.

References

[1] Oxtoby, J. C., Ergodic sets. Bull. Amer. Math. Soc. **58** (1952), 116–136.

[2] Parthasarathy, K. R., On the integral representation of the rate of transmission of a stationary channel. Ill. J. Math. **5** (1961), 299–305.

[3] Umegaki, H., General treatment of alphabet-message space and integral representation of entropy. Kōdai Math. Sem. Rep. **16** (1964), 18–26.

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